Fractal Spacetime from Quantum Mechanics at Ultrashort Distances

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Abstract

We give a pedagogical account on the origin of fractal spacetime stemming from the Heisenberg uncertainty relations at ultrashort distance scales.

Key words: uncertainty relations, fractal spacetime, fractional calculus, complexity, emergence, environmental decoherence, gravitational decoherence.

This brief report is a "down-to earth" heuristic explanation of energy fluctuations and the emergence of fractal spacetime on ultrashort distance scales (or, equivalently, at large energy scales).

At extremely short time scales, Quantum Mechanics enables large erratic fluctuations in energy due to the Heisenberg uncertainty principle. On intervals approaching the Planck time ($\approx 10^{-44}$ seconds), these fluctuations can become extreme in amplitude and/or frequency. According to General Relativity (GR), energy and momentum determine the curvature of spacetime through the Einstein field equations. It follows that “wild” and fast energy fluctuations necessarily induce equally “wild” and fast fluctuations in the *curvature of spacetime*.

This dynamic regime leads to a breakdown of the smooth geometric structure typically assumed in classical spacetime models. Geodesics—paths representing the shortest trajectories in curved spacetime—are no longer smooth or continuous in the classical sense. The violent, scale-dependent oscillations in curvature render these curves *non-differentiable*, and in many cases, singular.

This loss of differentiability has profound mathematical and physical consequences. The standard tools of differential geometry and calculus, which rely on smoothness and local continuity, are no longer adequate and break down. Instead, *fractional calculus*—extending the concept of derivatives and integrals to non-integer orders—becomes a natural substitute. Fractional derivatives and integrals are well-suited to describe dynamics on highly irregular and convoluted structures, and they inherently incorporate *nonlocal behavior* and *memory effects*—both of which are expected in extreme gravitational regimes.

The implication is that at large energy scales, spacetime itself is prone to exhibit fractal properties: it becomes a fractionally differentiable manifold, statistically self-similar, and governed by *continuous effective dimensions* that flow with the observation scale and differ substantially from the familiar integer values of classical geometry. This picture necessarily leads to the concept of ***fractal spacetime*** — a scale-dependent geometry in which traditional notions of distance, time, and motion must be reformulated.

By the same token, incessant fluctuations in energy and momentum *create* and *annihilate* quantum particles in and out of the quantum vacuum. It follows that fractal spacetime must be the primary source of both gravitational phenomena and particle phenomenology in conditions extending well beyond GR and Quantum Field Theory (QFT) into the realm of *complexity* and *emergence*. This perspective offers a compelling bridge between QFT, GR, and the novel mathematical framework of *complex dynamics* needed to describe the structure of spacetime at the most fundamental level (Fig. 1).

A key observation along these lines is that strongly fluctuating regimes tend to suppress quantum behavior on account of *environmental* and *gravitational decoherence.* From this standpoint, the emergence of fractal spacetime at ultrashort distance scales is *fundamentally different* from the (controversial) approach taken by Quantum Gravity, whereby the postulates of quantum physics are assumed to hold all the way up to the Planck scale [4].

In our interpretation, ultrashort phenomena evolve in far from equilibrium settings which favor the onset of chaos, critical behavior and self-organization, three fundamental manifestations of complex dynamics [1-3].

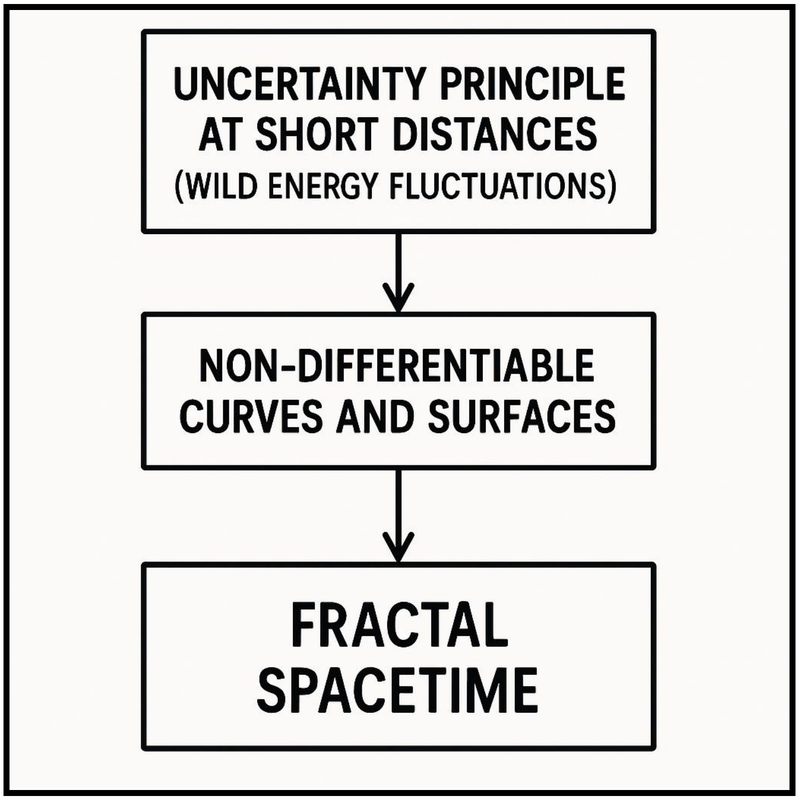

Fig. 1: From the uncertainty principle to fractal spacetime

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