

Proto-Unit Framework For Space-Time Derivation:

The ~~Real~~ Quadrature Of The Circle.

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Abstract

This paper introduces a proto-unit framework grounded in the duality of translation and rotation as fundamental space-time operators. Starting from first principles, we define energy as a temporal expression and explore a path between pure rotation (Euler-circle) to pure translation (geodesic), using a unified coordinate set (v, ω) . Through this, we derive clean expressions for physical constants and uncover the deep geometrical significance of $\pi/4$, revealing the structure of space-time as inherently balanced and quantized. The framework suggests a foundational mathematical unity that may underlie all physical law.

Keywords: Relativity, Quantum Mechanics, Complex Analysis, Zeta(s), Planck Units, Probability Theory, Number Theory

1 Introduction

The quest for a truly foundational framework—one that is simultaneously physical, mathematical, and philosophical—has animated inquiry since antiquity. In this work, we begin not with particles or fields, but with the abstraction of movement itself. We postulate that every entity in the universe can be understood as a combination of two fundamental modalities: *translation* (v) and *rotation* (ω), constrained by a maximal limit, the speed of light c , here normalized to unity ($c = 1$).

This framework emerges from a simple insight: energy is fundamentally a temporal phenomenon, and all observed structure—mass, charge, entropy—emerges from specific configurations in the space of motion.

1.1 Motivation

Inspired by principles of symmetry, information theory, and the interplay between linear and angular momentum, this model introduces the "proto-unit": a binary operator space (v, ω) representing translation and rotation probabilities or intensities. The duality of these components mirrors wave-particle duality, and the limit constraint $v^2 + \omega^2 = 1$ evokes a unit-circle geometry suggestive of complex numbers and spinor spaces.

The proto-unit encapsulates the beginning of motion, logic, and temporality—the root informational quanta from which all else may emerge.

2 Definition of the Proto-Unit

2.1 Temporal Root: $\sqrt{1} = 0t$

To formulate a foundational theory of space-time, we begin not with physical measurement, but with pure informational symmetry. The proto-unit is defined as a theoretical square of unit length (side = 1) in the virtual, complex plane—a space of pure potential and temporal logic. This unit square is not spatial, but temporal, informational and relational: a symbolic token of transformation. So we define the proto-unit not physically but informationally. Its root symmetry encodes no spatial displacement, only the capacity for temporal progression:

$$\sqrt{1} = 0t. \tag{1}$$

This implies that unity in this domain produces no extension—only the origin of time itself. Because 1 is the only integer defining itself ($\sqrt{1} = 1$) there is no internal temporal gradient whatsoever ($t = 0$).

2.2 Multi-Aspect Square: Forms of the Proto-Unit

The unit square manifests equivalently across multiple representations:

$$1 = 1^2 = c^2 = v^2 + \omega^2 = (i \cdot \text{Re})^2 = (\sqrt{-1} \cdot \sqrt{1})^2 \tag{2}$$

Whether expressed as a speed limit, a Pythagorean decomposition, or a rotation on the complex plane, the square preserves invariant unity. This equivalence across forms foreshadows a symmetry that governs space-time emergence. Each formulation encodes the same fundamental constraint: that any realization of motion, be it translation or rotation, must reside within this square boundary. This defines the proto-unit as a complex coordinate anchor between spatially-real and temporally-imaginary domains.

2.3 Balanced Dynamics and Constraint Geometry

Within the unit square constraint, we treat motion as a composition of two orthogonal modes: translation v and rotation ω , satisfying $v^2 + \omega^2 = 1$.

Let translation v and rotation ω be equally expressed:

$$v = \omega = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad v^2 = \omega^2 = \frac{1}{2} \tag{3}$$

Inserting into the constraint:

$$v^2 + \omega^2 = \frac{1}{2} + \frac{1}{2} = 1 \tag{4}$$

we confirm that the system remains internally complete and causally bounded. This configuration defines the internal balance point of the proto-unit—its most symmetric, least biased configuration. Observing the unit boundary itself we encounter four sides, each of length 1. That is the event horizon. It exists here in it's logical quadrature of the unit circle, a line enclosing a plane entirely and exclusively using 1's. It's the boundary condition of maximal compression and depending on the number of units dissolving into it, it may grow to truly gigantic scale. The horizon will grow it's circumference with every bit of information, another 1, dissolved from real values and merging into the boundary as pure temporal potential.

2.4 Defining the Lorentz-like Factor $\tilde{\gamma}$

In analogy to the Lorentz factor Einstein, 1905; Rindler, 2006 from special relativity, which describes how time and energy transform under velocity with:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \text{ for } v = c, \gamma \rightarrow \infty, \quad (5)$$

which we write as:

$$\tilde{\gamma} = \frac{1}{\sqrt{1 - (\tilde{v}^2 + \tilde{\omega}^2)}}, \text{ for } v = \tilde{c} = 1, \gamma \rightarrow \infty, \quad (6)$$

we define a generalized gamma factor that expresses the internal dynamic balance between v and ω .

Assuming $\tilde{c} = 1$, we may define $\tilde{\gamma}$ in terms of either component:

$$\tilde{\gamma} := \frac{1}{\sqrt{1 - v^2}} = \frac{1}{\omega}, \quad (7)$$

or equivalently,

$$\tilde{\gamma} := \frac{1}{\sqrt{1 - \omega^2}} = \frac{1}{v}. \quad (8)$$

This formulation implies that $\tilde{\gamma}$ serves as a dimensionless magnification factor that diverges as one component dominates and the other vanishes. In the balanced case, where $v = \omega = \frac{1}{\sqrt{2}}$, we find:

$$\tilde{\gamma} = \sqrt{2}. \quad (9)$$

Thus, $\tilde{\gamma}$ is not merely a relativistic scaling constant but reflects the geometric tension between rotational and translational modes of motion in a unified, normalized framework.

2.5 Emergence of $\pi/4$

On the unit quarter-circle, the angle theta (θ) associated with the balanced state is:

$$\theta = \tan^{-1}\left(\frac{\omega}{v}\right) = \tan^{-1}(1) = \frac{\pi}{4} = 45^\circ \quad (10)$$

This angle marks the diagonal of the (v, ω) configuration space. The appearance of $\pi/4$ signals the intrinsic symmetry of temporal-rotational partitioning: space and time share equal operational weight. Rotation in space-time is defining the logical separation of a temporal "storage" unit. These units stack through the full spectrum of scales. ω and v are relative to their observer. Flat translation to a small observer appears like rotation to a larger one. This is the localization and navigation vector of the real worldline, a logarithmic, fractal spiral, parameterized by (ω, v) , the critical line!

3 Zeta Stitching: The Onset of Space-Time

Having defined a unit of space and the logical root of temporal potential, we seek now to bind them. The Riemann Zeta Function emerges as a mathematical zipper, interlocking real and imaginary domains—stitching rotation to translation, time to space, potential to reality.

Through the periodic structure of complex exponentials and the analytical continuation of $\zeta(s)$, we conceive of this complex zipper interlacing discrete harmonic domains. Each contribution to $\zeta(s)$ represents a mode in the stitching—curling space and time together via the imaginary axis.

3.1 The Role of the Zeta Function

We introduce the Riemann zeta function $\zeta(s)$ [Edwards, 2001; Titchmarsh, 1986]:

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s}, \quad s = \frac{1}{2} + it \quad (11)$$

Its critical line, at $\text{Re}(s) = \frac{1}{2}$, mirrors our internal proto unit balance condition $v^2 = \omega^2 = \frac{1}{2}$. This resonance suggests that the zeta structure encodes a symmetry of temporal and spatial operation.

It is the diagonal in proto unit space, a geodesic trajectory γ along a path naturally created by pure number theoretic logic. The philosophical edge between being and not being, the logical cancelation between 0 and 1. In complex logic space, $0+1 \neq 0$, but $0+1 \neq 1$ either, instead $0+1 = \frac{1}{2}$ ish. It's a way of expressing, that if you add just 1 bit of information to a balanced number space, you can only make it as far as $+1/2$ and $-1/2$. To extend logic by one integer, we need 2 bits of information. Now, as 0 and 1 reach out to each other they create rotation. The complex plane allows the Re number line not to annihilate itself with opposing values from the positive and negative extensions, but balances the \pm potential AROUND ZERO. The motion is around 0 but the trajectory, the equator, is a diagonal at $\text{Re}\frac{1}{2}$ in flat logic, a circle with $r = 1/2$, an orbit around 0. The second orbit out in this temporal plane is at 1. It is the boundary condition of the system, the unit circle, the informational event horizon. These two orbits have a distinctive difference to them: The 1 orbit, the perfect **Circle**, never touches the origin, while the **Anti-Circle**, the inner zeta orbit at $\text{Re}\frac{1}{2}$ periodically touches 0. It has to - informationally speaking - because $\omega_{\text{Re}\frac{1}{2}}$ is constantly shifting between 0 and 1. Omega here is not constant like it is in rotation around the unit circle. $\omega_{e^{i\pi}} = \text{constant}$. What makes light so special, is that its oscillation ω is constantly balanced, but not by real space translation, instead v here is expressed as density oscillation in the direction of travel. The net velocity of light never changes because it exists on that special 1/2 geodesic. It lives in a place of complex projection where reality is logically squished in between $(0 < 1/2 < 1) \cdot i$, on the equilibrium center line of a probabilistic strip spiraling through logical number space. This is the critical strip, or as we came to call it: The Universe.

3.2 The Trajectory - γ from the Perspective of Light

From Riemann we write γ as:

$$\gamma(t) = \zeta\left(\frac{1}{2} + it\right) \in \mathbb{C}. \quad (12)$$

Taking derivative:

$$\gamma'(t) = \frac{d}{dt}\zeta\left(\frac{1}{2} + it\right) = i \cdot \zeta'\left(\frac{1}{2} + it\right) \quad (13)$$

This gives us a tangent vector at each point of the path.

The angle $\Omega(t)$ of this tangent is therefor given as:

$$\Omega(t) = \arg(\gamma'(t)) = \arg\left(\frac{d}{dt}\zeta\left(\frac{1}{2} + it\right)\right) = i \cdot \zeta'\left(\frac{1}{2} + it\right) \quad (14)$$

3.3 The Proto-Unit as a Zeta Kernel

Each proto-unit can be seen as a localized zeta kernel, resonant with particular values of s on the critical line:

$$s = \frac{1}{2} + i\Omega(t) \quad (15)$$

These inputs align with the balanced v - ω configurations, establishing a domain-specific encoding of reality. The proto-unit thus becomes the smallest stable structure where translation and rotation are zeta-bound.

3.4 Interference, Frequency, and Temporal Granularity

The spacing of zeta zeros along the critical line suggests a natural granularity in the temporal dimension. The interference patterns of stitched proto-units generate oscillatory modes—structures from which frequencies, energies, and quantization may emerge. The foundational beats of time itself are entangled with the rhythm of $\zeta(s)$.

3.5 Proto-Unit Normalization Operator for SI Conversion

To convert between proto-units and SI-units we seek define an operator $\Xi(\text{Xi})$. We let:

- \mathbb{S} be the set of SI-based measurements (meters, seconds, etc.) and
- \mathbb{L} the logical unit space where with the informational speed limit $\tilde{c} = 1$,
- where $\mathbb{L} \prec \mathbb{S}$ and
- $\widetilde{M}(\theta) = e^{i\pi}$ be a unified motion vector with:
 - $\theta = 0$: purely translational motion $\rightarrow \widetilde{M} = 1 + i0$,
 - $\theta = \frac{\pi}{2}$: purely rotational $\rightarrow \widetilde{M} = 0 + i1$, and
 - $\theta = \frac{\pi}{4}$: balanced motion $\rightarrow \widetilde{M} = \frac{1}{\sqrt{2}}(1 + i)$

We select to flag elements of \mathbb{L} with \sim above. We remember that any parameter with \sim has no SI based units, but is a pure numerical constant.

From complex number theory [Arfken and Weber, 2005] we borrow:

$$z = \frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}, |z|^2 = z \cdot \bar{z} = 1. \quad (16)$$

We say:

$$\text{Reality } \exists \iff 0 + 1 = \left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) \cdot \left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right)^2 = 1 \quad (17)$$

We're expressing how the number space transforms into the binary realm, where every real state requires two complex bits of information if we're extending the information space by one full square interger unit. After all, when we're extending the real number space with the complex degree of freedom, we are doubling the informational space.

We write therefor:

$$\Xi(\tilde{c}) \equiv \left(\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}\right) \cdot \left(\frac{1}{\sqrt{2}} - i\frac{1}{\sqrt{2}}\right) = (v + i \cdot \omega) \cdot (v - i \cdot \omega) = v^2 + \omega^2 = \frac{1}{2} + \frac{1}{2} = 1, \quad (18)$$

and define the complex motion vector \widetilde{M} as:

$$\widetilde{M}(v, \omega) = \frac{v}{\tilde{c}} + i\frac{\omega}{\tilde{c}}, \text{ with } \left|\widetilde{M}\right|^2 = \left(\frac{v}{\tilde{c}}\right)^2 + \left(\frac{\omega}{\tilde{c}}\right)^2 = 1, \quad (19)$$

with:

$$\widetilde{M}_0 = \cos(\alpha) + i \sin(\alpha) \Rightarrow \left| \widetilde{M} \right| = 1 \quad (20)$$

Now we can apply a **unitary rotation**:

$$\widetilde{M}(\theta) = \Xi(\tilde{c}) \widetilde{M}_0 = e^{i\pi} (\cos \alpha + i \sin \alpha) = \cos(\alpha + \theta) + i \sin(\alpha + \theta) \quad (21)$$

We therefor derive:

velocity:

$$\tilde{v} = \frac{v}{c} = \frac{\frac{m}{s}}{\frac{m}{s}} = 1, \quad (22)$$

space:

$$\tilde{x} = \frac{x}{ct} = \frac{m}{\frac{m}{s} \cdot s} = 1, \quad (23)$$

and time:

$$\tilde{t} = \frac{t}{t} = \frac{s}{s} = 1. \quad (24)$$

In the proto-unit system, SI-units now cancel out; space, time and motion become comparable - no separate units. Every object becomes defined by its c-normalized relation between v and ω .

4 Unified Field Equation

From the proto-unit infused with a zeta core and stripped from units, ready to directly converse between space-time and motion we can now attempt building the skeleton for a unified field equation, able to host reality. Something like:

$$Reality(x, t, v, \omega) = \sum_{n=1}^{\infty} \left(\frac{1}{n^{\frac{1}{2} + i \cdot \Omega(v, \omega)}} \right) \times e^{i \cdot \Theta(n, t)} \times \Lambda(n, t), \quad (25)$$

where:

- the $\sum_{n=1}^{\infty}$ counts the ongoing infinite sequence of building steps,
- $(1/n^s)$ makes every step a compressed oscillatory unit with $\Omega(v, \omega)$ recording the chaotic local time dependent phase shift, the maximally unpredictable state of balance at $Re(1/2)$,
- $e^{i \cdot \Theta}$ tracking cumulative rotational motion through time (ticks),
- and $\Lambda(n, t)$, the local temporal density modifier, regulating expansion and contraction against the external potential field.

If the proto-unit is the stitched kernel of balance between space and time, then the emergence of structure may follow from symmetry-breaking configurations of these proto-units. The balance point at $v = \omega = 1/\sqrt{2}$ defines a perfectly symmetric proto-unit—but in a universe of interactions, pure balance is rare. Local variations in v and ω create anisotropies: directional preferences that give rise to charge, spin, mass, or entropy. A chain of stitched proto-units, each slightly biased away from the $\pi/4$ balance point, can propagate structure through interference. Just as a standing wave emerges from constructive oscillations, a proto-geometry may arise from temporal-spatial patterns of imbalance. Gravity, in this view, is not a force but a curvature in the stitching density—space-time folds more tightly where the proto-units lean heavily toward either rotation or translation. The flattest geodesic in this manifold is zeta's critical line, the photonic equilibrium. Structure is not imposed on space-time by forces, but emerges from how proto-units break symmetry together.

4.1 Energy Equivalence and Zeta-Photon

First, we derive \hbar from proto unit logic as a bridge between:

- Energy E and
- Angular frequency $\omega = 2\pi v$.

From quantum physics we know:

$$E = \hbar\omega, \hbar \text{ in SI-units: } \hbar = \frac{kg \cdot m^2}{s}, \quad (26)$$

but because in our model:

$$E = mc^2 = \tilde{v}^2 + \tilde{\omega}^2 = 1, \quad (27)$$

\hbar must emerge from unit conversion between angular motion and energy.
Hence, when $\tilde{t} = 1$ (temporal tick):

$$E = \hbar \cdot \omega = \frac{1}{2\pi} \cdot 2\pi = 1 \quad (28)$$

We postulate therefor:

$$\boxed{\tilde{\hbar} = \frac{1 \text{ proto-energy unit}}{2\pi \text{ proto-angular cycles}} = \frac{E_{unit}}{\omega_{unit}} = \frac{\tilde{E}}{\tilde{\omega}}} \quad (29)$$

So, we can think of \hbar flipping it's identity, canceling it's units, and crystallizing it's true form $\tilde{\hbar}$ - a conversion factor between proto-motion geometry and measured physics of time.

Now, we may continue from Einsteins foundation and derive:

$$E = E_v + E_\omega, \quad (30)$$

with translational energy:

$$E_v = \gamma mc^2, \quad (31)$$

and rotational energy:

$$E_\omega = \hbar\omega. \quad (32)$$

Now we recall:

$$\tilde{v} = \frac{v}{c}, \quad (33)$$

and:

$$\tilde{\omega} = \frac{\omega}{c}, \quad (34)$$

such that the total logical motion is expressed as:

$$\tilde{v}^2 + \tilde{\omega}^2 = 1. \quad (35)$$

Hence we define a new normalized energy function:

$$E = mc^2(\tilde{v}^2 + \tilde{\omega}^2) = mc^2 \cdot 1 = mc^2. \quad (36)$$

This is the energetic boundary condition of space-time Albert Einstein already acknowledged. The physical encapsulation of motion, temporal potential within a perfect circular boundary, closing around zero and one. Riemann recognized the pivot point, the critical line, but science since has not managed to connect them logically.

When:

$$\tilde{v}^2 = \tilde{\omega}^2 = \frac{1}{\sqrt{2}}, \quad (37)$$

the energetic components become:

$$E_v = mc^2 \cdot \tilde{v}^2 = mc^2 \cdot \frac{1}{2}, \quad (38)$$

and:

$$E_\omega = mc^2 \cdot \tilde{\omega}^2 = mc^2 \cdot \frac{1}{2}, \quad (39)$$

so we can recall the path as $\gamma(t)$:

$$\tilde{\gamma}(t) = \zeta\left(\frac{1}{2} + it\right). \quad (40)$$

If we take:

- $\arg \tilde{\gamma}(t)$; recording angular displacement or phase direction and
- $|\tilde{\gamma}(t)|$; the probability amplitude,

then we may model an energetic photon along zeta's path:

$$\tilde{\gamma}(t) = c \cdot e^{i \arg \zeta(\frac{1}{2} + it)} \cdot f(|\zeta|). \quad (41)$$

In classical view, the photon is constantly traversing the boundary condition as pure translation through space and not moving in time. But, in reality we experiment with photons all the time, obviously they are present now and a second from now. Hence we assume that the photon is indeed moving through space **and** time. In this zeta balanced view, it's traversing a completely virtual, yet highly efficient geodesic through $Re1/2$, experiencing neither time nor space, yet both (the differentiation here feels more philosophical than physical to me and I leave it for the reader to answer).

At the critical line exists no preference to either pole, 0 or 1, such that there can exist total equilibrium. Concepts like force or inertia don't exist on this path, because it is not physical, but informational. It operates on probabilistic densities and motion in complex number space. The maximal amplitude in a sin-cos type oscillation experiences extreme logic space compression toward maximal amplitude (0° , 90°) and in real spiral freedom, the wave is actually constantly following the boundary condition maximal amplitude. It is constantly maximized, constantly at $\sum = 1$ state. It's rotational velocity is constantly shifting and to balance the equation, it's translational density is constantly pulsing, but despite this extreme energetic effort, it's total velocity can never change, because it is not able to overcome the density in informational space. It does not accept external informational bias from either space or time. It remains tightly locked on this informational geodesic carved by zeta.

The geodesic itself may be subject to constant external deformation of the real space and bend trajectory accordingly, but from the informational space perspective the geodesic holds firm.

5 Deriving the Fine Structure Constant α from Proto-Unit Logic

From here, we will attempt to illuminate forces, starting with the electromagnetic force. To understand it we call the fine structure constant α [Dirac, 1931; Jackson, 1998] given as:

$$\alpha = \frac{e^2}{4\pi\epsilon_0\hbar c} \approx \frac{1}{137.035999}, \quad (42)$$

where;

- ϵ_0 is the vacuum permittivity of electric fields and
- e is the elementary charge.

Using our $\tilde{\hbar} = 1/2\pi$ we write:

$$\alpha = e^2 \cdot \frac{2\pi}{4\pi\epsilon_0} = \frac{e^2}{2\epsilon_0}. \quad (43)$$

here we recover the logical proto square of 1unit charge e^2 , divided by the vacuums ability for information exchange.

ϵ_0 is reflected in SI as:

$$\epsilon_0 = \frac{1}{\mu_0 c^2}, \quad (44)$$

with μ_0 the magnetic constant.

We rewrite α :

$$\alpha = \frac{e^2 \mu_0 c^2}{2} = \frac{e^2 \mu_0}{2}. \quad (45)$$

If we define a proto-electromagnetic system with e^2 as a unit charge interaction and μ_0 as the logical vacuum resistance to electro-magnetic conversion, then $\tilde{\alpha}$ is literally the vacuum transfer function for electric energy. We can think of it as a “transparency constant” for electric logic to form physical light.

Suppose the proto electric charge unit $\tilde{e}^2 = 1$ and the proto vacuum permeability is defined by the geometric factor of the unit sphere as 4π steradians, then:

$$\tilde{\alpha} = \frac{1}{4\pi}. \quad (46)$$

The famous $\frac{1}{137}$ arises when we scale the proto charge down to \tilde{e} :

$$\tilde{e} \approx \sqrt{\frac{1}{137 \cdot 4\pi}}, \quad (47)$$

such that the actual charge e becomes a projection of a unit proto-charge \tilde{e} on the sphere of interaction:

$$\tilde{e}^2 = \tilde{\alpha} \cdot 4\pi \quad (48)$$

Therefore we infer that charge is not fundamental, but a reflection of topological coupling strength through the vacuum.

6 Deriving the Gravitational Constant G from Proto-Unit Logic

In SI units:

$$G = \frac{L^3}{MT^2} = \frac{m^3}{kg \cdot s^2} \quad (49)$$

Meaning: it scales energy per distance per time². But in proto logic, time and space are entangled, and energy is rotational so we guess:

$$G = \frac{1}{M_{Plank}^2} \quad (50)$$

with the Plank Mass M_P :

$$M_P \approx \sqrt{\frac{\hbar c}{G}}, \quad (51)$$

we write in proto units:

$$G = \frac{\tilde{\hbar}\tilde{c}}{M_P^2} \Rightarrow \text{if } \tilde{\hbar} = \frac{1}{2\pi}, \tilde{c} = 1, \text{ then:} \quad (52)$$

$$\boxed{\tilde{G} = \frac{1}{2\pi M_P^2}}. \quad (53)$$

So, G looks like the gravitational permeability of the vacuum in the same way ϵ_0 is the electric one.

7 Deriving the Boltzmann Constant k_B from Proto-Unit Logic

Boltzmann's entropy equation [Reif, 1965; Shannon, 1948] relates thermodynamic entropy S to the number of accessible microstates W .

It is given as:

$$S = k_B \cdot \ln W. \quad (54)$$

where:

- S = entropy in Joules/Kelvin
- W = number of microstates
- \ln = natural logarithm base e
- k_B = scaling constant that converts log counting into energy per temperature

But in proto units energy, time and frequency are 1:1 interchangeable and information (logarithmic state count) and entropy are unitless measures. So we define:

$$k_B \approx \frac{E}{\ln W} \rightarrow k_B = \text{energy per nat of entropy} \quad (55)$$

In proto units, we normalize temperature such that $\tilde{T} = 1$.

This implies:

$$E = k_B T \quad \Rightarrow \quad \tilde{E} = \tilde{k}_B. \quad (56)$$

So the Boltzmann constant becomes a direct measure of the energy per unit of entropy.

To determine its proto-unit value, we reinterpret k_B in terms of information. Recall Shannon entropy:

$$H = - \sum_i p_i \log p_i, \quad (57)$$

which becomes equivalent to thermodynamic entropy when multiplied by k_B :

$$S = k_B H. \quad (58)$$

Now, we assume a spherical state space where $W = 4\pi$, the number of distinguishable proto-units on a unit sphere. Each state occupies an area $\frac{1}{4\pi}$, and thus its information content is:

$$\ln \left(\frac{1}{4\pi} \right) = - \ln(4\pi). \quad (59)$$

If we take the entropy per proto-unit to be $S = 1$ (i.e., one nat of uncertainty), then solving for k_B we can write:

$$k_B = \frac{S}{\ln W} = \frac{1}{\ln(4\pi)}. \quad (60)$$

$$\boxed{\widetilde{k}_B = \frac{1}{\ln(4\pi)}} \quad (61)$$

Hence, the Boltzmann constant in proto units emerges as the reciprocal of the information capacity (in nats) of a spherical configuration space with 4π distinguishable states.

In proto units entropy is unitless, energy is fundamental and temperature is a relational curvature between state probabilities, such that k_B appears to encode the energy gradient per unit of logical uncertainty.

8 Deriving Planck Units

Using our new $\widetilde{c}, \widetilde{\hbar}, \widetilde{G}, \widetilde{k}_B$ we try to derive Planck units [Griffiths, 2018; Rovelli, 2004] from first principle.

- Planck Mass m_P

From:

$$G = \frac{1}{2\pi m_P} \Rightarrow m_P = \frac{1}{2\pi G} \quad (62)$$

In proto units:

$$\boxed{m_P = \frac{1}{2\pi G} = \frac{1}{2\pi l_P}} \quad (63)$$

- Planck Length l_P In SI:

$$l_P = \sqrt{\frac{\hbar G}{c^3}} \quad (64)$$

In proto units:

$$l_P = \sqrt{\frac{\frac{1}{2\pi} \cdot \frac{1}{2\pi m_P^2}}{1}} \Rightarrow \boxed{l_P = \frac{1}{2\pi m_P}} \quad (65)$$

Planck length is the inverse curvature radius of a unit mass m_P in unit-spin geometry.

- Planck Time t_P

In SI:

$$t_P = \sqrt{\frac{\hbar G}{c^5}} \quad (66)$$

In proto units:

$$t_P = l_P (\text{since } c=1) \Rightarrow \boxed{t_P = l_P = \frac{1}{2\pi m_P}} \quad (67)$$

Space and time unify at the Planck scale and under $c = 1$, a unit of time equals a unit of length: one quantum tick corresponds to one quantum step.

- Planck Temperature T_P In SI:

$$T_P = \frac{m_P c^2}{k_B} \Rightarrow \frac{m_P}{k_B} \quad (68)$$

In proto units:

$$T_P = \frac{m_P}{\frac{1}{\ln(4\pi)}} \Rightarrow \boxed{T_P = m_P \cdot \ln(4\pi)} \quad (69)$$

Planck temperature encodes mass curvature multiplied by the entropy surface count of a sphere.

- Planck Energy E_P

Standart:

$$E_P = m_P c^2 \quad (70)$$

So:

$$\boxed{E_P = m_P} \quad (71)$$

9 Conclusion

Reality must exist in a probabilistic state between 0 and 1. Everything emerges along the logic formed by complex information between rotation and translation, creating extension and self-containment of time and space.

Everything emerges from 0+1...

$$0 + 1 = 1^2 = c^2 = v^2 + \omega^2 = (i \cdot \text{Re})^2 = (\sqrt{-1} \cdot \sqrt{1})^2 = e^2 = T_{emp}^2 = T_{ime}^2 = \mathbf{1} \quad (72)$$

This is the living root of unity.

