Why MOND's fundamental acceleration is not a “fudge factor”

...but a consequence of the Heisenberg/Küpfmüller uncertainty principle

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Content

Abstract

The MOND theory (Modified Newtonian Dynamics) initiated by Milgrom has established itself as the most important model for explaining the measured rotation curves of galaxies without the aid of ominous dark matter. The core element of MOND is the so-called fundamental acceleration a_0 with a value of approx. `1.2·10^-10 m/s^2`, which results from the measurements of galaxy rotation velocities. At accelerations close to or below this value, neither Newton's nor Einstein's gravitational models work reliably.

Critics of the MOND theory argue that this value is an ad-hoc “fudge factor” that was not derived from a fundamental consideration of space and time. So Milgrom himself [1] as well as other proponents of MOND [2][3] have already shown that the value a_0 can very easily be brought into a numerical relationship with the age of the universe T_U and the speed of light c.

In this paper, I would now like to show that this numerical correlation is no coincidence, but can be derived by consistent application of Heisenberg's energy-time uncertainty relation on a cosmic scale. So I will show that $a_0 = c / (2\pi \cdot T_U)$ is the smallest possible acceleration for any rotation/orbital motion in an universe of age T_U and therefore not a “fudge factor”, but the counterpart of the Planck acceleration at the other, lower bound of the energy scale with which our universe can be described.

To warm-up: The minimum possible mass (the MOND mass)

The Wikipedia article on the Planck units succinctly describes the conciseness and significance of the Planck time ($5.391 \cdot 10^{-44}$ *s*) as follows: "*No current physical theory can describe timescales shorter than the Planck time, such as the earliest events after the Big Bang.*"

On the other hand, experts seem to have difficulty seeing **the age of the universe as the counterpart of Planck time** and simply formulating it clearly:
"*No current physical theory can describe timescales larger than the age of the universe*".

This thought came to me again when I read the rather well-known paper by Wesson from 2003 "Is mass quantized?" In this work [4], Wesson postulates a "quantum perturbation mass" of 2· 10^-68kg, which results from the following equation:

$$m_{min} = \frac{h}{c} \cdot \sqrt{\frac{\Lambda}{3}} \qquad (1)$$

h: Planck constant = $6.63 \cdot 10^{-34}$ Js

c: speed of light in vacuum = 299792458 m/s

Λ: Cosmological constant = $1.4657 \cdot 10^{-52} m^{-2}$

Reading through the accompanying text, one easily realizes that this equation is sophisticated numerology by a theoretical physicist. At this point, I would like to emphasize once again (as in previous work [5]) that I consider numerology to be an acceptable approach in principle, which I practice myself to a considerable extent. But if possible, it should be replaced by a more fundamental and easy to understand approach.

And that is exactly what can be done here: The question "Is mass quantized?" could easily be answered with "Sure, because the age T_U of the universe is finite."
So Wesson's value can also be arrived at by straightforward numerology-free considerations. The finite age of the universe T_U is a limiting factor in many respects: no particle, no photon, no process can have a longer lifetime than the universe itself. And according to Heisenberg's energy-time uncertainty principle,

$$\Delta E \cdot \Delta t >= \frac{h}{4\pi} \quad (2)$$

the energy of a quantum cannot be determined more precisely than

$$\Delta E_{min} >= \frac{h}{4\pi \cdot T_u} \quad (3)$$

T_U: age of the universe = 13.8 billion years

and with E = m· c^2 its corresponding mass cannot be determined more precisely than 2.7· 10^-69kg:

$$m_{min} >= \frac{h}{4\pi \cdot T_u \cdot c^2} = 1.35 \cdot 10^{-69} kg \quad (4)$$

So the first counter-Planck unit, the counter-Planck mass (let's calls it „MOND mass“), is thus derived without numerology. In his work, Wesson used h and not the reduced h (=h / 2pi) for his approach. If he had done so, he would also have arrived at the approx. 1.35· 10^-69kg.

To make it clear: Any body in the universe has a mass that is an integer multiple of this Counter-Planck mass of 1.35· 10^-69 kg. And accordingly, both the Planck mass and the total mass of the universe are integer multiples of this. I will come back to this aspect in note e) to Table 2 below.

Once you have internalized that the spectrum of possible masses in a finite universe is discrete, it is also easy to understand that the frequency spectrum of electromagnetic waves and gravitational waves is discrete. Let us now take a closer look at this.

The discrete gravitational spectrum leads to MOND

The starting point is therefore that Heisenberg's energy-time uncertainty relation is not only applied in the atomic world, but also and especially in the cosmic world, as we have already done above:

$$\Delta E \cdot \Delta t >= \frac{h}{4\pi} \qquad (2)$$

By applying the Planck formula E = h·f and shortening the Planck constant, we can see that the energy-time uncertainty principle is actually a frequency-time uncertainty principle:

$$\Delta f >= \frac{1}{4\pi \cdot \Delta t} \qquad (5)$$

In this representation, a bridge is built to signal processing and the equivalent Küpfmüller uncertainty relation [6], which roughly speaking says nothing more than that the frequency resolution of the total signal (i.e. the smallest possible measurable frequency within the signal) depends on the total duration of the signal. If we now consider *the age of the universe as the total duration of a signal*, then it becomes clear that the possible measurable frequencies in our universe cannot become arbitrarily small, but that their accuracy is limited by the age of the universe.

And this applies not only to electromagnetic waves/photons but also to gravitational waves/gravitons. The lifetime, or rather the formation time of each photon and each graviton, cannot be greater than the age of the universe. Otherwise, the definition of such a finite time epoch would be completely meaningless. Accordingly, all gravitational waves in our universe have a minimum frequency resolution of 1/(4· pi·T_u):

$$\Delta f_{min} >= \frac{1}{4\pi \cdot T_u} \qquad (6)$$

This also means that all gravitational waves in the universe have a frequency inaccuracy that is an integer multiple of 1/(4· pi·T_u):

$$\Delta f_{grav}(n) = \frac{n}{4\pi \cdot T_u},$$

$$where \quad (7)$$

$$n \in \mathbb{N}, n <= T_u \,/\, t_{Planck}$$

t_{Planck}: Planck time = 5.391 · 10^-44 s - For an explanation of theT_U / t_{Planck} limit, see later in Table 2, footnote e)

These discrete inaccuracies can each be attributed to a gravitational wave (n=1: wave with period = T_U, n=2: wave with period = $T_U/2$ and so on).This is also a consequence of the consistent application of the signal processing rules: **If the universe is the total signal, then the period of partial signals, e.g. those of gravitational waves or electromagnetic waves, can only be integer multiples of the total period.** So all frequencies of all gravitational waves in the universe are dependent on the age of the universe.

By establishing that gravitational waves do not have a continuous frequency spectrum, but can only be assigned to discrete frequency values, we are deviating from the assumptions of the general relativity. However, this does not mean that the entire theory is called into question. On the contrary, it merely adds a correction that only has an effect from galactic distances onwards anyway. In particular, we are now making use of an important insight from general relativity: The period of the dominant gravitational wave is exactly half of the orbital period required by a celestial body when orbiting the center of gravity:

$$f_{grav} = 2 \cdot f_{orbit} \quad (8)$$

In addition to this dominant gravitational radiation, there are also gravitational waves with higher frequencies, especially in very eccentric orbits, but all of these are also an integer multiple of the orbital frequency. In the following, I will only consider the dominant radiation with f(grav) / f(orbit) = 2. I will go into this in the later section *"But..."*. At this point, it should already be said that this consideration is an acceptable simplification in order to show in principle that there is a fundamental constant acceleration, which must lie approximately at $c / (2\pi \cdot T_U)$ and for ideal circular orbits even lies exactly at this value.

This fixed coupling of gravitational wave frequency and orbital frequency of celestial bodies means that all orbital frequencies of celestial bodies are also fixed in a discrete set of values that depend on the age of the universe! For each orbital frequency, an inaccuracy is added up that corresponds exactly to twice the inaccuracy of the associated dominant gravitational frequency:

$$\Delta f_{orbit} = 2 \cdot \Delta f_{grav} \tag{9}$$

From this, together with (7), it follows that **all orbital frequencies of celestial bodies in our universe can only assume discrete values whose accuracy is limited by 1/(2pi· T_U):**

$$\Delta f_{orbit}(n) = \frac{n}{2\pi \cdot T_u}, \quad where \quad n \in \mathbb{N}, n <= T_u \,/\, t_{Planck} \tag{10}$$

With ω =2pi·f we can thus derive a discrete spectrum of oribital angular velocities:

$$\Delta \omega_{orbit}(n) = \frac{n}{T_u}, \quad where \quad n \in \mathbb{N}, n <= T_u \,/\, t_{Planck} \tag{11}$$

The orbital velocities are also discrete

We are now halfway there. Now it remains to be shown that not only are all orbital frequencies/angular velocities in the universe discrete and their smallest inaccuracy depends on the age of the universe, but also that the corresponding orbital velocities can only have discrete values. And that the minimum possible discrete value for the respective orbital frequency ultimately also determines the minimum possible radial acceleration of this orbit. And we will see: All possible orbits of the universe always have the same value for this minimum possible radial acceleration !

With the fact that the universe has a finite age, we now know that the frequencies of gravitational waves/electromagnetic waves are discrete. **However, this does not mean that the wavelengths of these energy waves are discrete to the same extent.** The smallest possible wavenumber would be determined by the maximum size of the universe and here we only know that - if there is a finite expansion - it would have to be significantly greater than twice the distance that light has traveled during the time span T_U. **Therefore, we can say that for most wavenumber/wavelength values of energy radiation we do not have an associated frequency, which means that the speed of propagation of the energy radiation must be adjusted/rounded down so that the wavelength value meets a valid frequency value.**
Accordingly, we are dealing with an inaccuracy of the propagation speed of gravitational waves, which is accompanied by the inaccuracy of their frequencies:

$$\Delta v_{min} = \lambda_{grav} \cdot \Delta f_{min} \qquad (12)$$

Together with (6) this results in:

$$\Delta v_{min} = \frac{\lambda_{grav}}{4\pi \cdot T_u} \qquad (13)$$

This results in the maximum possible propagation of a gravitational/electromagnetic wave:

$$v_{max} = c - \frac{\lambda_{grav/em}}{4\pi \cdot T_u} \quad (14)$$

This means that gravitational waves (and thus also electromagnetic waves, see also my paper[7]) never travel at the speed of light (they could only do so in an infinitely old universe), but instead travel below the speed of light by the difference λ / (4pi·Tu), depending on their wavelength.
The following table shows that this difference is far too small for all the usual measurable ranges of electromagnetic waves and also for all gravitational waves that have been detected with LIGO [8] so far to be able to prove them in experiments. Only for the last row of the table, in which the longest possible wavelength is given, which hopefully can soon be detected with the LISA detector [9], is there a velocity difference value that should lead to a measurable transit time difference if the origin of the gravitational waves is far enough away.

Type of wave	**Wavelength in m**	**Δ v in m/s**
Red light	0.00000064	1.17· 10^-25
Longwave radio spectrum	5000	9.13· 10^-16
LIGO longest wave in GW170817 [10]	10,000,000	1.83· 10^-12 **a)**
LISA longest detectable wave (probably)	3·10^12	5.48· 10^-7 **b)**

a): The measured event GW170817 was 130 million light years away. With this Δ v of the gravitational wave, the somewhat faster gamma ray burst, which occurred approx. 2 seconds later, could only catch up by 7500 meters or 0.000025 seconds. Too small a difference to measure
b): Such a wave, if also 130 million light years away, would lose 2.2 million km in distance compared to a high-frequency-event, which amounts to a measurable transit time difference of 7.5 seconds. The only question is whether such an event would also be accompanied by a high-frequency-event. We shall see.

But we will now see: Indirectly, we can already measure such velocity differences today. Namely, those that occur on a galactic scale and beyond and have been causing us headaches for decades in the form of “dark matter”.

First, we now formulate equation (13) in such a way that reference is again made to the discrete values for all possible dominant gravitational waves in the universe and the associated orbital classes become visible:

$$\Delta v_{grav}(n) = \frac{\lambda_{grav}(n)}{4\pi \cdot T_u} \tag{15}$$

Just as we have defined the discrete values of all possible frequency inaccuracies by means of the minimum frequency inaccuracy resulting from the age of the universe, a maximum possible velocity inaccuracy results from the wavelength c·Tu associated with the age of the universe and the speed of light. This maximum possible velocity inaccuracy is the integer multiple of all possible velocity inaccuracies of gravitational waves.
So:

$$\lambda_{grav}(n) = \frac{c \cdot T_u}{n} \tag{16}$$

inserted in (15) results in:

$$\Delta v_{grav}(n) = \frac{c}{4\pi \cdot n} \tag{17}$$

Here you can already see that the greater the frequency inaccuracy of a gravitational wave (see equation (7)), the smaller its velocity inaccuracy becomes. The “inaccuracy product” of the two values therefore has a constant value for all n - and this product has the dimension of an acceleration. And this is exactly what is propagated in the associated orbits of a dominant gravitational wave.

Because It should be clear that the fixed coupling between gravitational wave frequency and orbital frequency only works if the inaccuracy of the propagation speed of a dominant gravitational wave is accompanied by an inaccuracy of the orbital speed of corresponding orbitals. Analogous to (9), we can therefore determine a minimum inaccuracy of all orbital velocities in the universe as follows:

$$\Delta v_{orbit}(n) = 2 \cdot \Delta v_{grav}(n) \quad (18)$$

This connection becomes particularly clear when you realize that every n-th orbit class has a theoretical orbit in which the orbiting body moves at (almost) the speed of light. The length of its orbit must therefore be twice as long as the wave length of its associated gravitational wave. This means that the “rate” at which its speed gains inaccuracy is the same as that of its dominant gravitational wave, which, with a period twice as long, also means twice the speed inaccuracy. Equation (17) inserted in (18) finally results in:

$$\Delta v_{orbit}(n) = \frac{c}{2\pi \cdot n} \quad (19)$$

For all those who only skim this paper, I would like to emphasize here that this velocity inaccuracy has nothing to do with Heisenberg's momentum-position uncertainty principle, from which a velocity inaccuracy is derived in many applications in the subatomic range. The velocity inaccuracy of the orbits results from the frequency-time uncertainty principle, which affects all gravitational waves in the universe. Due to the coupling of gravitational wave frequency and orbital frequency already derived by Einstein, all orbits “inherit” this velocity inaccuracy.

Increase times decrease equals constant

The radial acceleration results from the product of the orbital angular velocity and the orbital velocity. Accordingly, the product of their two inaccuracies is the inaccuracy of the radial acceleration:

$$\Delta\, a_{orbit}(n) = \Delta\, \omega_{orbit}(n) \cdot \Delta\, v_{orbit}(n) \tag{20}$$

Here we now insert the intermediate results of the last two sections, equation (11) for Δω and equation (19) for Δv. This results in:

$$\begin{aligned} \Delta\, a_{orbit}(n) &= \frac{n}{T_u} \cdot \frac{c}{2\pi \cdot n} \\ &= \frac{c}{2\pi \cdot T_u} \quad (for\ all\ n) \end{aligned} \tag{21}$$

So the decisive factor is:
With increasing orbital frequencies / decreasing orbital periods, the inaccuracy of the orbital frequency also increases, but the inaccuracy of the orbital velocity decreases to the same extent. The inaccuracy of the radial acceleration, which is a product of orbital frequency inaccuracy and orbital velocity inaccuracy, therefore remains the same for all possible orbits.

The following table is intended to illustrate this situation, where the frequency inaccuracy, velocity inaccuracy and the acceleration uncertainty resulting from both are shown for selected orbit classes:

n	Description	Min. possible orbital frequency $f_{min} = n/(2\pi \cdot T_U)$	Min. possible orbital velocity $v_{min} = c/(2\pi \cdot n)$	Min. possible radial acceleration = $2\pi \cdot f_{min} \cdot v_{min}$
1	Orbits with the minimum possible frequency uncertainty and corresponding maximum possible velocity inaccuracy	1 / (2π ·Tu) = 3.65· 10^-19 Hz	c / (2π) = 47713 km/s	1.1·10^-10m/s^2
5	Orbits with the fifth minimum possible frequency uncertainty and corresponding fifth maximum possible velocity inaccuracy	5 / (2π ·Tu) = 1.83· 10^-18 Hz	c / (10π) = 9543 km/s	1.1·10^-10m/s^2
193	Orbits whose min. possible frequency roughly corresponds to the *half* orbit frequency of the sun around the center of the galaxy	193 / (2π · T_U) ≈ 1 / (450 Myr)	c / (2π · 193) = 247.2 km/s **b)**	1.1·10^-10m/s^2
386	Orbits whose min. possible frequency roughly corresponds to the orbit frequency of the sun around the center of the galaxy	386 / (2π·Tu) ≈ 1 / (225 Myr)	c / (2π · 386) = 123.6 km/s **a)**	1.1·10^-10m/s^2
7.3·10^9	Orbits whose min. possible frequency roughly corresponds to the orbit frequency of the Jupiter around the sun	= 1 / (11.862 years)	c/ (2π · 7.3·10^9) = 0.00652 m/s **c)**	1.1·10^-10m/s^2
3.4·10^38 ≈2^128 **d)**	Orbits whose min. possible frequency roughly corresponds to the compton frequency of the electron	1.2356·10^20 Hz	1.4 ·10^-31 m/s	1.1·10^-10m/s^2
8·10^60 = T_U / t_{Planck} **e)**	Orbits with the maximum possible frequency uncertainty (Planck frequency) and corresponding minimum possible velocity inaccuracy	1 / (t_{Planck}·2π) = 2.952·10^42 Hz	= $a_0 \cdot t_{Planck}$ = 5.9·10^-54 m/s	1.1·10^-10m/s^2

a): The orbital speed of the sun around the center of the galaxy is estimated at 230 km/s. This means that the sun is still fast enough “on its own” not to violate the minimum speed that exists due to the speed inaccuracy of its orbit class. But the first deviations from Newton are already emerging, because with an orbital period of 225 million years, the orbital velocity granularity is already

124km/s. Only this value or an integer multiple is conceivable as the orbital velocity for the sun.

b): If the orbital period of the sun were twice as long (see at n=193), because its distance to the center of the galaxy would be greater, "dark matter" or a strict MOND regime would be determined for the sun. Its orbital velocity according to Newton should actually decrease, but we would find a velocity of at least 247 km/s.

c) Note how much the orders of magnitude change for both n and velocity inaccuracy when moving from galactic to stellar scale. The Jupiter, with an orbital velocity of 13 **k**m/s, does not have the slightest problem keeping to the minimum velocity inaccuracy of 0.0065 m/s. The orbital velocity of Jupiter can already be determined much more precisely. But that is not a contradiction. Only if, under the same measurement conditions and modeling assumptions, one were to measure systematic changes in its *mean* orbital velocity that are significantly less than 0.0065 m/s would my explanation begin to falter.

d) Considering the order of magnitude of the most important elementary particle, we should not be surprised that a value comes out for n that is once again related to 2^128. I refer here to my previous papers [7][5]

e) Here we have now reached the other end. The finitely small Planck time t_{Planck} and the Planck frequency associated with it mean that there must also be a finitely small minimum velocity $v_{min}=a_0 \cdot t_{Planck}$ - the counter-part to the maximum velocity c (Milgrom would probably call it "MOND velocity"). We discover how the maximum possible value for n = Tu/ t_{Planck} brings together the Planck units and their counter-Planck units (or MOND units). The Planck acceleration can also be expressed as follows using the fundamental acceleration a_0 (the counter-Planck acceleration):

$$a_{Planck} = \frac{c}{t_{Planck}} = a_0 \cdot 2\pi \cdot \frac{T_u}{t_{Planck}} \qquad (22)$$

As mentioned above, I assume that Tu/t_{Planck} is a natural number of the order of 10^60. Furthermore, I assume that the Planck units (and the quantities h, c and G contained therein) are not constant in time, but also "get older" with the age of the universe. The only things that are timeless are the dimensionless ratios such as Tu/ t_{Planck}, but also the dimensionless fine structure constant or the ratios of the masses of the various elementary particles and subatomic particles to each other, while the masses themselves also "age".

But...

At the beginning, I mentioned that it was initially completely sufficient to derive the fundamental acceleration only from the link with the dominant gravitational waves. For absolutely circular orbits, only these waves occur. And for slightly eccentric orbits, which is true for the majority, it is completely sufficient to consider only the dominant waves. But for very eccentric orbits, the higher orders must also be taken into account and the factors change. I intend to explain this in more detail in a follow-up version or continuation of this paper. But if I haven't overlooked anything, the following should emerge for the next higher order, i.e. for f(grav) / f(orbit) = 3:

$$\Delta\, a_{orbit,order3}(n) = \frac{3 \cdot n}{2 \cdot T_u} \cdot \frac{3 \cdot c}{4\pi \cdot n} = \frac{9 \cdot c}{8\pi \cdot T_u} = 2.46 \cdot 10^{-10}\, \frac{m}{s^2} \qquad (23)$$

As you can see, we are now still in the same order of magnitude, but clearly above the value of 1.2· 10^-10 m/s^2 known from rotational curve measurements. However, since the dominant waves generally have a much stronger influence, an experimental average value of 1.2· 10^-10 m/s^2 would even be quite conclusive.

Nevertheless, this would have to be calculated precisely and how exactly these different a_0 values are to be combined and weighted is a question that I cannot answer at the moment and for which I would be happy to receive feedback and suggestions (the AI chatbots are telling me something about the Peters-Mathews formula). This calculation is also important because changes in orbital velocity occur in all the non-circular orbits of our planets. And even with this discrete-values-approach, these changes must be able to occur so smoothly that they are consistent with the fairly accurate velocity measurements.

The assumption that a_0 is not just a threshold value at which acceleration values below it are smoothed out, but that it is actually the smallest possible acceleration value, contradicts the frequent observations that the stars at the edge of the galaxies are all traveling at the same velocity - and accordingly also the current MOND theory approaches. But if the radial acceleration did not decrease any further below a value of a_0, a = v^2/r would mean that the velocities at the edge of the galaxy would even increase[1]!

So this approach is all the more dependent on the “External Field Effect” (EFE), which can be justified by the fact that the gravitational waves of galaxy clusters or higher structures - insofar as they rotate - must also obey the discrete value corset presented and thus also have a_0 as a threshold value.

As far as we know today, clusters and superclusters do not rotate, but the last word has probably not yet been spoken. The flood of data generated by space telescopes (e.g. the JWST) is constantly bringing new surprises in this respect. After evaluating new JWST images, Shamir, for example, sees confirmation of his earlier analysis that the sense of rotation of all galaxies in the immediate vicinity is unevenly distributed (2/3 of all galaxies rotate in the opposite direction to the Milky Way), which may indicate that even larger structures in the universe have a common sense of rotation [11]. If this proves to be the case, then the stars would not only experience an acceleration due to rotation within the galaxy, but also due to rotation in an even larger structure. This should always be taken into account when using fundamental acceleration.

Furthermore even supporters of the ΛCDM model come to the conclusion that a_0 must have an effect on the structure of galaxy clusters [12].

1 There are a few galaxies where this is actually the case. The best known case is the nearby Triangulum Galaxy (M33)

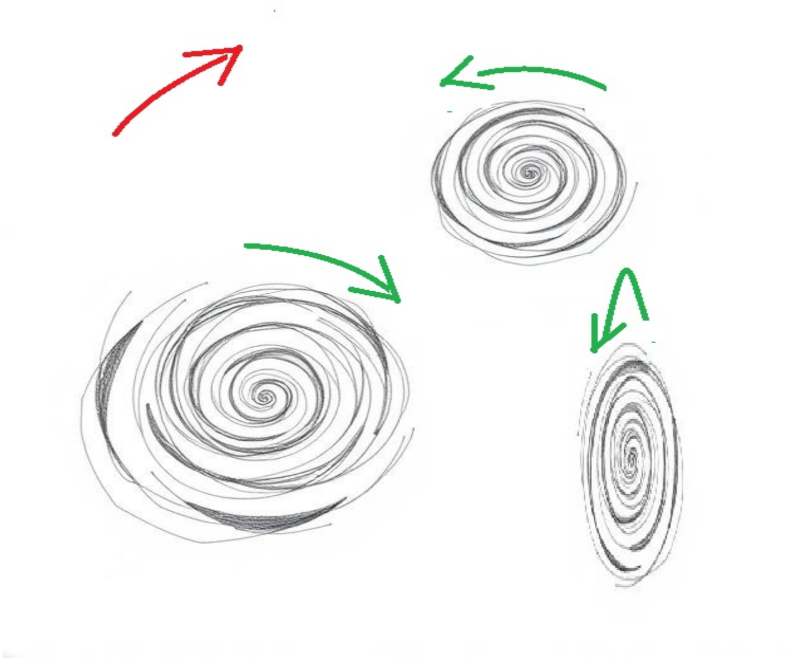

fig 1: „External Field Effect“: Three galaxies with different rotational directions of their own, but which all possibly rotate together in the same direction (see [11]). For stars on the edge of large galaxies, this „cosmic rotation“ has just as great an impact as the rotation of their own galaxy.

Summary and Discussion

In contrast to the previous MOND approaches, I come to the assumption that

$a_0 = c/(2pi \cdot T_U) \approx 1.1 \cdot 10\text{^-}10\ m/s^2$

is not just a limit value at which the acceleration drop are smoothed out, but that it is actually the minimum possible acceleration that exists in our finite universe. And all dynamic processes in our universe cannot fall below this value.

As explained in the previous section, this conclusion will now face even more challenges than the previous MOND approaches already have. But for now I see this as an incentive to refine the approach as also explained in the previous section.

In my view, the crucial mistake that the academic physics guild has made in recent decades is to see the field of application of Heisenberg's uncertainty principle only in the (sub)atomic realm and the related Küpfmüller uncertainty principle only in the practical environment of signal processing. The application of these principles to the cosmic realm is, I believe, the key to remedying the shortcomings of Newton and Einstein's description of gravity and to gaining a fundamental understanding of the cause of gravity.

Even though there may still be a few errors in reasoning and calculation in this paper, I think I was able to show in principle that there must be a constant minimum acceleration inaccuracy for all conceivable orbits in our universe, which simply results from the fact that our universe has a finite age. This age basically determines frequency resolution and “velocity resolution” for all orbital processes and, in combination, a fundamental “acceleration resolution”, which Milgrom was the first to discover in the 1980s by evaluating experimental data. In doing so, he performed a similarly decisive preliminary work as Kepler had done about 400 years earlier ...

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