# On a Remarkable Quadratic Identity

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**Abstrac**t− We prove the elementary identity

$$(n^2 + n + a)^2 + a = (n^2 + a)((n + 1)^2 + a)$$

for any real number $a$. The identity reveals a symmetric structure between shifted quadratic terms.

**Keywords**− quadratic identity, elementary algebra, polynomial factorization, symmetric expression.

## Proof

Expanding the left-hand side,

$$(n^2 + n + a)^2 + a = n^4 + 2n^3 + (2a + 1)n^2 + 2an + a^2 + a.$$

Expanding the right-hand side,

$$\begin{aligned}(n^2 + a)((n + 1)^2 + a) &= (n^2 + a)(n^2 + 2n + 1 + a) \\ &= n^4 + 2n^3 + (2a + 1)n^2 + 2an + a^2 + a.\end{aligned}$$

Thus, both sides are equal. ■

## Remark

The identity may serve as tool for constructing new factorization techniques and has potential connections with generalized quadratic forms.

## References

[1] Paulo Ribenboim, *Classical Theory of Algebraic Numbers*, Springer-Verlag, 2001.
[2] Arthur Engel, *Problem-Solving Strategies*, Springer-Verlag, 1998.