Relativity of mass
–Mass can decrease with velocity

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We present a simple approach for deriving velocity-dependent masses using the principle of relativity. Our analysis reveals that the transformations associated with Galilean, Lorentz, and other space-time frameworks between two inertial reference frames are fundamentally equivalent in the context of the relativity of mass. Consequently, the notion of velocity-dependent mass is not the exclusive characteristic of Special Relativity (Lorentz transformation). Among the notable conclusions drawn from our formalism are: mass can both increase and decrease with velocity, a particle can never be completely at rest, and superluminal signaling is in principle feasible. Furthermore, we discuss on the nature of mass and argue that a photon is not massless.

I. INTRODUCTION

To describe rest and motion (these being relative trems), we require a frame of reference which consists of a coordinate system and a time-measuring device (clock) [and also an observer to tell the coordinates and clock readings!]. A frame of reference in which Newton's (first) law of motion is valid, without any pseudo forces, is called an inertial frame of reference. In non-inertial (rotating or accelerating) frames, Newton's law is correct only if certain fictitious (pseudo) forces are added to the driving forces. Newtonian mechanics [1] assumes that both space (lengths) and time (durations) are absolute quantities, i.e., they are the same for all observers, regardless of their location or of their motion. According to the principle of relativity, originally proposed by Galileo, there are no priviledged inertial frames. Put differently, physical scenarios of (Newtonian) mechanics are indistinguishable for observers either at rest or in constant motion. Consider an inertial frame of reference $\mathcal{S}$ at rest and another inertial frame of reference $\mathcal{S}'$ moving relative to $\mathcal{S}$ with a velocity $\vec{v} = v\hat{x}$ (see Fig. 1). Then $\mathcal{S}$ moves with a velocity $\vec{v} = -v\hat{x}$ with respect to $\mathcal{S}'$. The space and time coordinates of a point in frames $\mathcal{S}$ and $\mathcal{S}'$ are denoted, respectively, by $x_\mu \equiv (x_1, x_2, x_3, x_0) \equiv (x, y, z, t)$ and $x'_\nu \equiv (x'_1, x'_2, x'_3, x'_0) \equiv (x', y', z', t')$. The relations between the coordinates of two inertial frames of reference are called *transformation* equations:

$$x'_k = x'_k(x, y, z, t) = x'_k(x_\mu) \leftrightarrow x'^T_\nu = \Lambda(v) x^T_\mu, \quad (1)$$

$$x_k = x_k(x', y', z', t') = x_k(x'_\nu) \leftrightarrow x^T_\mu = \Lambda^{-1}(v) x'^T_\nu, \quad (2)$$

where $\Lambda(v)$ is the transformation operator (matrix). If Eq.(1) is regarded as the direct transformation then Eq.(2) is the inverse transformation, and vice-versa. In classical relativity, the coordinates transformation between the different inertial systems are brought about

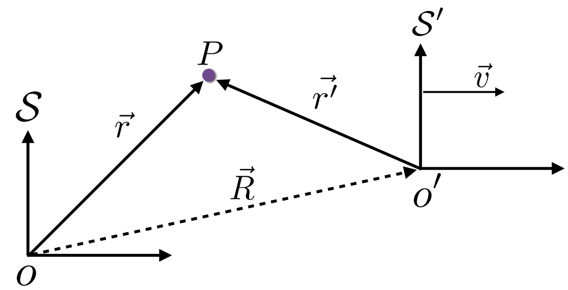

FIG. 1. Two inertial systems $\mathcal{S}$ and $\mathcal{S}'$ moving relative to each other with velocity $\vec{v}$. Position vector of a point P in $\mathcal{S}$ is $\vec{r} \equiv (x, y, z, t) \equiv x_\mu$ and that in $\mathcal{S}'$ is $\vec{r'} \equiv (x', y', z', t') \equiv x'_\nu$.

by Galilean transformation:

$$x' = x - vt, \;\; y' = y, \;\; z' = z, \;\; t' = t. \quad (3)$$

In classical (Galilean) relativity, positions and velocities are all dependent on the motion of the observer, but other quantities like length, time duration, mass, or the acceleration due to gravity are absolute quantites, i.e., remain invariant. Under Galilean transformation, force is invariant along the boost and momentum is conserved along the directions perpendicular to the boost.

The principle of relativity was found, however, not consistent with Maxwell's equations of electromagnetism. These equations are not covariant under Galilean transformation. Moreover, Maxwell's equations of electromagnetism predicted the speed of electromagnetic waves, including light, in vacuum based on the fundamental constants of the vacuum (the electric permittivity ϵ_0 and the magnetic permeability μ_0) as $c = 1/\sqrt{\mu_0 \epsilon_0}$. Note that the vacuum is not simply "empty space" but has intrinsic properties that influence the propagation of electromagnetic waves. The constants ϵ_0 and μ_0 are inherent properties of the vacuum itself, and thus, the calculated speed is also a constant. Hence, the constancy of the speed of light in vacuum is tied to these fundamental properties of the

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vacuum and is independent of the motion of the light source.

In the 19th century, physicists believed that light waves, like other waves, propagate through luminiferous aether, a hypothetical stationary medium that permeated all of space. The Michelson-Morley experiment [2] designed to detect the Earth's motion through this aether had failed to detect any "aether wind", which should have been apparent if the Earth was moving through this stationary ether. The experiment consisted of a Michelson interferometer, which split a beam of light into two perpendicular beams, reflected them back and then recombined them to produce an interference pattern. The interferometer was set up to detect any changes in the speed of light caused by the motion of the Earth through the aether. The idea behind this experiment was that if the Earth moves through the aether, then the speed of light in the direction of the Earth's motion should be different from the speed of light in the perpendicular direction. This would cause a shift in the interference pattern that could be measured. Contrary to the belief, the experiment yielded null result (the interference pattern remained unchanged) regardless of the orientation of the apparatus or the time of day or the season of year. This meant that the speed of light was the same in all directions and that there was no evidence of the existence of the aether. Lorentz [3] sought to reconcile the idea of a stationary aether with the constancy of speed of light regardless of the motion of observers. He introduced a set of transformations that related the space and time coordinates of two observers in relative motion, and could account for the observed behavior of electromagnetic phenomena. The notion of absolute space and absolute time prove to be untenable.

This paper is organised as follows. We review Einstein's Special Relativity briefly in Sec. II, and the associated issues in Sec. III. Thereafter, in Sec. IV, we provide an alternative approach to obtain the velocity-dependent mass followed by several remarks. In Sec. V, we argue that photons have mass. Finally, we conclude in Sec. VI.

II. EINSTEIN SPECIAL RELATIVITY

The theory of relativity is about the dependence or the invariance of physical statements on the reference system of the observer. The null result of Michelson-Morley experiment proved a major challenge to the prevailing scientific theory of the time. It played a key role in the development of Special Relativity (SR), which abandoned or eliminated the need for the aether and revolutionized our understanding of space and time. The theory of SR introduced by Einstein in 1905 [4–7] emerged from a confluence of perplexing observations and theoretical inconsistencies within classical physics, most notably the behavior of light. It becomes indispensable whenever the relative velocity between reference frames approach the velocity of light. The classical relativity turns out to be the limiting case of SR for small relative velocities. The theory of special relativity is based on two key postulates:

(P1) (The principle of relativity or equivalence) The laws of physics are the same for all observers in uniform motion relative to each other. Put differently, there is no preferred inertial frame. All inertial frames enjoy equal status.

(P2) (The principle of the constancy of speed of light) The speed of light in vacuum is always the same, regardless of the motion of the observer or the source of the light.

A. Lorentz Transformation

The transformation, for a boost $\vec{v} = v\hat{i}$ along x-axis, which is compatible with the above two postulates of SR is the Lorentz transformation [3, 7] (see also [8] and [9]), given by

$$\begin{aligned} x &= \gamma(v)(x' + vt'), \\ y &= y',\ z = z', \\ t &= \gamma(v)(t' + \frac{v}{c^2}x'), \end{aligned} \tag{4}$$

and

$$\begin{aligned} x' &= \gamma(v)(x - vt), \\ y' &= y,\ z' = z, \\ t' &= \gamma(v)(t - \frac{v}{c^2}x), \end{aligned} \tag{5}$$

where the Lorentz factor $\gamma(v) = (1 - kv^2)^{-\frac{1}{2}}$ with $k = 1/c^2$ depends on the relative velocity of the frames (observers) and c is the speed of light in vacuum. These transformation equations reduce to Galilean transformation when $v << c$ or $c \to \infty$. The Lorentz transformation refers only to uniformly straight-line relative motions of the considered systems and thus does not tell anything about systems which are accelerated relative to each other. The Lorentz transformations preserve the speed of light, lead to time dilation (moving clocks run slow) and length contraction (moving rods shorten along the boost), unifies or entangles space and time into four-dimensional continuum "spacetime", and form the basis of special relativity. Other notable milestones of SR are variation of mass with velocity and equivalence of mass and energy.

B. Velocity Composition and Acceleration

The velocities of a particle are given by $\vec{u} = \mathrm{d}\vec{r}/\mathrm{d}t$ in $\mathcal{S}$ and $\vec{u'} = \mathrm{d}\vec{r'}\Big/\mathrm{d}t'$ in $\mathcal{S}'$. Hence, using the Lorentz transformation Eq.(4) in their differential form, velocities transform as

$$u_1 = \frac{u_1' + v}{1 + u_1' v/c^2} = \frac{\gamma(u_1')\gamma(v)}{\gamma(u_1)}(u_1' + v), \tag{6}$$

$$u_2 = \frac{u_2'}{\gamma(v)(1 + u_1' v/c^2)} = \frac{\gamma(u_1')}{\gamma(u_1)} u_2', \tag{7}$$

$$u_3 = \frac{u_3'}{\gamma(v)(1 + u_1' v/c^2)} = \frac{\gamma(u_1')}{\gamma(u_1)} u_3', \tag{8}$$

where

$$\gamma(u_1) = \gamma(u_1')\gamma(v)(1 + u_1' v/c^2). \tag{9}$$

Similarly, using Eqs.(6, 7, 8) and Eq.(9), accelerations $\vec{a} = \mathrm{d}\vec{u}/\mathrm{d}t$ in $\mathcal{S}$ and $\vec{a'} = \mathrm{d}\vec{u'}\Big/\mathrm{d}t'$ in $\mathcal{S}'$ transform according to

$$a_1 = \frac{a_1'}{\gamma^3(v)(1 + u_1' v/c^2)^3} = \frac{\gamma^3(u_1')}{\gamma^3(u_1)} a_1', \tag{10}$$

$$\begin{aligned} a_2 &= \frac{1}{\gamma^2(v)(1 + u_1' v/c^2)^2} \left(a_2' - a_1' \frac{u_2' v/c^2}{1 + u_1' v/c^2} \right) \\ &= \frac{\gamma^2(u_1')}{\gamma^2(u_1)} \left(a_2' - a_1' \frac{u_2' v}{c^2} \frac{\gamma(u_1')\gamma(v)}{\gamma(u_1)} \right), \end{aligned} \tag{11}$$

$$\begin{aligned} a_3 &= \frac{1}{\gamma^2(v)(1 + u_1' v/c^2)^2} \left(a_3' - a_1' \frac{u_3' v/c^2}{1 + u_1' v/c^2} \right) \\ &= \frac{\gamma^2(u_1')}{\gamma^2(u_1)} \left(a_3' - a_1' \frac{u_3' v}{c^2} \frac{\gamma(u_1')\gamma(v)}{\gamma(u_1)} \right). \end{aligned} \tag{12}$$

C. Variation of Mass with Velocity

The relationship between mass of an object and its speed were investigated and discovered by early physicists [3, 10]. Einstein SR also posits that mass varies with velocity. Three widely-known expressions of variation of mass with velocity are:

$$m = m_0 \left(1 - u^2/c^2\right)^{-\frac{3}{2}}, \tag{13}$$

$$m = m_0 \left(1 - u^2/c^2\right)^{-\frac{1}{2}}, \tag{14}$$

$$m = m_0\, e^{\frac{Qu^2}{2c^2}}, \quad (Q > 0), \tag{15}$$

where $m_0 \equiv m(\vec{u} = 0)$ is the rest mass and $m \equiv m(\vec{u} \neq 0)$ is the moving or relativistic mass of a particle. Conventionally, mass in Eq.(13) is called *longitudinal mass* and that in Eq.(14) is called *transverse mass*. These two masses have a singularity at $u = c$. The mass varying exponentially with velocity in Eq.(15) diverges when $u \to c$ and Q is unusually high. The expressions of velocity-dependent masses in Eqs.(13, 14) have been obtained in literature using different approaches [4, 11–18] with and without Lorentz transformations, such as (i) relativistic invariance of laws of electrodynamics, (ii) energy and momentum conservation in elastic and ineleastic collisions of two particles, (iii) momentum conservation in splitting or fission process, (iv) force moment balance method, (v) relativity and symmetry principles, (vi) Hamilton's principle of least action and Lagrangian. Nevertheless, these appraches have inherent limitations. In another approach, the velocity-dependent mass is obtained using momentum conservation to a single particle subject to a force [15]. Sharma [18], in order to avoid divergence (singularity) at $u = c$, arrived at Eq.(15) using the assumption that the variation of mass with velocity ($\frac{\mathrm{d}m}{\mathrm{d}u}$) is proportional to velocity (u) of body and its mass (m).

D. Consequences of Velocity-dependent Mass

Two remarkable conclusions follow from the velocity-dependent mass in Eqs.(13, 14). (i) Since mass increases with velocity, relativistic mass m will be exceptionally large when $u \to c$ for a massive particle ($m_0 \neq 0$). But it will require an infinite amount of energy to accelerate a particle to speed $u \approx c$ which is not possible. Due to this energy constraint, a massive particle has a upper speed limit which is c. (ii) Since light travels with speed c, its rest mass is assumed zero. Another argument for masselss photon is given by the uncertainty principle according to which the range of the force is inversely proportional to the force-mediating particle mass. Since the electromagnetic interaction is long-ranged, the photon must be massless.

III. ISSUES WITH EINSTEIN SPECIAL RELATIVITY

The theory of special relativity has proved revolutionary, has had profound implications for our understanding of the universe, and has refined our understanding of space and time considerably among other notable developments. Regardless of its tremendous acclaim, Einstein theory of special relativity is viewed with skepticism and has drawn much criticism [19–25]. There are concerns on the bases of conceptual, scientific, mathematical, and phlosophical arguments. We remark a few issues below.

1. *Aether and null result of Michelson-Morley experiment:* The Michelson-Morley experiment failed to detect the stationary luminiferous aether. There could be several possible reasons of this null result [26–28]: (i) the aether simply does not exist, (ii) the Earth drags the aether along with it, (iii) the experiment was not sensitive enough, (iv) there were experimental methodological errors, i.e., experimenters did not exhaust all the operation modes or settings, (v) the experiment was affected by external factors such as variations in temperature or

air pressure, (vi) the calculation was made on the wrong premises. The concept of zero-point energy fluctuations [29, 30] reveals that space is not truly empty. This "neo-aether" (quantum field) is, however, different from the early concept of the static aether.

2. *Constancy of speed of light:* There is a general consensus within the scientific community that the speed of light in vacuum is a fundamental constant. However, there are theoretical considerations and cosmological implications such as horizon problem, cosmic inflation, etc. that question this assumption and have prompted investigations into the variable speed of light (VSL) theories [31]. A few arguments against the constancy of speed of light are: (i) The speed of light $c = 1/\sqrt{\mu_0\epsilon_0}$ in vacuum is characteristic of the fundamental properties of vacuum. Light slows down when passing through a denser medium ($v = c/\sqrt{\mu_r\epsilon_r}$). Extending this logic, it is possible that in the early universe conditions were such that light travelled with a speed greater than c. This could be true even today in some corners of the universe. Hence, speed of light appears to be characteristic of the properties of the underlying medium (substratum of matter) in which it travels. (ii) Until now the speed of light has been measured in a round trip. It is possible that one-way light is anisotropic and its speed has different value from c. (iii) Light is affected by gravity. It bends around massive stars and stops at (or is completely absorbed by) black holes.

3. *Lorentz transformations:* It is not necessary to postulate the constancy of speed of light in vacuum to arrive at Lorentz transformation. It can be derived even without light [32–52]. Moreover, the paradoxial nature and redundancy of Lorentz transformation has also been reported [53–55]. Not only Lorentz transformations but other space-time transformations such as Voigt [56, 57] and Selleri [58, 59] also render spherical wavefront covariant.

4. *Velocity composition rule:* Using Eq.(6) we see that for $p \neq 0$ (i) if $v = \frac{c}{p}$ and $u_1' = -\frac{c}{p}$ then $u_1 = \frac{0}{1-\frac{1}{p^2}} = 0$, (ii) if $v = c$ and $u_1' = -c$ then $u_1 = \frac{0}{0}$ is undetermined, and (iii) if $v = \frac{c}{p}$ and $u_1' = \pm c$ then $u_1 = \pm c$ seems to be independent of the relative velocity between the frames. In another case let $v \neq 0$, $u_1' = 0$ and $u_2' = c$ (say, a laser light is shown along y-axis in frame $\mathcal{S}'$). Then, $u_1 = v$ from Eq.(6) and $u_2 = \frac{c}{\gamma(v)}$ from Eq.(7). Thus, an observer in frame $\mathcal{S}$ does not see light moving with speed c along the y-direction but in the xy-plane.

5. *Variation of mass with velocity:* Relativistic mass in Eq.(14) increases with increasing velocity. Since photon (light) travels with speed c, its rest mass m_0 is assumed zero and hence its relativistic mass is undefined. But it has nonzero momentum and energy. A neutrino has negligible rest mass and it travels at nearly the speed of light. Does its relativisic mass is exceptionally large? Does mass really increase with velocity [61–63]? What happens to the enhanced mass of a moving particle when it is brought to rest?

6. *Equivalence of mass and energy:* Is Lorentz transformation inevitable to arrive at the mass-energy equivalence [64–68]? What is its real meaning or interpretation? Is it true in the sense that mass and energy are interconvertible freely? If it were true then given an energy E, one should be able to create a matter particle of mass $m = \frac{E}{c^2}$. Has this ever been achived? Conversely, for a nonzero mass m to have an equivalent energy $E = mc^2$, it must travel with speed c which is impossible for a massive particle. Even if mass and energy are equivalent and they are convertible into one another, there has to be a systematic mechanism behind that. Actually, the binding (or disassociation) energy and the mass defect are equivalent [69, 70].

IV. ALTERNATIVE DERIVATION OF VELOCITY-DEPENDENT MASSES

In this section, we present a simple yet elegant approach to obtain velocity-dependent masses, in particular Eqs.(13, 14), using the principle of relativity alone under Lorentz transformation. Newton's second law of motion $F = \mathrm{d}p/\mathrm{d}t \overset{m=\text{const}}{\longrightarrow} ma$ is a physical law. And conservation of momentum is another physical law in the absence of a net external force. Therefore, for a given boost, we require from the first postulate that under Lorentz transformation, as in Galilean transformation, force is invariant along the boost and momentum is invariant (conserved) along the directions perpendicular to the boost. The physical quantities in $\mathcal{S}$ are *unprimed* and those in $\mathcal{S}'$ are *primed*.

Case 1. First, we demand invariance of momentum perpendicular to the direction of boost in the two inertial frames, and obtain using Eqs.(7, 8),

$$\begin{aligned} m_k' u_k' &= m_k u_k = m_k \frac{\gamma(u_1')}{\gamma(u_1)} u_k' \\ \Rightarrow \frac{m_k'}{\gamma(u_1')} &= \frac{m_k}{\gamma(u_1)} = \frac{m_{k0}}{\gamma(0)} \text{ (say)} \\ \Rightarrow m_k &= m_{k0}\gamma(u_1) \quad (k = 2, 3). \end{aligned} \tag{16}$$

Here the velocity-dependent mass, following from invariance of momentum, along $y-$ or $z-$axis depends only on the velocity along x. Here m_{k0} denotes the rest mass of a particle along the spatial coordinate x_k. It must be invariant ($m_{k0} = m_0$).

Case 2. By demanding invariance of force $F = ma$ along the x-axis in two inertial frames, we get the longitudinal mass

$$\begin{aligned} m' a_1' &= m a_1 = m \frac{\gamma^3(u_1')}{\gamma^3(u_1)} a_1' \\ \Rightarrow \frac{m'}{\gamma^3(u_1')} &= \frac{m}{\gamma^3(u_1)} = m_0 \text{ (say)} \\ \Rightarrow m &= m_0 \gamma^3(u_1) = m_0 \left(1 - u_1^2/c^2\right)^{-\frac{3}{2}}. \end{aligned} \tag{17}$$

Case 3. Demanding invariance of force $F = \mathrm{d}p/\mathrm{d}t$ along the x-axis in two inertial frames, we get the transverse mass. Moreover, we get a new expression of relativistic mass which is inversely proportional to velocity of the particle. First, note that

$$F = \frac{\mathrm{d}(mu)}{\mathrm{d}t} = \left(m + u\frac{\mathrm{d}m}{\mathrm{d}u}\right)a. \tag{18}$$

Then $\frac{\mathrm{d}(m'u'_1)}{\mathrm{d}t'} = \frac{\mathrm{d}(mu_1)}{\mathrm{d}t}$ yields

$$\left(m' + u'_1\frac{\mathrm{d}m'}{\mathrm{d}u'_1}\right)a'_1 = \left(m + u_1\frac{\mathrm{d}m}{\mathrm{d}u_1}\right)\frac{\gamma^3(u'_1)}{\gamma^3(u_1)}a'_1 \tag{19}$$
$$\Rightarrow \frac{1}{\gamma^3(u'_1)}\left(m' + u'_1\frac{\mathrm{d}m'}{\mathrm{d}u'_1}\right) = \left(m + u_1\frac{\mathrm{d}m}{\mathrm{d}u_1}\right)\frac{1}{\gamma^3(u_1)}.$$

While the expression on lhs is a function of u'_1, that on rhs is a function of u_1. This is possible only when each of them is separately equal to a constant, say, m_0. Consequently, we get

$$\frac{\mathrm{d}m}{\mathrm{d}u_1} + \frac{m}{u_1} = m_0\frac{\gamma^3(u_1)}{u_1} \quad (0 < u_1 < c). \tag{20}$$

Eq.(20) is an ordinary first-order linear nonhomogenous differential equation. Its *complementary* solution is obtained by equating the lhs of Eq.(20) to zero. That is,

$$\frac{\mathrm{d}m}{\mathrm{d}u_1} + \frac{m}{u_1} = 0$$
$$\Rightarrow \int_{m_0}^{m}\frac{dm}{m} = -\int_{u_{10}}^{u_1}\frac{du_1}{u_1}$$
$$\Rightarrow mu_1 = m_0u_{10} \equiv \chi, \tag{21}$$

where χ, we believe, depends on composition of the particle and medium (substratum of matter). It is easy to verify that the *particular* solution of Eq.(20) is

$$m = m_0\left(1 - u_1^2/c^2\right)^{-\frac{1}{2}}. \tag{22}$$

Hence, the general expression of velocity-dependent mass is given by

$$m = \frac{\chi}{u_1} + m_0\left(1 - u_1^2/c^2\right)^{-\frac{1}{2}}. \tag{23}$$

Based on the above developments, a few remarks are in line.

Remark 1. In our derivation above, m_0 is not the *rest* mass of Einstein because $u_1 = 0$ is not permissible by Eq.(20). Rather, it is the mass corresponding to some threshold or critical speed $m_0 = m(u_1 = u_{10} \neq 0)$ by Eq.(21). Thus, a particle is never completely at rest (though its net displacement might be zero). Moreover, every existing particle has an *intrinsic (or existential)* mass m_0 based on its origin and composition. The mass m of a particle at any instant depends on its intrinsic mass and velocity $\vec{u}$ at that instant, and is given by $m := m(m_0, \vec{u})$.

Remark 2. While m is always greater than m_0 according to Eq.(22), it can be smaller or greater than m_0 according to Eq.(21). Thus, mass can decrease with speed. Eq.(23) is the extended expression of mass varying with velocity.

Remark 3. Because there is no singularity (divergence) issue as seen from Eq.(21), speed of the particle can, in principle, exceed the speed of light in vacuum. We, therefore, propose that there exists an ultimate (maximum theoretical possible) speed $C \geq c$. Correspondingly, the Lorentz factor modifies to $\gamma(u) = \left(1 - u^2/C^2\right)^{-\frac{1}{2}}$ and the speed of a particle is constrained to $0 < u < C$. The issue of singularity is also automatically resolved with this conception. Moreover, $m \to 0$ when $u_{10} << u_1 \to C$. This result is quite interesting and reasonable as it explains why neutrinos travelling nearly at the speed of light have negligible masses. We also argue below that a photon has a nonzero (existential) mass.

Remark 4. We are familiar with Galilean and Lorentz transformations. There exist, however, several other space-time transformations depending on different physical (and mathematical) requirements. The principle of relativity together with Galilean transformation also provides velocity-dependent mass. For velocity-dependent masses obtained using Galilean and other space-time transformations, see Table I. Thus, Lorentz transformation or SR is not sacred anymore in this regard.

Remark 5. Does mass vary with velocity? In our formulation m_0 is not the rest mass. It is invariant and is characteristic of the existence of a particle. The velocity of a particle can depend on several factors such as its composition, medium in which it resides, and the external forces imparted to it. We are of the opinion that the change in mass with velocity is apparent, and is a measure of inertia (analogous to workfunction or barrier potential) [70, 71] and/or energy. While the solution of homogeneous equation ensures that the boundary conditions are satisfied, the particular solution ensures that the inhomogeneous equation is satisfied. The particular solutions are characteristics of the space-time transformations.

V. ON PHOTON MASS

In this section, we argue that a photon also has a nonzero (existential) mass depending on its origin [72], though it may be quite insignificant.

Photons are regarded as quanta (elementary particles) of light. Because a photon is never found at rest, its rest mass is postulated zero. Its relativistic mass, $m = m_0\gamma(c)$, is then undetermined. It is believed that photons follow a unique set of rules that are different

Transformation	$m'a' = ma$	$\frac{\mathrm{d}(m'u')}{\mathrm{d}t'} = \frac{\mathrm{d}(mu)}{\mathrm{d}t}$
Galilean $x' = x - vt$ $y' = y, \quad z' = z$ $t' = t$	$m'_k = m_k$ $(k = 1, 2, 3)$	complementary: $m_k u_k = m_{k0} u_{k0}$ particular: $m_k = m_{k0}\left(1 + \frac{u_{k0}}{u_k}\right)$ $(k = 1, 2, 3)$
Lorentz [3] $x' = \gamma(v)(x - vt)$ $y' = y, \quad z' = z$ $t' = \gamma(v)(t - \frac{v}{C^2}x)$	$m_1 = m_{10}\gamma^3(u_1)$	complementary: $m_1 u_1 = m_{10} u_{10}$ particular: $m_1 = m_{10}\gamma(u_1)$
Voigt [56] $x' = x - vt$ $y' = y/\gamma(v), \quad z' = z/\gamma(v)$ $t' = t - \frac{v}{C^2}x$	$m_1 = m_{10}\gamma^3(u_1)$	complementary: $m_1 u_1 = m_{10} u_{10}$ particular: $m_1 = m_{10}\gamma(u_1)$
Selleri [58] $x' = \gamma(v)(x - vt)$ $y' = y, \quad z' = z$ $t' = t/\gamma(v)$	$m'_1 = m_1\gamma^3(v)$ $m'_k = m_k\gamma^2(v)$ $(k = 2, 3)$	$m'_1 + u'_1\frac{\mathrm{d}m'_1}{\mathrm{d}u'_1} = \left(m_1 + u_1\frac{\mathrm{d}m_1}{\mathrm{d}u_1}\right)\gamma^3(v)$ $m'_k + u'_k\frac{\mathrm{d}m'_k}{\mathrm{d}u'_k} = \left(m_k + u_k\frac{\mathrm{d}m_k}{\mathrm{d}u_k}\right)\gamma^2(v)$ $(k = 2, 3)$
Edwards [60] $x' = \gamma(v)(x - vt)$ $y' = y, \quad z' = z$ $t' = \gamma(v)[(1 + a\frac{v}{C})t - (\frac{v}{C^2} + \frac{a}{C})x]$	...	...
Generalized $x' = a_1(x - v_1 t)$ $y' = a_2(y - v_2 t)$ $z' = a_3(z - v_3 t)$ $t'_1 = a_0\left[\left(1 + \frac{\vec{b}.\vec{v}}{C}\right)t - \sum_{k=1}^{3} e_k\left(\frac{v_k}{C^2} + \frac{b_k}{C}\right)x_k\right]$ $t'_2 = a_0\left[\left(1 + \frac{\vec{b}.\vec{v}}{C}\right)t - \sum_{k=1}^{3}\left(\frac{e_k v_k}{C^2} + \frac{b_k}{C}\right)x_k\right]$	...	...
Symmetric [92] $x'_k = \gamma(v_k)(x_k - v_k t_k)$ $t'_k = \gamma(v_k)(t_k - \frac{v_k}{C^2}x_k)$	$m_k = m_{k0}\gamma^3(u_k)$ $(k = 1, 2, 3)$	complementary: $m_k u_k = m_{k0} u_{k0}$ particular: $m_k = m_{k0}\gamma(u_k)$ $(k = 1, 2, 3)$

TABLE I. Velocity-dependent masses obtained using invariance of Newton's second law for various space-time transformations. Here $\gamma(\xi) = \left(1 - \xi^2/C^2\right)^{-\frac{1}{2}}$ with $C \geq c$ and c is the speed of light in vacuum. A careful observation reveals that space and time coordinates of these transformations are related. Note that Lorentz $\equiv \gamma(v)$Voigt, and Edwards transformations reduce to Lorentz and Selleri transformations, respectively, for $a = 0$ and $a = -\frac{v}{C}$. Furthermore, Galilean, Lorentz, Voigt, Selleri and Edwards transformations are special cases of the generalized transformation for appropriate values of parameters (a_1, a_2, a_3, a_0), (b_1, b_2, b_3), (e_1, e_2, e_3), and velocities $(v_1 = v, v_2 = 0, v_3 = 0)$. m_{k0} is the intrinsic mass of a particle along the spatial coordinate x_k. It must be invariant ($m_{k0} = m_0$). The velocity-dependent mass m_k of the particle along different spatial coordinates may be different suggesting that relativistic mass should be treated as a vector quantity. The particular solutions are characteristics of the space-time transformations. The Galilean and Lorentz transformations are equivalent with regard to the relativity of mass, and hence the concept of velocity-dependent mass is not unique to Lorentz transformation or SR. While Galilean, Lorentz, Voigt, and symmetric transformations suggest the presence of an inherent (existential) mass, Selleri transformation relates the masses in two inertial frames.

from those governing massive particles. But light can be slowed down, captured or freezed [73–76]. Moreover, it can be converted into a *supersolid* [77]. A supersolid acts as both a solid and a fluid and has zero viscosity.

The four fundamental forces (interactions) in nature can be understood in terms of the exchange of force-carrying particles (spin-1 bosons). The symmetry property of the gauge interactions requires that spin-1 gauge bosons must be massless. This is fine with photons (electromagnetic force) and gluons (strong nuclear force) but is in conflict with massive $W-$ and $Z-$bosons of weak nuclear force. To preserve the symmetries of gauge theories, Higgs introduced the mechanism of spontaneous symmetry breaking and predicted the existence of a spin-0 uncharged particle, the Higgs boson, which is responsible for masses of the particles. But what if all elementary particles, including the force-mediating ones, have nonzero mass? All gauge interactions will be on equal footing.

A photon has energy ($E = h\nu$) and momentum ($p = E/c$). Note that this expression for momentum cannot be obtained from the energy-momentum relation

$$E^2 = p^2c^2 + m_0^2c^4 \tag{24}$$

by substituting $m_0 = 0$. This is because $m_0 = 0$ implies $p = mv = \gamma(v)m_0 v = \frac{0}{0}$ and $E = \frac{0}{0}$. That a photon has momentum is derived by using the de Broglie wave-particle duality: every matter particle is associated with a wave whose wavelength is given by $\lambda = \frac{h}{p}\left(= \frac{h}{mv}\right)$.

Thus, $E = h\nu = pc$. This suggests that photon is a matter particle and should have an inherent mass. Now considering the equivalence of mass and energy (assuming there exists a mechanism and a machine with 100 percent efficiency to convert matter into energy!), photon has a nonzero mass given by

$$m_0(\lambda) = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{\lambda c}. \tag{25}$$

The mass of a photon corresponding to the wavelength $\lambda = 7000$ nm is $m_0 = 3.159 \times 10^{-36}$ kg (roughly one million times smaller than the electron's mass). Light is "heavy" and several theoretical limits on photon mass have been proposed using various approaches [78–91]. For such a vanishingly small limiting mass, $(m_0c^2)^2 \to 0$, and Eq.(24) essentially yields $E \sim pc$. Photons will respect Eq.(24) if they have a nonzero mass.

VI. CONCLUSION

In summary, we obtained the velocity-dependent masses using the invariance of Newton's second law employing various space-time transformations. We found that even Galilean transformation renders mass to vary with velocity. The concept of velocity-varying mass is thus not unique to Special Relativity. As far as relativity of mass is concerned, all inertial transformations are equivalent. As byproducts, we observed that a particle is never entirely at rest, and has an intrinsic (existential) mass that remains invariant. The change in mass with velocity is apparent, and superluminal communication is possible. We also argued that photons (and all other fundamental entities) possess mass. Apart from shedding valuable insights in foundations of physics, our approach is also useful from the pedagogic viewpoint.

[1] I. Newton, *Philosophiae Naturalis Principia Mathematica* (G. Brookman, 1833).

[2] A. A. Michelson and E. H. Morley, *On the Relative Motion of the Earth and the Luminiferous Ether*, Am. J. Sci. **34**, 333-345 (1887).

[3] H. A. Lorentz, *Electromagnetic Phenomena in a System Moving with Any Velocity Smaller than that of Light*, Proc. KNAW **6**, 809-831 (1904).

[4] A. Einstein, *Zur Elektrodynamik Bewegter K´orper [On the Electrodynamics of Moving Bodies]*, Ann. Phys. **322(10)**, 891-921 (1905).

[5] M. Saha and S. Bose, *The Principle of Relativity: Original papers by A. Einstein and H. Minkowski* (1920).

[6] P. G. Bergmann, *Introduction to the Theory of Relativity* (Courier Corporation, 1976).

[7] R. Resnick, *Introduction to Special Relativity* (John Wiley & Sons, Singapore, 1998).

[8] Lorentz transformations for an arbitrary boost: Let $\vec{v} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$ be the relative velocity and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} = \vec{r}_\parallel + \vec{r}_\perp = (\vec{r}.\hat{v})\,\hat{v} + \hat{v} \times (\vec{r} \times \hat{v})$ be the position vector in $\mathcal{S}$, and $\vec{r'}$ be the position vector in $\mathcal{S}'$. Then the Lorentz transformations is given by

$$\vec{r}_\parallel = \gamma(v)\left(\vec{r'}_\parallel + t'\vec{v}\right), \quad \vec{r}_\perp = \vec{r'}_\perp,$$
$$t = \gamma(v)\left(t' + \frac{\vec{v}.\vec{r'}}{c^2}\right),$$
$$\vec{r} = \vec{r}_\parallel + \vec{r}_\perp = \vec{r'} + \gamma(v)t'\vec{v} + (\gamma(v) - 1)\left(\vec{r'}.\hat{v}\right)\hat{v}.$$

[9] F. Goy, *A derivation of three-dimensional inertial transformations*, Found. Phys. Lett. **10**, 329-346 (1997).

[10] J. T. Cushing, *Electromagnetic mass, relativity, and the Kaufmann experiments*, Am. J. Phys. **49**, 1133-1149 (1981).

[11] M. Planck, *Das Prinzip der Relativitt und die Grundgleichungen der Mechanik [The Principle of Relativity and the Fundamental Equations of Mechanics]*, Verhandlungen Deutsche Physikalische Gesellschaft **8**, 136-141 (1906).

[12] A. Beiser, *Perspectives of modern Physics* (Singapore: McGraw-Hill Book Company, 1987).

[13] R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Vol. 1, pp 16-6 to 16-8 (New Delhi: Narosa Publishing House, 1989).

[14] D. Landau and E. M. Lifshitz, *The classical Theory of Fields*, pp 24-25 (Oxford: Pergamon Press, 1971).

[15] B. Dikshit, *A new approach in the derivation of relativistic variation of mass with speed*, Eur. J. Phys. **36**, 035005 (2015).

[16] J. M. Hill and B. J. Cox, *Generalised Einstein mass-variation formulae: I Subluminal relative frame velocities*, Results Phys. **6**, 112-121 (2016).

[17] J. M. Hill and B. J. Cox, *Generalised Einstein mass-variation formulae: II Superluminal relative frame velocities*, Results Phys. 6, 122-130 (2016).

[18] A. Sharma, *The Various Equations of Variation of Mass with Velocity*, Ind. J. Sci. Tech. 10(21), 1-10 (2017).

[19] P. Beckmann, *Einstein Plus Two* (Golem Press, Boulder, CO 1987).

[20] I. McCausland, *The Persistent Problem of Special Relativity*, Phys. Essays **18(4)**, 530-532(3) (2005).

[21] R. Hatch, *Escape from Einstein* (Kneat Kompany, Wilmington, CA, 1992).

[22] F. Selleri, *Open Questions in Relativistic Physics* (Apeiron, Montreal, 1998).

[23] H. Dingle, *Science at the Crossroads* (Martin Brian and Okeeffe, London, 1972).

[24] R. A. Monti, *Theory of Relativity: A Critical Analysis*, Phys. Essays **9(2)**, 238-260 (1996).

[25] A. Kelly, *Challenging Modern Physics: Questioning Einstein's Relativity Theories* (Brown Walker Press, Boca Raton, 2005).

[26] P. Marmet, The Overlooked Phenomena in the Michelson-Morley Experiment (2004).

[27] I. Perez, *The Physics Surrounding the Michelson-Morley Experiment and a New Æther Theory*, arXiv:1004.0716 (2010).

[28] M. Consoli and A. Pluchino, *Michelson-Morley Experiments: at the crossroads of Relativity, Cosmology and Quantum Physics*, Int. J. Mod. Phys. A **38(35n36)**, 2330017 (2023).

[29] M. Maggiore, *Zero-point quantum fluctuations and dark energy*, Phys. Rev. D **83**, 063514 (2011).

[30] A. Clerk, *Seeing the "Quantum" in Quantum Zero-Point Fluctuations*, Physics **5**, 8 (2012).
[31] J. Magueijo, *New varying speed of light theories*, Rept. Prog. Phys. **66**, 2025 (2003).
[32] W. von Ignatowsky, *Das Relativittsprinzip*, Arch. Math. Phys. **17(1)**, (1911).
[33] R. Weinstock, *New approach to special relativity*, Am. J. Phys. **33**, 540-645 (1965).
[34] V. Mitavalsky, *Special relativity without the postulate of constancy of light*, Am. J. Phys. **34**, 825 (1966).
[35] V. Berzi and V. Gorini, *Reciprocity principle and the lorentz transformations*, J. Math. Phys. **10**, 1518 (1969).
[36] A. R. Lee and T. M. Kalotas, *Lorentz transformations from the first postulate*, Am. J. Phys. **43**, 434-437 (1975).
[37] J.-M. Levy-Leblond, *One more derivation of the Lorentz transformation*, Am. J. Phys. **44**, 271-277 (1976).
[38] A. M. Srivastava, *Invariant speed in special relativity*, Am. J. Phys. **49**, 504-505 (1981).
[39] N. D. Mermin, *Relativity without light*, Am. J. Phys. **52**, 119-124 (1984).
[40] H. M. Schwartz, *Deduction of the general Lorentz transformations from a set of necessary assumptions*, Am. J. Phys. **52**, 346-350 (1984).
[41] H. M. Schwartz, *A simple new approach to the deduction of the Lorentz transformations*, Am. J. Phys. **53**, 1007-1008 (1985).
[42] S. Singh, *Lorentz transformations in Mermins relativity without light*, Am. J. Phys. **54**, 183-184 (1986).
[43] A. Sen, *How Galileo could have derived the special theory of relativity*, Am. J. Phys. **62**, 157-162 (1994).
[44] P. B. Pal, *Nothing but Relativity*, Eur. J. Phys. **24**, 315-319 (2003).
[45] J. Q. Shen, *Lorentz, Edwards transformations and the principle of permutation invariance*, arXiv:physics/0509012.
[46] M. J. Feigenbaum, *The Theory of Relativity–Galileo's Child*, arXiv:0806.1234 (2008).
[47] F. R. Tangherlini, *Galilean-Like Transformation Allowed by General Covariance and Consistent with Special Relativity*, J. Mod. Phys. **5(5)**, 230-243 (2014).
[48] A. Pelissetto and M. Testa, *Getting the Lorentz transformations without requiring an invariant speed*, Am. J. Phys. **83**, 338-340 (2015).
[49] A. Galiautdinov, *Derivation of the Lorentz transformation without the use of Einstein's second postulate*, arXiv:1701.00270 (2017).
[50] H. Guo, *A new paradox and the reconciliation of Lorentz and Galilean transformations*, Synthese **199**, 81138142 (2021).
[51] S. Datta, *A Revisit to Lorentz Transformation without Light*, arXiv:2212.03706 (2022).
[52] D. V. Redžić, *Relativity with or without light and Maxwell*, arxiv:2404.19566 (2024).
[53] R. M. Kassir, *The Critical Error in the Formulation of the Special Relativity*, Int. J. Phys. **2(6)**, 197-201 (2014).
[54] R. J. Buenker, *Proof That the Lorentz Transformation is Incompatible with the Law of Causality*, East African Scholars J. Eng. Comput. Sci. **5(4)**, 53-54 (2022).
[55] S. P. Asokan, *Special Relativity sans Lorentz Transformation (OR) Perceptional Relativity*, Research Square (2021).
[56] W. Voigt, *Ueber das Dopplersche Princip*, Nachr. Ges. Wiss. Gottingen **8**, 41-51 (1987).
[57] R. Heras, *A review of Voigt's transformations in the framework of special relativity*, arXiv:1411.2559.
[58] F. Selleri, *Noninvariant One-Way Velocity of Light*, Found. Phys. **26(5)**, 641-664 (1996).
[59] F. Selleri, *Inertial Transformations: A Review* in S. Jeffers, S. Roy, J. P. Vigier, G. Hunter (eds) The Present Status of the Quantum Theory of Light, Fundamental Theories of Physics **80**, pp 413-435, Springer, Dordrecht, 1997.
[60] W. F. Edwards, *Special relativity in anisotropic space*, Am. J. Phys. **31**, 482-490 (1963).
[61] C. G. Adler, *Does mass really depend on velocity, dad?*, Am. J. Phys. **55**, 739-743 (1987).
[62] C. Whitney, *Does Mass Really Increase?*, Galilean Electrodyn. **16(3)**, 57-58 (2005).
[63] L. B. Okun, *The Einstein formula: $E_0 = mc^2$ "Isn't the Lord laughing?"*, Phys.-Usp. **51(5)**, 513-527 (2008).
[64] A. Einstein, *An Elementary Derivation of the Equivalence of Mass and Energy*, Bull. Amer. Math. Soc. **41(4)**, 223-230 (1935).
[65] D. J. Steck and F. Rioux, *An elementary development of mass-energy equivalence*, Am. J. Phys. **51(5)**, 461-462 (1983).
[66] F. Rohrlich, *An elementary derivation of $E = mc^2$*, Am. J. Phys. **58**, 348-349 (1990).
[67] L. Peliti, *Elementary derivation of the expressions of momentum and energy in special relativity*, arXiv:1504.07287.
[68] A. V. Nenashev, S. D. Baranovskii, and F. Gebhard, *A pedestrian approach to Einstein's formula $E = mc^2$ with an application to photon dynamics*, arXiv:2308.02612.
[69] A. Sharma, *The Origin of the Generalized Mass-Energy Equation $E = Amc^2$ and Its Applications in General Physics and Cosmology*, Phys. Essays **17**, 195-222 (2004).
[70] V. M. Sabarish, *Apparent Weight of a Photon Box–Revisiting Mass-Energy Equivalence*, Resonance **25(7)**, 911-914 (2020).
[71] Inertia is defined as a fundamental property of matter that describes an object's resistance to changes in its state of motion (velocity). It is regarded inherent to mass. The more mass an object has, the more inertia it possesses. But this notion needs to be refined. The resistance to acceleration of a particle by its surrounding (including external forces) should be regarded as inertia. Put differently, inertia is the measure of interactions with its surrounding.
[72] In fact, we postulate that every existing entity has a nonzero existential mass!
[73] A. B. Matsko, O. kocharovskaya, Y. Rostovtsev, G. R. Welch, A. S. Zibrov, and M. O. Scully, *Slow, Ultraslow, Stored, and Frozen Light*, Adv. At. Mol. Opt. Phys. **46**, 191-242 (2001).
[74] J. B. Khurgin, *Slow light in various media: a tutorial*, Adv. Opt. Photon. **2(3)**, 287-318 (2010).
[75] I. Georgescu, *Frozen light switch*, Nature Phys. **8**, 252 (2012).
[76] M. Fleischhauer and G. Juzelinas, *Slow, Stored and Stationary Light* In: M. Al-Amri, M. El-Gomati, M. Zubairy (eds) Optics in Our Time, pp 359-383, Springer, Cham (2016).
[77] D. Trypogeorgos, A. Gianfrate, M. Landini *et al.*, *Emerging supersolidity in photonic-crystal polariton condensates*, Nature (2025).
[78] I. Y. Kobzarev and L. B. Okun, *On the photon mass*, Soviet Physic Uspekhi, Usp. Fiz. Nauk **95**,131-137 (1968).
[79] P. A. Franken and G. W. Ampulski, *Photon Rest Mass*, Phys. Rev. Lett. **26**, 115 (1971).
[80] L. B. Okun, *The Concept of Mass*, Phys. Today **42(6)**, 31-36 (1989).
[81] R. Lakes, *Experimental limits on the photon mass and cosmic magnetic vector potential*, Phys. Rev. Lett. **80(9)**, 1826-1829 (1998).

[82] L.-C. Tu and J. Luo, *Experimental tests of Coulomb's Law and the photon rest mass*, Metrologia **41**, S136-S146 (2004).

[83] L.-C. Tu, J. Luo, and G. T. Gillies, *The mass of the photon*, Rep. Prog. Phys. **68**, 77-130, (2005).

[84] D. D. Ryutov, *Using plasma physics to weigh the photon*, Plasma Phys. Control. Fusion **49(12B)**, B429-B438 (2007).

[85] A. S. Goldhaber and M. M. Nieto, *Photon and Graviton Mass Limits*, arXiv:0809.1003v5.

[86] A. D. Roy, *On the massive nature of photon*, Int. J. Sci. Eng. Res. **5(3)**, (2014).

[87] G. G. Nyambuya, *Are Photons Massless or Massive?*, J. Mod. Phys. **5**, 2111-2124 (2014).

[88] M. B. van der Mark and G. W. 't Hooft, *Light is Heavy*, arXiv:1508.06478 (2015).

[89] C. Marchal, *LIGO: The hypothesis of photons with nonzero rest mass*, J. Phys.: Conf. Series **1251**, 012034 (2019).

[90] A. M. K. Ahmed, M. A. M. Yousif, Z. M. Kurawa *et al.*, *Determination of Photon and Elementary Particles Rest Masses Using Maxwells Equations and Generalized Potential Dependent Special Relativity*, Nat. Sci. **12(8)**, 588-598 (2020).

[91] M. Goray and R. N. Annavarapu, *Rest mass of photon on the surface of matter*, Results Phys. **16**, 102866 (2020).

[92] Symmetric transformations and three temporal dimensions: The invariance of momentum along the directions perpendicular to the boost results in a velocity-dependent mass which is surprisingly independent of the velocity along that direction. Because space and time are entangled as spacetime and are treated on equal footing, there should be three temporal dimensions (apart from past, present and future) corresponding to three spatial dimensions. The idea of multiple temporal dimensions has been introduced before [93–95]. These arguments lead us to propose a symmetric space-time framework.

[93] I. Bars and C. Kounnas, *Theories with two times*, Phys. Lett. B **402(1-2)**, 25-32 (1997).

[94] J. G. Foster and B. Müller, *Physics With Two Time Dimensions*, arXiv:1001.2485 (2010).

[95] S. Zahir, *Extra Time Dimension: Deriving Relativistic Space-Time Transformations, Kinematics, and Example of Dimensional Compactification Using Time-Dependent Non-Relativistic Quantum Mechanics* , J. Mod. Phys. **14**, 1333-1354 (2023).