Article

Seismic Activation Modeling with Statistical Physics

Daniel Brox †,*

† Independent Researcher; brox@alumni.caltech.edu
* Correspondence: dbrox@ece.ubc.ca

Abstract: Starting from fault dynamic equations, it is explained how real time evolution of a seismic activation region's elastic parameters preceding a major earthquake can be modeled in terms of statistical physics. Initial evidence for model validity is provided by deriving previously reported deviation of seismic activation earthquake occurence statistics from Gutenberg-Richter statistics in time intervals preceding a major earthquake.

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Introduction

An increase in the number of intermediate sized earthquakes ($\mathrm{M} > 3.5$) in a seismic region preceding the occurrence of an earthquake with magnitude $\mathrm{M} > 6$, referred to as seismic activation, has been documented by various researchers [7]. For example, seismic activation was observed in a geographic region spanning $21^\circ N - 26^\circ N \times 119^\circ E - 123^\circ E$ for a period of time between 1991 and 1999 preceding the magnitude 7.6 Chi-Chi earthquake [11]. Figure 1 shows a schematic plot of the cumulative distribution of earthquakes of different magnitudes in a seismic activation region in two different time intervals of equal duration preceding occurrence of a major ($7 < \mathrm{M} < 8$) earthquake at time $\tau = \tau_0$. In this figure, τ is a real time parameter, and τ_0 is the characteristic time of major earthquake recurrence assuming an earthquake of similar magnitude occurred in the same region at $\tau = 0$ [20,29]. Importantly, the cumulative distribution of earthquakes in a time interval of fixed width increasingly deviates away from a Gutenberg-Richter linear log-magnitude plot as the end of the time interval approaches τ_0.

As a means of predicting the time $\tau = \tau_0$ at which a major earthquake preceded by seismic activation occurs, it has been hypothesized that the average seismic moment $\langle \mathrm{M} \rangle_\tau$ of earthquakes occuring in intervals of time $(\tau, \tau + \Delta\tau)$ preceding a major earthquake obeys an inverse power of remaining time to failure law:

$$\langle \mathrm{M} \rangle_\tau \propto \frac{1}{(\tau_0 - \tau)^{\gamma_1}} \tag{1}$$

and that the cumulative Benioff strain $\mathcal{C}(\tau)$, defined as:

$$\mathcal{C}(\tau) = \sum_{i=1}^{n(\tau)} \mathrm{M}_{0,i}^{1/2}, \tag{2}$$

where $\mathrm{M}_{0,i}$ is the seismic moment of the i^{th} earthquake in the region starting from a time $\tau = 0$ preceding the major earthquake, and $n(\tau)$ is the number of earthquakes occurring in the region up to time τ, satisfies [27]:

$$\mathcal{C}(\tau) = a - b(\tau_0 - \tau)^{\gamma_2}, \ \gamma_2 = 1 - \gamma_1/2. \tag{3}$$

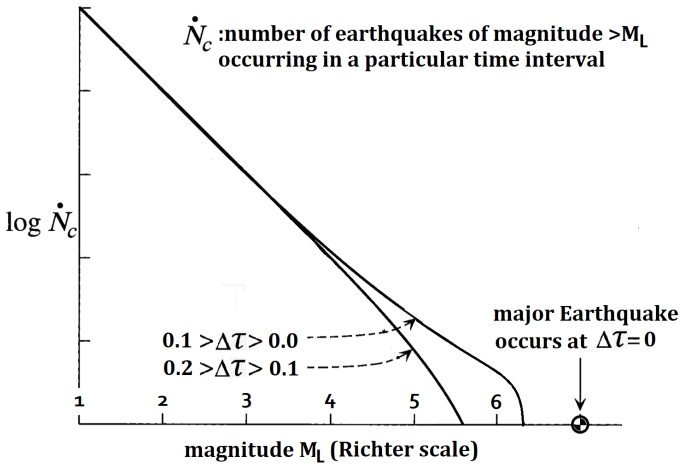

Figure 1. Plot of the cumulative distribution of earthquakes of different magnitudes in a seismic zone in two different time intervals of equal width preceding occurrence of a major earthquake at $\Delta\tau = \tau_0 - \tau = 0$ [20,29].

The exponent selection of $1/2$ in equation (2) is not necessary to derive formula (3) with a different arithmetic relation between γ_1 and γ_2, but appears to have been selected by previous researchers based on resulting predictions of major earthquake occurrence time when formula (3) is fit to real seismic data, which suggest a typical value of γ_2 is 0.3 [7,28]. Notably, the validity of the accelerating seismic moment release hypothesis (1) has been questioned by some researchers who claim normal foreshock and aftershock can account for seismic measurements without moment acceleration [15,31].

A model of seismic activation based on fault damage mechanics (FDM) has been used to derive equation (3) with a value $\gamma_2 = 1/3$ [4]. In this derivation, the occurrence of seismic activation earthquakes progressively decreases the average shear modulus of fault material in the seismic region where subsequent seismic activation earthquakes occur, and the result $\gamma_2 = 1/3$ is obtained using a Boltzman kinetic type description of the rupture nucleation process in which ruptured faults of different lengths at different positional locations grow and join together [26].

In addition to the FDM model of seismic activation, an empirical statistical physics model of seismic activation known as the Critical Point (CP) model has been put forth to derive equation (3) with a value $\gamma_2 = 1/4$ [20]. In this derivation, the inverse power of remaining time to failure law:

$$\langle \mathrm{M} \rangle_\tau \propto \frac{1}{(\tau_0 - \tau)^{3/2}} \tag{4}$$

is asserted based on identifying the mean rupture length $\mathcal{L}(\tau)$ of earthquakes occuring at time τ with the correlation length of a statistical physical system described by Ginzburg-Landau mean field theory with a τ-dependent temperature parameter, whereby:

$$\mathcal{L}(\tau) \propto \frac{1}{(\tau_0 - \tau)^{1/2}}, \tag{5}$$

and relation (4) follows from the scaling relation $\langle \mathrm{M} \rangle_\tau \propto \mathcal{L}(\tau)^3$ which holds when the fault material shear modulus is constant [21].

Importantly, previous work on the CP model has not explained why it is physically reasonable to describe seismic activation earthquake occurrence statistics with thermal equilibrium statistical physics formalism [24]. Therefore, the first objective of this article is to clarify how the FDM and CP models of seismic activation can be in correspondence with each other. The second objective of the article is to use this correspondence to advance

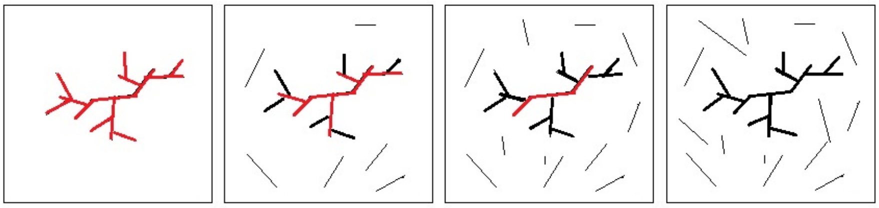

Figure 2. Schematic illustration of seismic activation in a 2D geometry at four different times τ in which each black line represents an earthquake fault rupture that has already occured, and the red lines represent earthquake faults along which shear stress is increasing prior to rupture [18].

rigorous testing of seismic activation model predictions against seismic measurements, and in the event of positive experimental verification, advance earthquake prediction technology.

Motivating the presented correspondence between FDM and CP seismic activation models is previous work suggesting that the real time evolution of the elastic model of a seismic activation region, expressed in terms of a finite element method stiffness matrix, can in certain cases be described with statistical physics renormalization group flow equations [2,13]. This theoretical work may have computational utility to seismic activation modeling if dimensional reduction of statistical physics models at critical points can be used to systematize dimensional reduction of seismic activation region stiffness matrices in windows of time preceding a major earthquake.

The outline of the article is as follows. Section 2 explains how fault rupture dynamics can be described in terms of soliton equations, and how these soliton equations can be used to characterize critical points of statistical physics models whose mean field values at criticality correspond to unstable seismic displacements. Section 3 further claims that seismic activation earthquake occurrence statistics are expressible in terms of the Yang-Lee zero distribution of a statistical physics model partition function, and uses this claim to account for deviation of occurrence statistics from Gutenberg-Richter statistics before a major earthquake. Section 4 concludes by commenting on how validity of statistical physics modeling of seismic activation can be tested against seismic measurements.

Materials and Methods

Seismic Activation Fault Dynamics

Figure 2 shows a 2D schematic of earthquake occurrence in a seismic activation region [18]. In this figure, the activation region is shown at 4 different times up to and including the moment after a major earthquake has occurred. At each time, black lines indicate fault ruptures associated with earthquakes that have occurred, and red lines indicate faults where stress is accumulating prior to earthquake occurrence. Qualitatively, the picture suggests the occurrence of successively larger earthquakes, associated with successively longer rupture lengths, leads to increased strain along the major earthquake fault as seismic activation proceeds. From an FDM point of view, this increased strain occurs with a reduction in the average shear modulus of material in the vicinity of the fault, until fault rupture occurs at time $\tau = \tau_0$, when fault material is marginally stable with respect to material displacement perturbation [10].

Quantitatively, this picture of seismic activation leading to rupture along a major earthquake fault is supported by modeling of earthquake fault dynamics in 1+1 spacetime dimensions, whereby the differential equation:

$$A\partial_\tau^2 U(\tau,z) - B\partial_z^2 U(\tau,z) + C\partial_\tau U(\tau,z) = -\sin(U(\tau,z)/D). \tag{6}$$

has been used to model both creep along a major earthquake fault and rupture propagation, depending on whether or not frictional forces dominate the fault dynamics and shear stress evolution along the fault is more appropriately described with a reaction diffusion equation or a solitary wave equation [9]. In this equation, τ is real time, z coordinates a direction of creep or slip along an earthquake fault, $U(\tau, z)$ is the local displacement of elastic material across the earthquake fault, $A\partial_\tau^2 U(\tau, z)$ is the local inertial force acting on the fault material, $B\partial_z^2 U(\tau, z)$ is the local elastic restoring force acting on the fault material, and $C\partial_\tau U(\tau, z)$ and $\sin(U(\tau, z)/D)$ are local frictional forces acting on the fault material attributed to contact of the material with tectonic plates on either side of the fault. For $C = 0$, an (anti-kink) soliton solution to equation can be interpreted as propagation of earthquake fault rupture [30].

To generalize this description of fault creep and rupture in 1 spatial dimension to 3 spatial dimensions, first note that if the seismic activation region resides in an elastic half space $\mathcal{H}$, then real time evolution of the elastic displacement of material in the region is specified by a path $\gamma(\tau)$ in the Lie group $\mathcal{G} = Diff(\mathcal{H})$ [1]. For $\tau < \tau_0$, this path specifies a gradual deformation of the activation region's quasi-elastostatic equilibrium configuration in which strain energy is minimized, whereby a displacement $\vec{\mathbf{u}}$ of the region's equilibrium configuration at tme τ increases the strain energy of the region by:

$$\Delta\mathcal{E} = \frac{1}{2}\vec{\mathbf{u}}^T K(\tau)\vec{\mathbf{u}}, \tag{7}$$

for $K(\tau)$ equal to a positive definite stiffness matrix of the region at time τ. At $\tau = \tau_0$, this stiffness matrix has at least one zero eigenvalue corresponding to a marginally stable seismic displacement $\vec{\mathbf{u}}_0$ that describes the major earthquake faulting mechanism. For $\tau > \tau_0$, when the path $\gamma(\tau)$ specifies fault rupture propagation, the equation of motion of the tangent vector $\gamma'(t)$, pulled back to the Lie algebra g of vector fields on $\mathcal{H}$ by left (or right) translation, is a soliton equation describing parallel transport of the initial unstable seismic displacement $\vec{\mathbf{u}}_0$.

Seismic Activation Region Finite Element Model

In finite element method terms, the Lie algebra g is approximated by the vector space of nodal displacements associated with a mesh of $\mathcal{H}$. More specifically, suppose a major earthquake hypocenter resides in a 3D elastic half space $\mathcal{H}$ in such a way that the elastic parameters of the half space are constant outside a hemisphere of diamater $\mathcal{L}_0$ centered at the earthquake epicenter. Then, if each fracture within the region is defined as a thin low elastic impedance layer, a Dirichlet-to-Neumann map is defined at the hemisphere boundary, and a finite element mesh accounting for fracture and boundary geometry is defined, the elastic model of the region at time τ can be written as a frequency dependent stiffness matrix $K(\omega; \tau)$ with dimension equal to the number of finite element nodes [5,25]. Similarly, using the density of the activation region, a time dependent lumped mass matrix $M(\tau)$ can be written with dimension equal to the number of finite element nodes. Together, the stiffness and mass matrices define a nonlinear eigenvalue problem:

$$\Big(K(\omega; \tau) - \omega^2 M(\tau)\Big)\vec{\mathbf{u}} = 0, \tag{8}$$

at each time τ, whose non-zero solution vectors $\vec{\mathbf{u}}$ specify nodal displacements associated with elastic resonant frequencies ω of the activation region.

Statistical Physics Mean Field Theory

To introduce the relevance of statistical physics to modeling real time evolution of the seismic activation region elastic model, suppose that in a window of time preceding $\tau = \tau_0$,

$K(\omega, \tau)$ is independent of ω, and $W(\tau) = K(\tau)M(\tau)^{-1}$ has (τ-dependent) real eigenvalues λ_i associated with orthonormal eigenvectors $\vec{\mathbf{u}}_i$. In this event, writing:

$$\vec{\mathbf{u}} = \sum c_i \vec{\mathbf{u}}_i, \tag{9}$$

it follows that:

$$\vec{\mathbf{u}}^T W(\tau)\vec{\mathbf{u}} = \sum \lambda_i(\tau) c_i^2, \tag{10}$$

and assuming each $\lambda_i(\tau) > 0$ for $\tau < \tau_0$, the onset of instability of the seismic activation region at $\tau = \tau_0$ coincides with vanishing $\lambda_1(\tau_0) = 0$ of at least one of the eigenvalues.

Now suppose that a statistical physics mean field theory is defined in such a way that its Landau free energy is given by expression (10) plus higher order terms in mean field values c_i [Goldenfeld]. Also suppose that the temperature of the system is determined by the parameter τ in such a way that the sign change of $\lambda_1(\tau)$ at $\tau = \tau_0$ corresponds to ordering of the statistical physics system with a non-zero value of c_1 for $\tau > \tau_0$. With these suppositions, the stiffness matrix $K(\omega, \tau)M(\tau)^{-1}$ is a matrix coefficient of a statistical field theory with a critical point at $\tau = \tau_0$, and the order parameter fields of this theory have a classical physics interpretation as magnitudes of activation region nodal displacement from mechanical equilibrium. Moreover, if the statistical physics model is defined so that a discontinuous gap in the coefficient $\lambda_1(\tau)$ occurs at $\tau = \tau_0$, as known to occur for the 2D XY model, the mean field condition $c_1^2 \propto -\lambda_1(\tau)$ implies the quantity $\sqrt{-\lambda_1(\tau_0^+)}$ is proportional to the rupture length of the major earthquake.

Results

To relate the discussion in the previous chapter to seismic activation earthquake occurrence statistics, now suppose the negative eigenvalues of the stiffness matrix $K(\tau)M(\tau)^{-1}$ are the Yang-Lee zeroes of the statistical physics model partition function [Bena et al.]. With this supposition, Yang-Lee zero statistics should describe the cumulative distribution of seismic activation earthquakes with rupture length $\sqrt{-\lambda}$, a prediction that is now verified to the extent that it accounts for the deviation of seismic activation earthquake occurrence statistics from Gutenberg-Richter statistics.

In the time interval $(\tau, \tau + \Delta\tau)$, let ω be the corner frequency of an activation earthquake with rupture length $\sqrt{-\lambda}$, where λ is an eigenvalue of the stiffness matrix that changes sign during the time interval. Then, assuming the earthquake occurs within the time interval with probability proportional to $\omega d\tau$, and $\rho(\omega)$ is the density of corner frequencies in the interval $(\omega, \omega + d\omega)$ associated with activation earthquakes occurring in the time interval, the number of earthquakes with corner frequency less than or equal to ω occurring during the time interval is:

$$\dot{N}_c d\tau = \int_{\omega_c(\tau)}^{\omega} \bar{\omega}\rho(\bar{\omega}) d\tau d\bar{\omega}, \tag{11}$$

where $\omega_c(\tau)$ is the corner frequency of the largest activation earthquake occurring up until time τ.

To specify the mathematical form of the integral in equation (11), recall that the Gutenberg-Richter law implies the total number of earthquakes of Richter magnitude in the interval $(M_R, M_R + dM_R)$ occurring in the seismic activation region in the time interval $(\tau, \tau + d\tau)$ is proportional to:

$$10^{-bM_R} dM_R d\tau, \tag{12}$$

which according to the relation between Richter magnitude and seismic moment:

$$M_R = \big(\log_{10}(M_s) - 9\big)/1.5, \tag{13}$$

and scaling relation $M_s \propto \omega^{-3}$, satisfies:

$$10^{-bM_R} dM_R d\tau \propto M_s^{-1-b/1.5} dM_s d\tau \propto \omega^{2b-1} d\omega d\tau. \tag{14}$$

Therefore, assuming the Gutenberg-Richter law is valid, it follows that:

$$\rho(\omega) \propto \omega^{2b-2}. \tag{15}$$

To account for modification of the Gutenberg-Richter law in time intervals preceding a major earthquake, now assume that for corner frequencies ω satisfying:

$$\omega \approx \omega_c(\tau_0) \equiv \omega_0, \tag{16}$$

with ω_0 equal to the corner frequency of the largest seismic activation earthquake preceding the major earthquake at time $\tau = \tau_0$, the quantity $\rho(\omega)$ is determined by a distribution of the eigenvalues λ satisfying:

$$\int_{\omega_0}^{\omega} \rho(\bar{\omega}) d\bar{\omega} \propto (\omega - \omega_0)^{\beta_0},\ 1 > \beta_0 > 0. \tag{17}$$

With this assumption, equation (11), modified to account for occurrence of an earthquake at corner frequency ω_0, implies:

$$\dot{N}_c = 1 + \int_{\omega_0}^{\omega} \bar{\omega}\rho(\bar{\omega}) d\bar{\omega} \approx 1 + c(\omega - \omega_0)^{\beta_0}. \tag{18}$$

Consequently:

$$\log_{10} \dot{N}_c \approx \log_{10}\Big(1 + c(\omega - \omega_0)^{\beta_0}\Big), \tag{19}$$

when plotted against Richter magnitude $M_R \propto -2\log_{10}\omega$ for $\beta_0 < 1$, can have either of the cumulative distribution curve shapes shown in Figure 1 for different time intervals, depending on the value of β_0.

In passing, it is also noted that in accordance with previous statistical physics models of seismic activation, the identification $\beta_0 = \beta(\tau_0)$, where $\beta(\tau)$ is a parameter in a τ-dependent statistical physics model such as the 2D XY model, is logical. From this point of view, the parameters of the statistical physics models, including $\beta(\tau)$, are related by renormalization group flow, and an increase in the value of $\beta(\tau)$ as $\tau \to \tau_0$ accounts for increasing steepness of the cumulative distribution curve shown in Figure 1.

Discussion

Previous research has identified predicting the time of occurrence of major earthquakes as a possible application of statistical physics models of seismic activation, but this application has not yet been realized [7]. In more recent times, the earthquake early warning algorithm Virtual Seismologist has been developed which can in principle use previous earthquake occurrence statistics as input to improve warning accuracy, and the artificial intelligence algorithm QuakeGPT has been developed for predicting the occurrence of major earthquakes using seismic event records created with stochastic simulator training data [6,12,22]. Therefore, a practical applied science goal for the statistical physics model presented in this article appears to be improving statistical characterization of earthquake precursors for use in earthquake warning and/or forecasting technologies, acknowledging that preliminary tests of the model's validity against real seismic data must be passed before achieving this application objective can be considered a realistic possibility.

From a geophysical testing point of view, if it is true that renormalization of a 2D sine-Gordon model describes real time evolution of the elastic model of a seismic activation region and, as a result, a nonlinear dynamical system of finite phase space dimension characterizes the elastic model during nucleation of shear stress in a seismic region preceding a major earthquake, a geophysical signal processing technique known as singular spectrum analysis should apply to determine this phase space dimension [8]. Specifically, it is suggested that measurements of relative changes in seismic surface wave and/or body wave velocity be performed between pairs of seismic stations in a seismic region over a duration of time during which seismic activation is known to have occurred, and used as input to a time domain multichannel singular spectrum analysis algorithm [19]. The number of channels of this algorithm would equate to the number of station pairs, and the number of singular values output by the algorithm in different time windows preceding occurrence of a major earthquake should categorize the region's elastic model if the statistical physics model of seismic activation is correct in principle. With reference to previous geophysical application of singular spectrum analysis, performed in the frequency domain, the signal processing algorithm suggested here is different in that it should be carried out in the time domain τ rather than the frequency domain [23].

In conclusion, work towards improving current earthquake early warning systems can proceed in two directions. Firstly, as an initial check on whether or not the statistical physics modelling approach presented here could be of practical utility, work can be done to determine whether or not observed changes of the Earth's elastic velocity model preceding major earthquakes can be processed to extract an integer identifiable as the phase space dimension of a nonlinear dynamical system. Secondly, work can be done to elaborate upon the statistical physics mathematical model of seismic activation presented in this article to determine other tests of its scientific validity and potential for practical application.

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Abbreviations

The following abbreviations are used in this manuscript:

MDPI	Multidisciplinary Digital Publishing Institute
DOAJ	Directory of open access journals
TLA	Three letter acronym
LD	Linear dichroism

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