Galilean Transformations and Extended Reflectors: A Theoretical Analysis of Feist's Acoustic Michelson-Morley Experiment

Version 2 — June 2026

Correction notice: the path-selection assumption used in Case 2 of version 1 is incorrect, and the conclusions derived from that case are withdrawn. The original text is retained below for the record.

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Abstract

Version 2 — June 2026. Correction notice. The path-selection assumption used in Case 2 of Version 1 is incorrect, and the conclusions derived from it are withdrawn. This abstract states the corrected result; the original text is retained, unchanged and marked as withdrawn, in the body below for the record.

This paper analyses Norbert Feist's acoustic Michelson–Morley experiment within a Galilean framework, treating the reflector first as a point (Case 1) and then as an extended planar surface (Case 2).
Version 1 concluded that reflector extension yields an angle-independent round-trip time, $t_{tot}(D) = 2Lc/(c^2 - v^2)$, and that this provides a classical, Lorentz-free account of the reported null result.
That conclusion rested on identifying the first-contact point ***D*** — the first point of the reflector reached by the incident wavefront — with the reflection point relevant to the received signal. This identification is incorrect.
For a point-like receiver, the relevant reflection point is the one that minimises the total round-trip time T(s) over the reflecting surface, where s is a signed coordinate with s = 0 at the geometrical centre ***C***. The minimising coordinate is $s\_min = Lv^2 \sin\varphi \cos\varphi / (c^2 - v^2\cos^2\varphi)$, and the corresponding minimum round-trip time is $t_{tot,min} = 2Lc / [\sqrt{(c^2 - v^2)} \cdot \sqrt{(c^2 - v^2\cos^2\varphi)}]$.
Whereas $t_{tot}(D)$ is exactly independent of the orientation angle φ, $t_{tot,\,min}$ depends on φ, and the two differ already at second order in $\beta = v/c$ — the same order as the anisotropy under study.
Reflector extension therefore does not restore isotropy in a Galilean framework, and the suggested transfer of the argument to the optical experiment, and against Lorentzian kinematics, does not hold.

Case 1 is unaffected.

A corrected and extended analysis is in preparation.

Correction Notice for Version 2

Version 1 argued that the finite extension of a planar reflector could produce an angle-independent round-trip propagation time in a Galilean framework.
That conclusion was based on an incorrect path-selection assumption and is withdrawn.

The error concerns Case 2, in which the reflector is spatially extended.
Version 1 identified the first point ***D*** reached by the incident wavefront with the reflection point relevant to the signal subsequently received.
The final expression obtained in Version 1 for the round-trip time along the path through ***D*** is correct:

$$t_{tot}(\boldsymbol{D}) = \frac{2L\mathrm{c}}{c^2 - v^2}$$

This expression is independent of the orientation of the apparatus.
The error lies in assuming that the point reached first during outward propagation must also determine the earliest returned signal.
To formulate the correct path-selection problem, let s be a signed coordinate measured along the reflector surface, with $\mathrm{s} = 0$ at its geometrical centre ***C***.
Let ***L*** be the constant distance between the source/receiver and $\boldsymbol{C}$, φ the angle between the source-reflector axis and the direction of motion, v the velocity of the rigid apparatus through the propagation medium, and c the propagation speed in that medium.
For each possible reflection point s, let $t_{tot}(s)$ denote the total outward-and-return travel time to a point-like receiver.
The total outward-and-return travel time must be evaluated as a function of the reflection point and minimized over the reflector surface.
The relevant reflection point in this time-of-flight model is not determined by first contact, but by minimizing $t_{tot}(s)$ over the reflector surface. This result assumes that the reflector is sufficiently extended for the minimizing point to lie on its surface.
The minimizing coordinate is

$$s_{\min} = \frac{Lv^2 \sin(\varphi)\cos(\varphi)}{c^2 - v^2\cos^2(\varphi)}$$

and the corresponding minimum round-trip time is

$$t_{tot,min} = t_{tot}(s_{\min}) = \frac{2L\mathrm{c}}{\sqrt{c^2 - v^2}\sqrt{c^2 - v^2\cos^2(\varphi)}}$$

The corrected minimum round-trip time depends explicitly on the orientation angle φ.
Therefore, reflector extension does not restore isotropy within a Galilean framework.
The resulting minimum-time point generally differs both from the first-contact point ***D*** and from the geometrical centre ***C***.

The conclusion that reflector extension provides a Galilean explanation of the reported acoustic null result is withdrawn. The further suggestion that the same construction can be transferred to the optical Michelson–Morley experiment, or that it weakens the inferential case for Lorentzian kinematics, is also withdrawn.

Introduction

The acoustic Michelson-Morley experiment conducted by Norbert Feist has provided a unique perspective on the behavior of sound waves in a moving reference frame. Feist's experiment, which utilized an ultrasonic range finder, yielded a null result for the anisotropy of the two-way speed of sound (Feist, 2010/2019). This outcome invites a re-examination of the conventional interpretation of the Michelson-Morley experiment and its implications for the nature of light and the existence of a preferred reference frame. In a series of experiments, Feist demonstrated that the two-way velocity of sound is isotropic in a moving system, similar to the optical Michelson-Morley experiment. In this paper, we delve into the theoretical foundations of Feist's experiment, aiming to provide a comprehensive explanation for the observed results while adopting standard assumptions about sound propagation.

Feist's experiment can be described as follows: an ultrasonic range finder and a reflector were mounted on a rigid rail, ensuring a constant distance, L, between them. This rail was placed horizontally on the roof of a car, allowing the entire source-reflector system to move with the car at a constant velocity, v, in a straight line. The rail was designed to rotate horizontally, enabling the experimenter to vary the angle, φ, between the source-reflector joining line and the direction of the car's motion.

This allowed for measuring the round-trip time of the ultrasonic signal at different orientations relative to the car's motion. For each angle φ, Feist measured the round-trip time of the ultrasonic signal, determining the two-way velocity of sound as a function of the car's velocity and the orientation of the source-reflector system. Surprisingly, the results showed that the two-way velocity of sound, c_2, was isotropic, independent of the angle φ.

This result is analogous to the famous optical Michelson-Morley experiment, which is widely regarded as demonstrating the isotropy of the speed of light in all inertial reference frames. However, as this paper will argue, such a conclusion is not necessarily implied by the experimental results.

Assumptions To theoretically analyze Feist's experiment, I make the following assumptions:

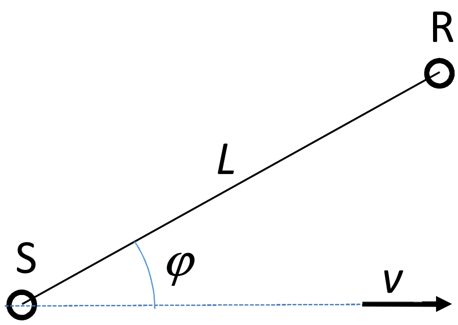
- Galilean transformations are valid for linking observations made by different inertial observers.
- The standard theory of sound propagation is applicable.
- The ultrasonic waves emitted by the source have isotropic propagation velocities only in the privileged reference frame where the medium (air) is at rest.

Analysis I consider two cases: a point-like reflector and an extended reflector.

Case 1: Point-like Reflector

Under the simplifying assumption that the reflector can be treated as a point-like object, which requires its dimensions to be much smaller than the wavelength of the sound waves, I derive the expression for the two-way velocity of sound.

Let us graphically represent the system consisting of an ultrasound source (S) and a reflector (R).



Let ***L*** be the source-reflector distance.

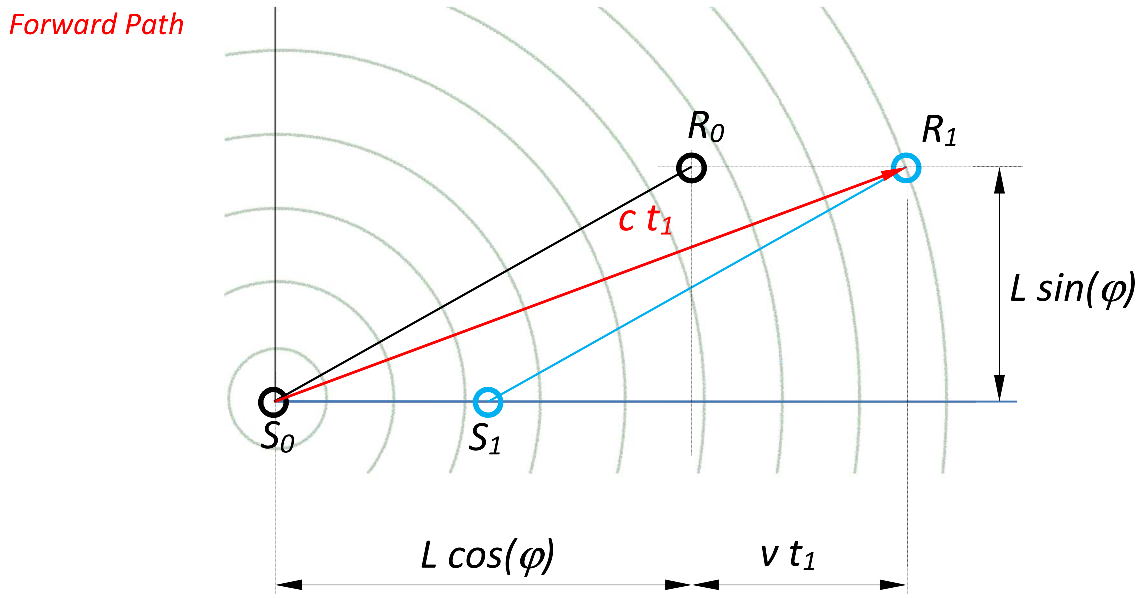
Let **v** be the system's translational velocity in the medium (air).

Let φ be the angle between the source-reflector joining line and the direction of motion of the system.

Let ***c*** be the speed of sound in the medium.

We can assume that the sound waves emitted are spherical. This approximation is reasonable if the source-reflector distance is large compared to the source size.

Let us perform this analysis in two phases; in the first we consider the forward path from source to reflector; in the second we consider the return path from reflector to source.



Suppose that, at a given initial instant (*t = 0*), a wave front is emitted from the source.
Let S_0 and R_0 be the source and reflector positions, respectively, at the initial instant.
Let t_1 be the time interval needed by the wave front to reach the reflector.
Let S_1 and R_1 be the source and reflector positions, respectively, at instant t_1.
In the inertial reference frame in which the air is still, the sound has a constant and isotropic speed ***c***.
In the time interval t_1, the wave front has propagated a distance equal to $c\ t_1$.
In the same time interval, the source-reflector system, moving away from the incoming wave front, has moved horizontally a distance equal to $v\ t_1$.
Therefore, applying the Pythagorean theorem, we can write:

$$[L\cos(\varphi) + v\,t_1]^2 + [L\sin\varphi]^2 = [c\,t_1]^2$$

It follows that:

$$L^2\cos^2(\varphi) + v^2{t_1}^2 + 2\,L\,v\,t_1\cos(\varphi) + L^2\sin^2(\varphi) = c^2{t_1}^2$$

It follows that:

$$L^2 + v^2{t_1}^2 + 2\,L\,v\,t_1\cos(\varphi) - c^2{t_1}^2 = 0$$

It follows that:

$$(c^2 - v^2){t_1}^2 - 2\,L\,v\,t_1\cos(\varphi) - L^2 = 0$$

The roots of this second degree equation are:

$$t_{1\pm} = \frac{L\,v\cos(\varphi) \pm L\sqrt{v^2\cos^2(\varphi) + (c^2 - v^2)}}{c^2 - v^2}$$

The positive root equation provides the propagation time for the forward path.

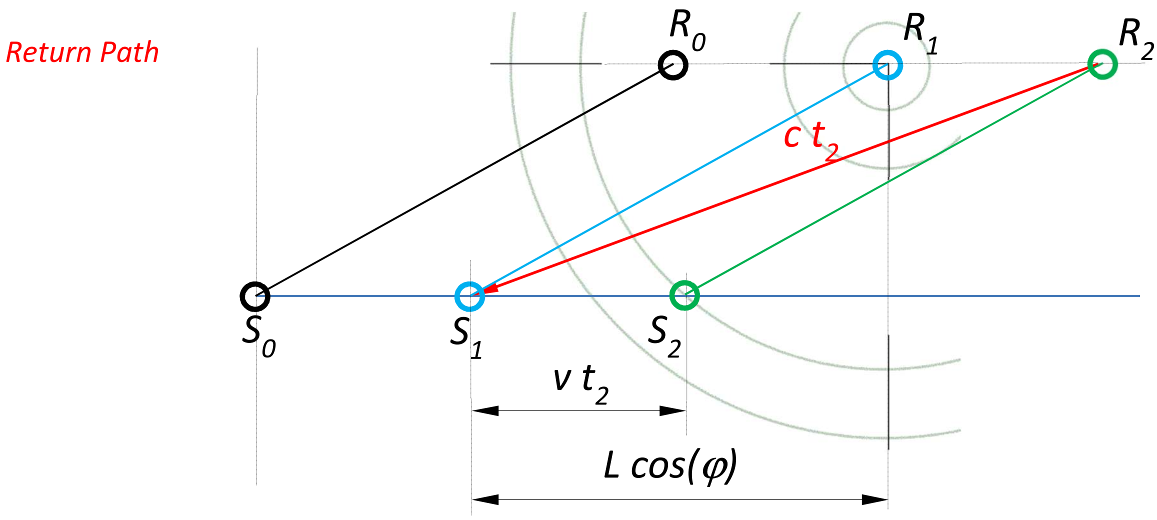
$$t_1 = L\frac{\sqrt{c^2 + v^2\cos^2(\varphi) - v^2} + v\cos(\varphi)}{c^2 - v^2}$$

which we can rewrite in the following form:

$$t_1 = L\frac{\sqrt{c^2 - v^2\sin^2(\varphi)} + v\cos(\varphi)}{c^2 - v^2}$$

It follows that, in the reference frame fixed to the source-reflector system, the speed associated with the forward path is:

$$c_1 \equiv \frac{L}{t_1} = \frac{c^2 - v^2}{\sqrt{c^2 - v^2\sin^2(\varphi)} + v\cos(\varphi)}$$



At the instant t_1 the reflected wave front starts from the position R_1.
Let us denote t_2 as the time interval necessary for the reflected wave front to reach the source.
Let S_2 and R_2 be the source and reflector positions, respectively, at the time $t_1 + t_2$.

In the inertial reference frame in which the air is still, during the time interval t_2 the reflected wave front has propagated a distance equal to $c\,t_2$. In the same time interval the source-reflector system, moving towards the reflected wave front, has moved horizontally a distance $v\,t_2$.

Therefore, applying the Pythagorean theorem, we can write:

$$[L\cos(\varphi) - v\,t_2]^2 + [L\sin\varphi]^2 = [c\,t_2]^2$$

It follows that:

$$L^2\cos^2(\varphi) + v^2{t_2}^2 - 2\,L\,v\,t_2\cos(\varphi) + L^2\sin^2(\varphi) = c^2{t_2}^2$$

It follows that:

$$L^2 + v^2{t_2}^2 - 2\,L\,v\,t_2\cos(\varphi) - c^2{t_2}^2 = 0$$

It follows that:

$$(c^2 - v^2){t_2}^2 + 2\,L\,v\,t_2\cos(\varphi) - L^2 = 0$$

The positive root of this equation provides the propagation time for the return path.

$$t_2 = L\frac{\sqrt{c^2 + v^2\cos^2(\varphi) - v^2} - v\cos(\varphi)}{c^2 - v^2}$$

$$t_2 = L\frac{\sqrt{c^2 - v^2\sin^2(\varphi)} - v\cos(\varphi)}{c^2 - v^2}$$

It follows that, in the reference frame fixed to the source-reflector system, the speed associated with the return path is:

$$c_2 \equiv \frac{L}{t_2} = \frac{c^2 - v^2}{\sqrt{c^2 - v^2\sin^2(\varphi)} - v\cos(\varphi)}$$

At this point we can calculate the total round-trip time:

$$t_{tot} = t_1 + t_2 = L\frac{\sqrt{c^2 - v^2\sin^2(\varphi)} + v\cos(\varphi)}{c^2 - v^2} + L\frac{\sqrt{c^2 - v^2\sin^2(\varphi)} - v\cos(\varphi)}{c^2 - v^2}$$

Therefore:

$$t_{tot} = 2L\frac{\sqrt{c^2 - v^2\sin^2(\varphi)}}{c^2 - v^2}$$

The two-way velocity is, then:

$$c_{two-way} \equiv \frac{2L}{t_{tot}} = \frac{c^2 - v^2}{\sqrt{c^2 - v^2\sin^2(\varphi)}}$$

This expression is equal to (2) shown in the article and it is valid only if the reflector can be represented as a point.

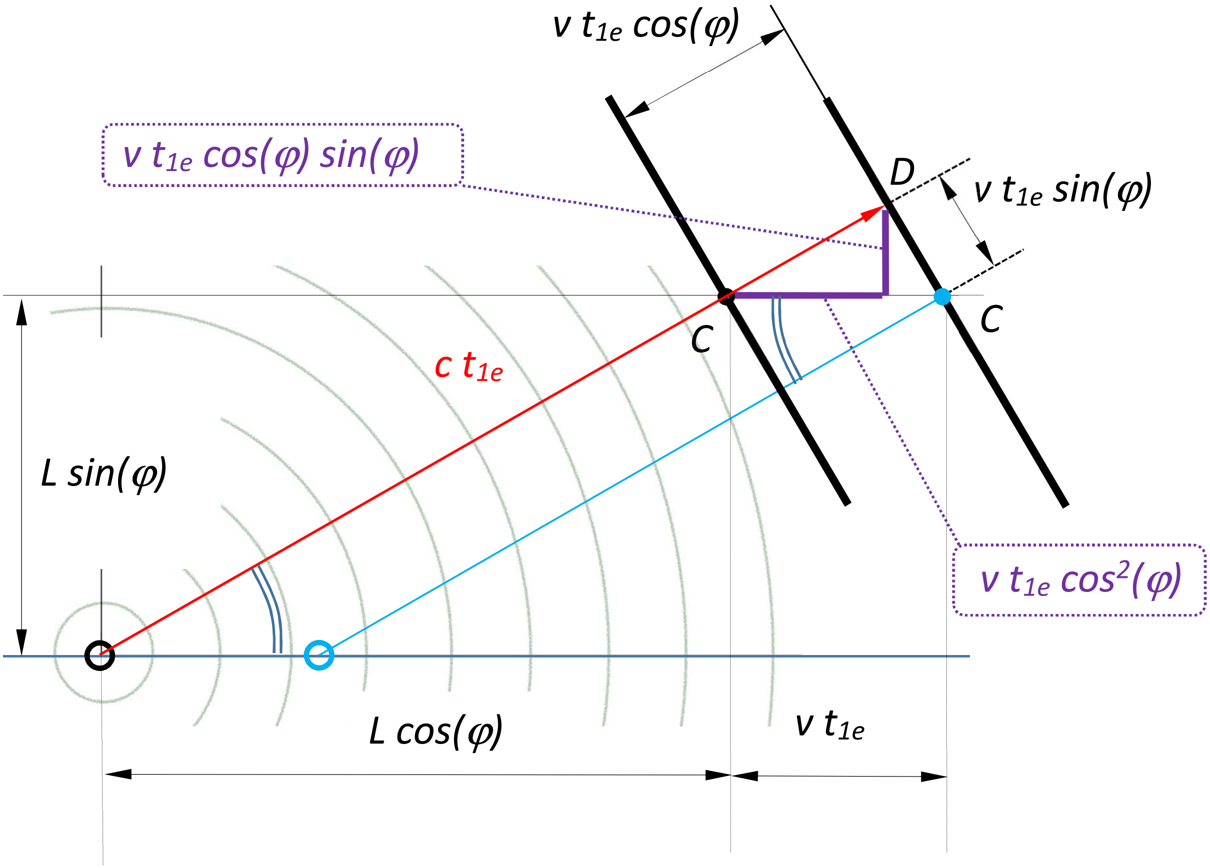
The point-like reflector hypothesis assumes that the linear dimensions of the reflecting object are much smaller than the wavelength of the incident signal. In the actual experiment, the reflector used was a flat surface with dimensions much larger than the wavelength of the ultrasonic signal.

Case 2: Extended Reflector

I derive the expression for the two-way velocity of sound, considering an extended reflector.

Here we will treat the reflector as an extended flat surface.
We will assume that the reflecting surface is perpendicular to the line joining the source and the centre of the reflector

Forward Path



Suppose that, at a given initial instant ($t = 0$), a wave front is emitted from the source.
Let t_{1e} be the time interval needed by the wave front to reach the reflector.
At instant t_{1e}, as is easy to see by observing the geometric construction represented above, the wave front will touch the reflector at a point different from the centre (point D instead of C).
In the time interval t_{1e} the wave front has propagated a distance $c\, t_{1e}$.
In the same time interval the source-reflector system has moved horizontally a distance equal to $v\, t_{1e}$.

Therefore, applying the Pythagorean theorem, we can write:

$$[L\cos(\varphi) + v\, t_{1e}\, cos^2(\varphi)]^2 + [L\sin(\varphi) + v\, t_{1e}\cos(\varphi)\sin(\varphi)]^2 = [c\, t_{1e}]^2$$

By expanding and rearranging the terms according to the degree of t_{1e}, and utilizing the trigonometric identity $cos^2(\varphi) + sin^2(\varphi) = 1$, we simplify the equation to a quadratic form.

$$L^2 cos^2(\varphi) + v^2\, t_{1e}{}^2 cos^4(\varphi) + 2\, L\, v\, t_{1e}\, cos^3(\varphi) + \\ + L^2 sin^2(\varphi) + v^2\, t_{1e}{}^2 cos^2(\varphi) sin^2(\varphi) + 2\, L\, v\, t_{1e}\, sin^2(\varphi) \cos(\varphi) = c^2\, t_{1e}{}^2$$

Therefore:

$$c^2\, t_{1e}{}^2 - v^2\, t_{1e}{}^2 cos^4(\varphi) - v^2\, t_{1e}{}^2 cos^2(\varphi) sin^2(\varphi) + \\ -2\, L\, v\, t_{1e}\, sin^2(\varphi) \cos(\varphi) - 2\, L\, v\, t_{1e}\, cos^3(\varphi) - L^2 sin^2(\varphi) - L^2 cos^2(\varphi) = 0$$

Therefore:

$$c^2\, t_{1e}{}^2 - v^2\, t_{1e}{}^2 cos^2(\varphi)[cos^2(\varphi) + sin^2(\varphi)] + \\ -2\, L\, v\, t_{1e}\, sin^2(\varphi) \cos(\varphi) - 2\, L\, v\, t_{1e}\, \cos(\varphi)\, cos^2(\varphi) - L^2[sin^2(\varphi) + cos^2(\varphi)] = 0$$

Therefore:

$$c^2\, t_{1e}{}^2 - v^2\, t_{1e}{}^2 cos^2(\varphi) - 2\, L\, v\, t_{1e}\, sin^2(\varphi) \cos(\varphi) - 2\, L\, v\, t_{1e}\, \cos(\varphi)\, cos^2(\varphi) - L^2 = 0$$

Therefore:

$$c^2\, t_{1e}{}^2 - v^2\, t_{1e}{}^2 cos^2(\varphi) - 2\, L\, v\, t_{1e}\, sin^2(\varphi) \cos(\varphi) - 2\, L\, v\, t_{1e}\, \cos(\varphi)\, [1 - sin^2(\varphi)] - L^2 = 0$$

Therefore:

$$[c^2 - v^2\, cos^2(\varphi)] t_{1e}{}^2 - 2\, L\, v\, \cos(\varphi)\, t_{1e} - L^2 = 0$$

This equation is a quadratic in t_{1e}, the time for the forward path in the case of an extended reflector.
We can solve it to find the propagation time.
The roots of this equation are:

$$t_{1e\pm} = \frac{L\, v\, \cos(\varphi) \pm \sqrt{L^2\, v^2 cos^2(\varphi) + L^2[c^2 - v^2\, cos^2(\varphi)]}}{c^2 - v^2\, cos^2(\varphi)}$$

$$t_{1e\pm} = \frac{L\, v\, \cos(\varphi) \pm L\, c}{[c - v\, \cos(\varphi)]\, [c + v\, \cos(\varphi)]} = \begin{cases} \dfrac{L}{c - v\, \cos(\varphi)} \\ \dfrac{-L}{c + v\, \cos(\varphi)} \end{cases}$$

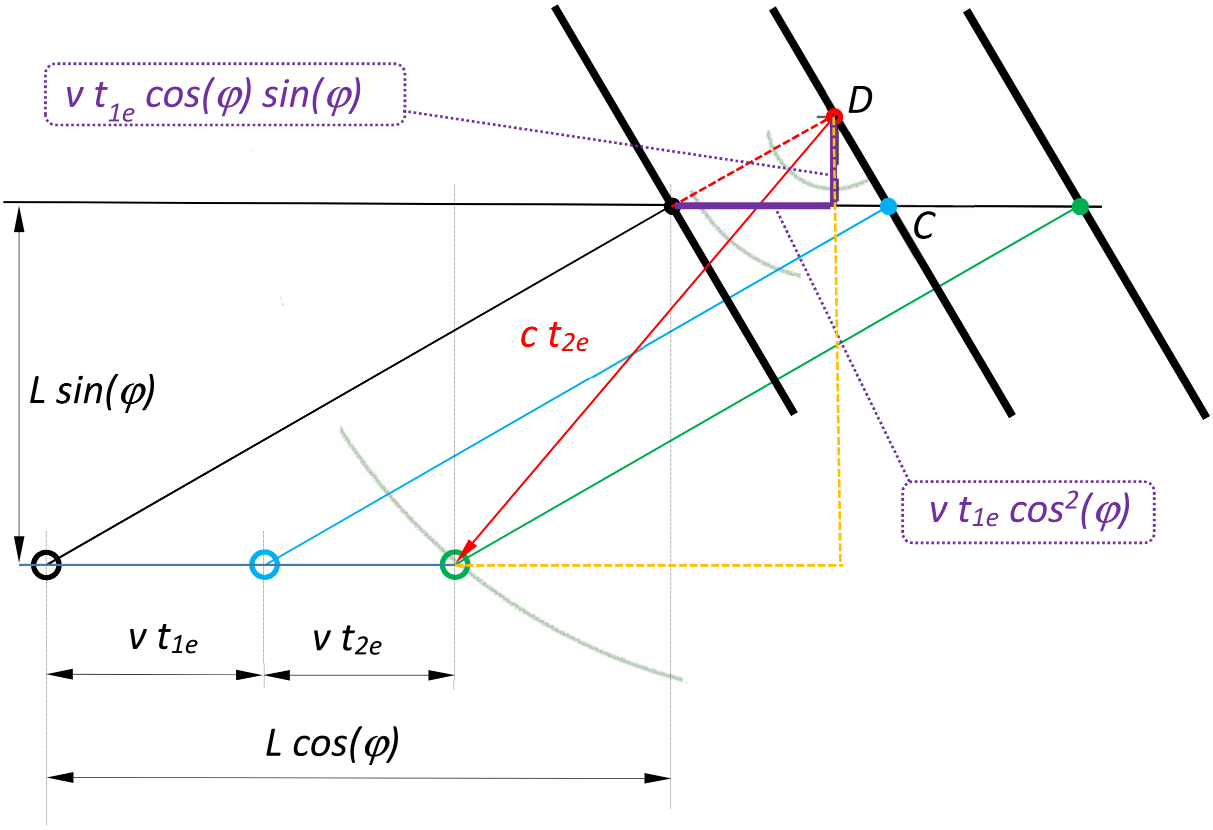
The positive root provides the propagation time for the forward path.

$$t_{1e} = \frac{L}{c - v\, \cos(\varphi)}$$

In the reference frame constrained to the source-reflector system, the speed associated with the forward path is:

$$c_{1e} \equiv \frac{L}{t_{1e}} = c - v\, \cos(\varphi)$$

Return Path



At the instant t_{1e} the reflected wave front starts (from point D).
Let us denote t_{2e} as the time interval necessary for the reflected wave front to reach the source.

Proceeding in a similar way to the previous case, applying the Pythagorean theorem to the right triangle with hypotenuse $c\ t_{2e}$and the two catheti drawn in orange, we can write:

$$[L\ \cos(\varphi) - v\ t_{1e} - v\ t_{2e} + v\ t_{1e}\ cos^2(\varphi)]^2 + [L\ \sin(\varphi) + v\ t_{1e} \cos(\varphi) \sin(\varphi)]^2 = [c\ t_{2e}]^2$$

Indicating:

$A = [L\ \cos(\varphi) - v\ t_{1e} - v\ t_{2e} + v\ t_{1e}\ cos^2(\varphi)]^2 \ = [v\ t_{1e}\ (\cos^2(\varphi) - 1) + L\ \cos(\varphi) - v\ t_{2e}]^2,$

$B = [L\ \sin(\varphi) + v\ t_{1e} \cos(\varphi) \sin(\varphi)]^2,$

the previous expression can be rewritten:

$$c^2 {t_{2e}}^2 - A - B = 0$$

After substituting the expressions for A and B and simplifying, we obtain a quadratic equation in t_{2e}.
This involves expanding the terms, rearranging them, and simplifying the resulting expression.

Introducing the expression of t_{1e}, in A we obtain:

$$A = \left[\frac{v\,L\,(\cos^2(\varphi) - 1)}{c\ - v\ \cos(\varphi)}\ + L\cos(\varphi) - v\,t_{2e}\right]^2 = \left[\frac{v\,L\cos^2(\varphi) - v\,L}{c\ - v\ \cos(\varphi)}\ + L\cos(\varphi) - v\,t_{2e}\right]^2$$

$$A = \left[\frac{v\,L\cos^2(\varphi) - v\,L + c\,L\cos(\varphi) - v\,L\cos^2(\varphi)}{c\ - v\ \cos(\varphi)}\ - v\,t_{2e}\right]^2 = \left[\frac{c\,L\cos(\varphi) - v\,L}{c\ - v\ \cos(\varphi)}\ - v\,t_{2e}\right]^2$$

$$A = \frac{L^2\ [c\ \cos(\varphi) - v\ ]^2}{[c\ - v\ \cos(\varphi)\ ]^2} + v^2\ {t_{2e}}^2 - \frac{2\,v\,L\,[c\ \cos(\varphi) - v\ ]}{c\ - v\ \cos(\varphi)}\ t_{2e}$$

Introducing the expression of t_{1e}, in B we obtain:

$$B = \left[L\ \sin(\varphi) + \frac{v\,L\ \cos(\varphi)\sin(\varphi)}{c\ - v\ \cos(\varphi)}\right]^2$$

$$B = \left[\frac{c\,L\ \sin(\varphi) - v\,L\ \cos(\varphi)\sin(\varphi) + v\,L\ \cos(\varphi)\sin(\varphi)}{c\ - v\ \cos(\varphi)}\right]^2 = \frac{[c\,L\ \sin(\varphi)]^2}{[c\ - v\ \cos(\varphi)]^2} = \frac{c^2\ L^2\ sin^2(\varphi)}{[c\ - v\ \cos(\varphi)]^2}$$

So the original expression becomes:

$$c^2{t_{2e}}^2 - \left\{\frac{L^2\ [c\ \cos(\varphi) - v\ ]^2}{[c\ - v\ \cos(\varphi)]^2} + v^2\ {t_{2e}}^2 - \frac{2\,v\,L\,[c\ \cos(\varphi) - v\ ]}{c\ - v\ \cos(\varphi)}\ t_{2e}\right\} - \left\{\frac{c^2\ L^2\ sin^2(\varphi)}{[c\ - v\ \cos(\varphi)]^2}\right\} = 0$$

Therefore:

$$(c^2 - v^2)\ {t_{2e}}^2 + \frac{2\,v\,L\,[c\ \cos(\varphi) - v\ ]}{c\ - v\ \cos(\varphi)}\ t_{2e} - \left\{\frac{L^2\ [c\ \cos(\varphi) - v\ ]^2}{[c\ - v\ \cos(\varphi)]^2} + \frac{c^2\ L^2\ sin^2(\varphi)}{[c\ - v\ \cos(\varphi)]^2}\right\} = 0$$

Therefore:

$$(c^2 - v^2)\ {t_{2e}}^2 + \frac{2\,v\,L\,[c\ \cos(\varphi) - v\ ]}{c\ - v\ \cos(\varphi)}\ t_{2e} - \frac{L^2[c^2cos^2(\varphi) + v^2\ - 2\,v\,c\ \cos(\varphi) + c^2sin^2(\varphi)]}{[c\ - v\ \cos(\varphi)]^2} = 0$$

$$(c^2 - v^2)\ {t_{2e}}^2 + \frac{2\,v\,L\,[c\ \cos(\varphi) - v\ ]}{c\ - v\ \cos(\varphi)}\ t_{2e} - \frac{L^2[c^2 + v^2\ - 2\,v\,c\ \cos(\varphi)]}{[c\ - v\ \cos(\varphi)]^2} = 0$$

$$(c^2 - v^2)\ {t_{2e}}^2 + 2\,v\,L\frac{c\ \cos(\varphi) - v}{c\ - v\ \cos(\varphi)}\ t_{2e} - \frac{L^2[c^2 + v^2 - 2\,v\,c\cos(\varphi)]}{[c\ - v\ \cos(\varphi)]^2} = 0$$

To find the propagation time for the return path, t_{2e}, we solve this quadratic equation.
The roots are:

$$t_{2e\pm} = \frac{-\frac{v\,L\,[c\ \cos(\varphi) - v\ ]}{c\ - v\ \cos(\varphi)} \pm \sqrt{v^2L^2\frac{[c\ \cos(\varphi) - v\ \ ]^2}{[c\ - v\ \cos(\varphi)\ ]^2} + \frac{(c^2 - v^2)\,L^2[c^2 + v^2 - 2\,v\,c\cos(\varphi)]}{[c\ - v\ \cos(\varphi)\ ]^2}}}{(c^2 - v^2)}$$

$$t_{2e\pm} = \frac{-v\,L\,[c\,\cos(\varphi) - v] \pm L\sqrt{v^2[c\,\cos(\varphi) - v\,]^2 + (c^2 - v^2)[c^2 + v^2 - 2\,v\,c\cos(\varphi)]}}{(c^2 - v^2)[c\,-v\,\cos(\varphi)]} =$$

$$= \frac{-v\,L\,[c\,\cos(\varphi) - v] \pm L\sqrt{v^2[c^2\cos^2(\varphi) + v^2 - 2\,v\,c\cos(\varphi)] + (c^2 - v^2)[c^2 + v^2 - 2\,v\,c\cos(\varphi)]}}{(c^2 - v^2)[c\,-v\,\cos(\varphi)]}$$

Let's develop the mathematical expression under the square root sign:

$$\sqrt{v^2[c^2\cos^2(\varphi) + v^2 - 2\,v\,c\cos(\varphi)] + (c^2 - v^2)[(c^2 + v^2) - 2\,v\,c\cos(\varphi)]} =$$

$$= \sqrt{v^2c^2\cos^2(\varphi) + v^4 - 2\,v^3c\cos(\varphi) + (c^4 - v^4) - 2\,v\,c\,(c^2 - v^2)\cos(\varphi)} =$$

$$= \sqrt{v^2c^2\cos^2(\varphi) + v^4 - 2\,v^3c\cos(\varphi) + c^4 - v^4 - 2\,v\,c^3\cos(\varphi) + 2\,v^3c\cos(\varphi)} =$$

$$= \sqrt{v^2c^2\cos^2(\varphi) + c^4 - 2\,v\,c^3\cos(\varphi)} = \sqrt{[c^2 - v\,c\,\cos(\varphi)]^2}$$

By simplifying the expression under the square root, we can see that it is a perfect square.
This allows us to simplify the solution for t_{2e}.
Therefore:

$$t_{2e\pm} = \frac{-v\,L\,[c\,\cos(\varphi) - v] \pm L[c^2 - v\,c\,\cos(\varphi)]}{(c^2 - v^2)[c\,-v\,\cos(\varphi)]}$$

Therefore:

$$t_{2e\pm} = \frac{-L\,v\,c\,\cos(\varphi) + L\,v^2 \pm L\,c^2 \mp L\,v\,c\,\cos(\varphi)}{(c^2 - v^2)[c\,-v\,\cos(\varphi)]}$$

Therefore:

$$t_{2e\pm} = \begin{cases} \dfrac{-L\,v\,c\,\cos(\varphi) + L\,v^2 + L\,c^2 - L\,v\,c\,\cos(\varphi)}{(c^2 - v^2)[c\,-v\,\cos(\varphi)]} \\ \dfrac{-L\,v\,c\,\cos(\varphi) + L\,v^2 - L\,c^2 + L\,v\,c\,\cos(\varphi)}{(c^2 - v^2)[c\,-v\,\cos(\varphi)]} \end{cases}$$

Therefore:

$$t_{2e\pm} = \begin{cases} \dfrac{-2\,L\,v\,c\,\cos(\varphi) + L\,v^2 + L\,c^2}{(c^2 - v^2)[c\,-v\,\cos(\varphi)]} \\ \dfrac{L\,v^2 - L\,c^2}{(c^2 - v^2)[c\,-v\,\cos(\varphi)]} \end{cases} = \begin{cases} \dfrac{L\,[c^2 + v^2 - 2\,v\,c\,\cos(\varphi)]}{(c^2 - v^2)[c\,-v\,\cos(\varphi)]} \\ \dfrac{-\,L}{[c\,-v\,\cos(\varphi)]} \end{cases}$$

The positive root provides the propagation time for the return path.

$$t_{2e} = \frac{L\,[c^2 + v^2 - 2\,v\,c\,\cos(\varphi)]}{(c^2 - v^2)[c\,-v\,\cos(\varphi)]}$$

In the reference frame constrained to the source-reflector system, the speed associated with the return path is:

$$c_{2e} \equiv \frac{L}{t_{2e}} = \frac{(c^2 - v^2)(c - v\,\cos(\varphi))}{c^2 + v^2 - 2\,v\,c\,\cos(\varphi)}$$

Now that we have the expressions for the forward and return propagation times, we can calculate the total round-trip time by summing t_{1e} and t_{2e}.

$$t_{tot\_e} = t_{1e} + t_{2e} = L\left\{\frac{1}{c\ - v\ \cos(\varphi)} + \frac{[c^2 + v^2 - 2\ v\ c\ \cos(\varphi)]}{(c^2 - v^2)[c\ - v\ \cos(\varphi)]}\right\}$$

$$t_{tot\_e} = L\left\{\frac{(c^2 - v^2) + [c^2 + v^2 - 2\ v\ c\ \cos(\varphi)]}{(c^2 - v^2)[c\ - v\ \cos(\varphi)]}\right\} = L\left\{\frac{2\ c^2 - 2\ v\ c\ \cos(\varphi)}{(c^2 - v^2)[c\ - v\ \cos(\varphi)]}\right\} = \frac{2\ L\ c}{c^2 - v^2}$$

The two-way velocity is, then:

$$c_{two-way} \equiv \frac{2L}{t_{tot\_e}} = \frac{c^2 - v^2}{c}$$

[Version 2: the following two sentences belong to Version 1 and are withdrawn — see Correction Notice, p. 2]

~~The final expression for the two-way velocity in the case of an extended reflector is independent of the angle φ, which is consistent with Feist's experimental results.~~
~~This highlights the importance of considering the extended nature of the reflector in the analysis.~~

Conclusions

Version 2 notice — the text of this section belongs to Version 1 and is withdrawn in its entirety.
The central claim below — that reflector extension provides a Galilean, Lorentz-free explanation of the acoustic null result, transferable to the optical Michelson–Morley experiment — is incorrect.
It follows from identifying the first-contact point ***D*** with the reflection point relevant to the received signal, an identification refuted in the Correction Notice (p. 2).
The corrected result is that the minimum-time reflection point yields an angle-dependent round-trip time $t_{tot,\ min}$, so reflector extension does not restore isotropy in a Galilean framework.
The original text is reproduced below, struck through, solely for the record.

~~The analysis presented in this paper provides a convincing explanation for the experimental results obtained by Norbert Feist in his acoustic Michelson–Morley experiment. By using Galilean transformations and considering the extended nature of the reflector, we demonstrate that the null result can be theoretically justified. The commonly used expression for the bidirectional speed of sound is valid only under the simplifying assumption of a point-like reflector. By deriving the correct expression for an extended reflector, we show that the experimental results can be explained using classical physics.~~

~~This reasoning can be extended to the original electromagnetic Michelson–Morley experiment, suggesting that also in that case, considering the extended nature of the reflectors, it might be possible to explain the null result without resorting to Lorentz transformations.~~
~~The analysis developed in this paper invites a re-examination of the traditional interpretation of the Michelson–Morley experiment. Historically, the null result has been taken to exclude the existence of a luminiferous aether and to necessitate the adoption of Lorentz transformations as the only viable framework to explain the constancy and isotropy of the speed of light. However, the reasoning proposed here — grounded in Galilean kinematics and a detailed consideration of the reflector's geometry — demonstrates that the null result can also be understood within a framework that retains the notion of a preferred reference frame (i.e., the rest frame of the medium) and adheres to classical principles of motion. This underscores an important epistemological point: the Michelson–Morley experiment does not logically compel the adoption of Lorentz transformations.~~

References

Feist, N. (2010). Acoustic Michelson-Morley Experiment with an Ultrasonic Range Finder. Proceedings of the NPA, 6(2), 1-3.

Feist, N. (2019). Acoustic Michelson-Morley Experiment with an Ultrasonic Range Finder. Global Journal of Science Frontier Research: A Physics and Space Science, 19(8), 1-6.