Lobachevsky's Imaginary Geometry as Specular and Hyperdimensional Structure

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Abstract

This article proposes a reinterpretation of Lobachevsky's imaginary geometry as a hyperdimensional, specular structure arising from the intersection of two three-dimensional Euclidean spaces. The model describes non-Euclidean parallelism as emerging dynamically from oscillating curvatures, leading to a topological system with four subspaces, two transverse and two vertical, whose behavior is governed by synchronized or opposing phases.

1 Conceptual Model

In the 19th century, Russian mathematician Nikolai Lobachevsky revolutionized the concept of parallelism by demonstrating that, given a point outside a line, multiple lines can pass through it without intersecting the original line, behaving as parallels. This assertion marked a radical break from the traditional Euclidean geometry, which had prevailed for over twenty centuries, and whose fifth postulate states that only one parallel line can be drawn from a point external to a given line.

Before Lobachevsky, various methods had been attempted to prove Euclid's postulate. Lobachevsky's proposal, however, was based on an imaginary exercise: assuming that rotating a line parallel to another would still result in both not intersecting, something impossible in Euclidean geometry. Hence, he called this new geometry "imaginary."

Although initially ignored, Lobachevsky's geometry (also independently developed by Bolyai and not unknown to Gauss) was later reinterpreted as a geometry of curved lines. Mathematicians like Beltrami, Klein, and Poincaré developed visual models in which Lobachevskian "straight lines" are represented as arcs or geodesics on surfaces of negative curvature. This new formulation became known as hyperbolic geometry.

Coinciding with the 13th Bolyai–Gauss–Lobachevsky Conference on non-Euclidean geometry in physics and mathematics, this article proposes a new approach to Lobachvesky's imaginary geometry as a specular geometry emerging as a hyperdimensional substructure from the intersection of two three-dimensional Euclidean spaces. This alternative framework includes a topological model as a physical expression of such geometry, focused on the connectivity and invariance of the structure under continuous deformations.

This interaction involves the intersection of the curvature of the two Euclidean spaces possessing a property we call "curvature," analogous to the curvature of spacetime in general relativity. This curvature is dynamic, in the sense that it oscillates and periodically varies in intensity through time. The topological transformations experienced by these spaces (their contraction and expansion) modify this curvature, altering its amplitude and its spatial "shape" or distribution.

It is the interaction of these dynamic curvatures, when the Euclidean spaces intersect, that gives rise to the emergence of the non-Euclidean subspaces, whose geometry is determined by the way in which the dy-

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namic curvatures of the source spaces combine.

In that way, each subspace is composed of two sectors: one determined by the curvature of source 1 and the other by the curvature of source 2. These sectors may be positive and negative (in the transverse subspaces), or double negative/double positive (in the vertical ones). The point of intersection between curvatures generates an inflection, a singularity that determines an abrupt change in the direction of curvature.

The transverse subspaces expand or contract following the phase of their host space when the two spaces are desynchronized, or inversely to the phase of their host space when both spaces are synchronized.

Taking the case of opposite phases as reference, we consider a line lying in one of the transverse planes. Its parallel appears as a mirror reflection in the opposite subspace, both being external to point P, which is common to the entire system. This point shifts according to the phases: sideways in opposite phases, and up/down in equal phases.

When the right space contracts and the left expands, the right transverse subspace contracts (accelerating its internal dynamics), while the left expands (decelerating). Both tilt toward the contracting side, preserving dynamic parallelism.

All these subspaces are equally influenced by the curvature of both the contracting and the expanding space: when one of the spaces contracts, it lifts one edge of each subfield upward, and when the other expands, it lowers the opposite edge downward, generating the observed inclination in all of them that preserves parallelism.

This tilt recalls the complex plane, where some points are imaginary due to displacement from the Euclidean metric.

The transverse subspaces are hyperdimensional because they cannot be described using the three spatial and one temporal coordinate of the Euclidean system. When tilted, their vertical coordinate projects as a diagonal, distorting the metric. One may also be temporally offset from the other.

Describing this geometry requires additional spatial and temporal coordinates. This offset is comparable to what occurs in general relativity between non-aligned frames of reference.

Vertical subspaces also tilt, and at a transition moment, when the transverse subspaces equalize in volume and the vertical crosses the symmetry center, the entire system adopts Euclidean geometry. At that moment, the X and Y axes switch roles.

The transverse subspaces are cobordant with the vertical ones. For instance, the positive sector of the left transverse subspace corresponds to the convex side of the right space, its concave side acts as the negative sector of the concave vertical subspace. In turn, the negative sector of the left transverse subspace corresponds to the concave side of the left space, its convex side becomes the positive sector of the convex vertical subspace.

Convexity or concavity depends on the frame of observation (from inside or outside). The subspace geometry expresses these aspects functionally, without intrinsic curvature change.

Topological transformations do not alter the dual-sector structure, but determine which sector is active at any given time, influencing the system physically. For example, when the right space contracts and the left expands, the concave side of the right space exerts pressure on the negative sector of the left vertical subspace, while the convex left side transmits decompression. In that moment, the positive sector of the left transverse subspace feels no force, while the right one does.

From this perspective, non-Euclidean geometry is not an external alternative, but an internal and dynamic expression of Euclidean space. Lobachevsky would not have invented an exotic geometry, but anticipated a specular structure integrating the complex plane.

In equal-phase cases, subspaces expand or contract in unison, maintaining chiral symmetry. However, when both transverse subspaces expand simultaneously, their planes tilt toward the positive vertical axis, causing the lines in those planes to intersect near the Y^+ axis and diverge as they move away toward Y^-. The reverse occurs during simultaneous contraction, with planes opening outward.

This behavior is an exception within the non-Euclidean framework, a kind of inversion that violates its internal logic of non-Euclidean parallelism by uniting the real and imaginary planes into a complex system. (Here, “imaginary” is used in the sense of the complex plane, not Lobachevsky’s usage.)

This violation is observed from the perspective of the complex plane, where tilted transverse subspace planes cross when projected onto the Y axis.

However, in XY coordinates of the real plane, dynamic parallelism persists in the vertical subspaces, which shift upward or downward along Y, implying translation.

This dynamic parallelism in the vertical subspaces can be described, on the concave side, as real translation in Euclidean coordinates and real time, since that vertical subspace follows the synchronized phase of the intersecting spaces. When both spaces contract, the concave vertical subspace contracts with inward pressure and accelerates its internal orbital motion. When both expand, it descends and its orbital energy slows.

On the convex side, the structure of double-positive curvature does not produce direct mirror symmetry with the concave side, but rather an inverse functional correspondence. The descending expansion on the concave side, implying loss of energy and force, has its counterpart on the convex side, where the lost force and energy are expressed in reverse.

It is worth noting that the non-Euclidean parallel lines situated in the planes of the vertical subspaces (in equal-phase cases) or transverse ones (in opposite-phase cases), though straight, lie in orbital planes, thus rotating around the axis of their respective subspace. This axis is shared by both vertical subfields (in equal phases) or both transverse ones (in opposite phases). Since the contracting subspace accelerates and the expanding one decelerates, their lines are rarely aligned in the same direction.

This angular discrepancy does not imply intersection, but redefines parallelism dynamically, as topological coherence without intersection in a rotational system. This non-Euclidean parallelism, influenced by asymmetry in internal orbital phases, is relative to the system and its phase structure, not to a fixed spatial metric.

This increasing complexity intensifies when considering that the Euclidean spaces giving rise to the non-Euclidean subspaces may periodically synchronize and desynchronize, generating full cycles of symmetry and specular rupture.

During these cycles, the entire system may also rotate around a shared central axis, introducing a global precessional dynamic. Between each expansion and contraction phase, moments of stasis arise, from the moment a space reaches its maximum expansion or contraction until it begins to contract or expand again.

Finally, it is possible to establish conceptual connections between the visual and physical geometry of the model and some abstract algebraic developments:

In this sense, the dual-sector curvature and the singular point may be related to the Gorenstein liaison.

With four subspaces undergoing four transformations each, one can consider sixteen singularities or states of point P, associating the model with Kummer surfaces.

Further connections may be drawn with Hodge cycles, modular theory in the Takesaki framework, and octonions, as developed in other articles.

This model has also been applied as an alternative proposal for the atomic nucleus.

In this context, the model offers a mechanical interpretation of the Pauli exclusion principle as the result of a specular regime with opposite phases, and provides an explanation of quark charge and color based on the activation or deactivation of pressure forces, depending on whether they arise from the concave side of a contracting field or the convex side of an expanding one.

It also reinterprets the neutron as the moment when Euclidean geometry briefly emerges within the non-Euclidean system, when the transverse subspaces equalize and the verticals cross the system’s symmetry center.

At this point, the central subspaces would act as gluons, transmitting forces and energy between the doubly decompressing expanding transverse subspace and the doubly compressing one.

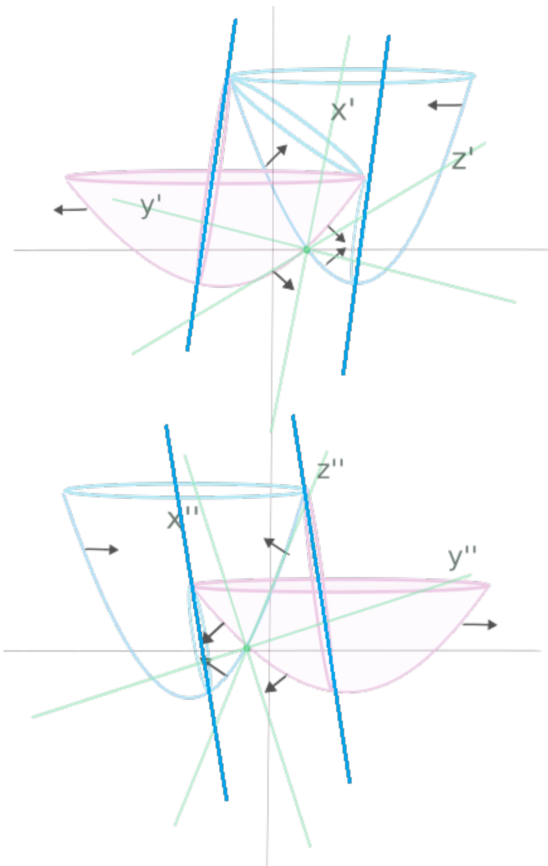

Figure 1: *Non-Euclidean parallel lines in the opposite-phase system. This diagram illustrates the non-Euclidean parallel lines that emerge in the opposite-phase system. Note the specular reflection of the blue lines traced on the transverse plane of both contracting and expanding transverse subspaces.*

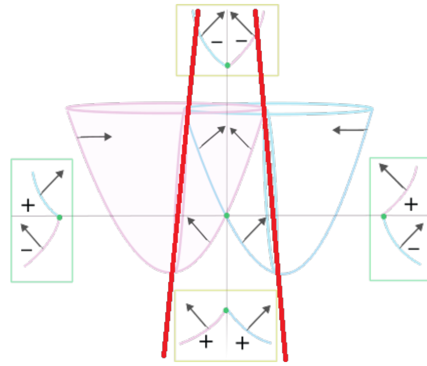

Figure 2: *Non-Euclidean non-parallel lines in the equal-phase system. This diagram illustrates the non-Euclidean non-parallel lines observed in the equal-phase system. Observe how the two red lines converge towards the Y+ axis when both transverse subspaces expand while the source spaces contract.*

References

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