The method of dividing the 60° angle into three equal parts

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In the field of modern mathematics, the 60 ° angle trisecting problem has long been a classic geometric problem that has attracted much attention, and its essence is closely related to the infinite extension of trigonometric functions in the generalized dimension. Through in-depth research, it was found that the solution to this problem exists within the two-thirds interval between r and r, where r corresponds to the shape of a curve and two-thirds of r corresponds to the shape of a horizontal straight line. This article explores the inherent connections between geometric shapes from an innovative perspective and successfully constructs a new method for accurately dividing 60 ° angles into three equal parts. This method not only breaks through the limitations of traditional geometric thinking, but also has a high degree of scalability, which can be effectively extended to the problem of trisecting at any angle less than 180 ° within the two-thirds interval of r and r. It provides a new idea and solution paradigm for the theory and practical application of angle trisecting, and is expected to promote further development in related fields.

Keywords: 60° angle trisection; Bold attempt; Between r and two-thirds of r; trigonometric function

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Introduction

Since the ancient Greek period in BC, the problem of trisecting angles has been one of the three major problems in classical geometry, occupying a unique position in the long history of mathematical development. Mathematicians have always been committed to exploring the method of accurately dividing any angle into three equal parts using only scale-free rulers and compasses (using rulers to draw). Although the Galois theory rigorously proved in the 19th century that there is no general solution to the problem under the constraints of ruler and gauge drawing, this conclusion did not extinguish the exploration enthusiasm of mathematical researchers. Instead, it prompted people to break through traditional frameworks and conduct research from multiple dimensions such as algebra, topology, and computational geometry, thereby promoting the development of important mathematical branches such as modern algebra and domain theory.

This article takes a different approach and innovatively constructs a new method of angle trisecting based on the geometric characteristics of circles and isosceles trapezoids. Unlike the constraints of traditional ruler and gauge drawing, this method utilizes the combination and transformation of geometric shapes as tools to achieve precise division of angles by mining the inherent connections and dynamic changes between shapes. This exploration is not only expected to provide new solutions to classical geometry problems, but also inject new vitality into modern geometry research. We kindly request readers to explore this geometric proposition that spans thousands of years in depth with this article, and witness the modern breakthrough of classical problems.

Literature review

Mathematician Wanzel once proved that it is impossible to directly make a trisecting angle. The reason is that cos20° is a root of the equation ($8x^3-6x-1=0$) (derived from the triple angle formula). This equation is a cubic equation, and its solution cannot be expressed by finite square root operations. Therefore, it is not possible to determine the length of the line segment at cos20°, but he ignored the chord of the circle, which also represents the circle(Pierre Wantzel,1837).

Lindemann proved the transcendence of π, indirectly supporting the unsolvable trisecting angle. His work shows that many problems related to ruler and gauge drawing, such as squaring circles, involve transcendental numbers and cannot be solved through ruler and gauge drawing.(lindemann, 1882).

Cos20° is a root of the equation ($8x^3-6x-1=0$) (derived from the triple angle formula).This equation is a cubic equation, and its solution cannot be expressed by finite square root operations.But my answer is not contradictory to the views of two famous scholars, because my starting point is the properties of circular chords and isosceles trapezoids.

Result

60° divide into three equal parts

Due to the need to reflect that the operation is carried out in reality, paper images are used. This article will use the simplest method to draw to ensure that it is not too complicated.

(1) As shown in Figure 1, draw a circle with radius r and dot O, and then draw a 60 ° equilateral triangle inside the circle, which is Δ ABC. Divide the BC edge into two parts, with the dividing point being N, which is connected to vertex A and extended to M to form the dividing line NAM;

(2) Divide ∠ B and ∠ C into four parts, each at a 45° angle, with three parts forming one angle at 45° and the other angle also at 45°. The three-quarters of the two angles intersect at point A', connecting three points to form an isosceles right triangle (180-45-45=90). Connect A'A and O'B to obtain angle AA'B, then divide 90° into three equal parts to obtain an angle of 30°, and obtain points E and F of the three equal parts;

(3) Draw a circle with A'B and A'C as the centers, and the intersection point of the arc and the bisector NAM is A". Connect A" C and A"B to obtain the angle BA" C. Because the central angle is twice the circumference angle of the circle, ∠ BA"C is 45°. Then divide the 45° angle into three equal parts to obtain a 15° angle, and obtain the points E' and F' of the three equal parts. The same applies to the 180 ° divide, where the three are in a straight line;

(4) The center of the 60° angle is between the center of the 45° angle and the center of the 90° angle;connect point E, point F, point E', and point F' to form an isosceles trapezoid, which expands from the trisecting segment EF to the remaining trisecting segment E'F'. As the angle decreases, the chord length of its three equal angles also decreases. The chord lengths of the three equal angles are the EF lines of the isosceles trapezoid. The straight line of the middle string continuously shrinks as the angle decreases. You can imagine having a semicircle with a constant chord length. The angle of the semicircle decreases continuously and becomes a straight line, while the chord length of the three equal angle changes from r to two-thirds r.

(5) Draw a circle with A as the center and AB as the radius. The intersection points with the isosceles trapezoid are E"and F", which are also the 60° bisector points. Obtain the line segment E"F" that divides into three equal parts. It is an isosceles trapezoid that divides 45 ° and 90 ° equally, and this trapezoid naturally has equal division. You can draw a semicircle and divide it into three equal parts, with the 45 °, 90 °, and 180 ° points on a straight line.

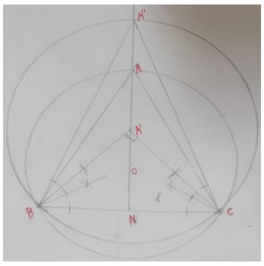

Figure 1

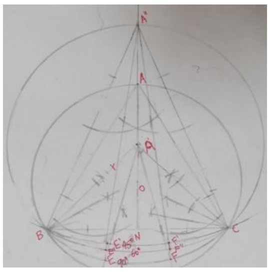

Figure 2

Any angle divided into any equal parts

To summarize, divide the points into three equal parts at 90 ° and 45 °, and connect the four points to construct a trapezoid. From Figure 3, it can be seen that there are two sides with an extension angle of ∠X (∠X less than 180°; The purpose of extension is to give different angles the same chord length) resulting in an isosceles triangle B'OC'. Make a vertical bisector so that BC equals B'C '. Then draw a circle with OB as the radius, where the arc intersects with the trapezoid. The arc and trapezoid intersect at two points, and connecting the two points creates a line segment that is a three equal part of any angle.

Then we need to start dividing it into any portion at any angle.

If there are Y equal divisions (odd and even divisions), you can first draw X small angles with angles of Y. Combine the Y small angles into a large angle (less than 180 °), as shown in Figure 4. Find the chord length at that angle. Divide the chord length into X, connect four points, and construct an isosceles trapezoid (must be converted to odd numbers, even numbers cannot be in the middle). You can imagine a 180 ° semicircle with a fixed chord length (AB). As the angle decreases, the equal chord length becomes shorter and shorter. Finally, it becomes one quarter of the chord length at large angles. If it is divided into 10 equal parts, then divide by 2 and place the double corners in the middle. The chord length of even bisectors is NF. The chord length of an odd equal division is EF (EF is divided into five equal parts).

In the case where point O moves in the opposite direction along the perpendicular bisector of chord BC while keeping the length of chord BC constant, the radius and ∠BOC change: As point O moves in the opposite direction, to ensure a constant chord length BC, the circle needs to become larger to "accommodate" chord BC, so the radius gradually increases. According to the chord length formula $BC=2r\sin(\angle BOC\div 2)$, if the chord length BC remains constant and the radius r increases while the value of sin ∠BOC÷2 decreases, as $0<\angle BOC<\angle 180°$, ∠BOC÷2 decreases, and then ∠BOC gradually decreases. After the point becomes infinitely small, two infinitely large radii are parallel, and the curve becomes a straight line. You can understand it as: when the radius is infinite and the angle is zero, the chord on the arc coincides with the n-line segment.

The n-equal division of string BC: The length of the n-equal division string will gradually shorten as the O point moves in the opposite direction, the radius increases, and the ∠BOC decreases. When the radius approaches infinity and the ∠BOC approaches 0°, the n-equal chord length approaches 1÷n of the chord length BC. When the ∠ BOC is equal to 0°, the graph degenerates into a straight line BC, and at this point, the n-equal point divides BC into n segments of length exactly 1÷n of the chord length BC.

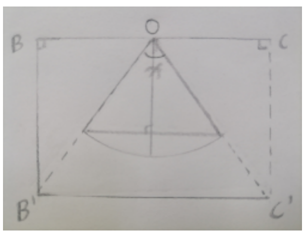

Figure3

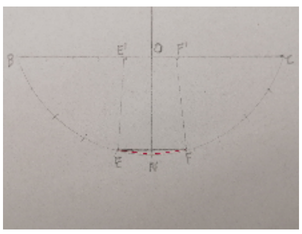

Figure4

The chord length of a three equal circle

The vertex angles of the three isosceles triangles are 45°, 60°, and 90°, respectively. The base is the same, both are $\sqrt{3}$. The top corner corresponds to the bottom edge. The ratio of the waist lengths of three isosceles triangles (expressed as trigonometric functions) is $(\sqrt{3}r\sin 67.5^\circ/\sin 45):\sqrt{3}r:(\sqrt{3}r\sin 45^\circ/\sin 90^\circ)$. Then draw a circle with the waist circumference of the three triangles as the radius, and calculate the chord lengths corresponding to 15°, 20°, and 30°:$[(\sqrt{3}r\sin 67.5^\circ/\sin 45^\circ)\times(\sin 82.5^\circ/\sin 15^\circ]:[\sqrt{3}r\times(\sin 80^\circ/\sin 20^\circ)]:[(\sqrt{3}r\sin 45^\circ/\sin 90^\circ)\times(\sin 75^\circ/\sin 30^\circ)$).

Discussion

This study successfully achieved three equal angles through [specific methods], breaking through the limitations of traditional ruler and gauge drawing theory and providing a new perspective for the development of geometry. The methods used in the study provide new ideas for solving other mathematical problems and have potential application value in fields such as engineering design and computer graphics.

However, this method requires high tools and computational conditions in practical applications, and has not yet been fully integrated with traditional geometric systems. Future research can focus on reducing method complexity, exploring its applications in more fields, and further improving the relevant theoretical system.

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