The contradiction between the law of universal gravitation and the second law of Newton. Revision 2

Victor Strohm

independent researcher

vfstrohm@yahoo.de

Abstract

The motion of bodies along an ellipse with a constant sector velocity is considered. At perihelion, the velocity is greater than at aphelion. The radii are the opposite: at perihelion, the radius is smaller than at aphelion. Let's calculate the tangential and centripetal acceleration. Let's calculate the force using Newton's second law and the law of universal gravitation. The forces are equal in magnitude, but opposite in direction, Fig. 2. Contradictions are eliminated if we consider the momentum of the system.

Keywords: Kepler's and Newton's laws, force, acceleration.

According to Kepler's first and second laws, planets move along an ellipse with a constant sector velocity relative to the center of mass, point *C*, Fig. 1. This means that at perihelion, point *P*, the velocity is greater than at aphelion, point *A*, and $r_a > r_p$, where r_a is the radius from the center of mass at aphelion, r_a is the radius from the center of mass at perihelion.

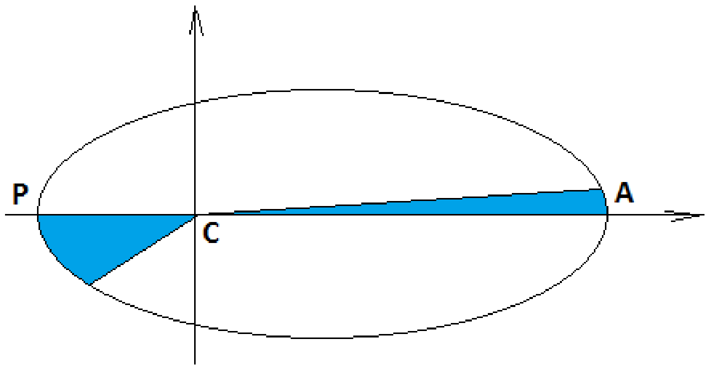


Fig. 1

There are two pairs of objects in the Solar system with a barycenter significantly removed from the centers of the objects. These are Jupiter - the Sun, the Moon - the Earth.

Let's calculate the parameters of the motion of a mathematical point along second-order curves in accordance with Kepler's laws using equation (1) for the Moon - Earth system:

$$\ddot{\varphi} = \frac{2*e*\sin(\varphi(t))*\dot{\varphi}^2}{1-e*\cos(\varphi(t))} \quad (1)$$

The barycenter is in the left focus. The point starts moving from aphelion.

The acceleration formulas from equation (1) are shown in Appendix [A1]. The properties of the equation are discussed in the article [1].

We construct a graph of forces, Fig. 1, based on the calculation results, Table [A3].

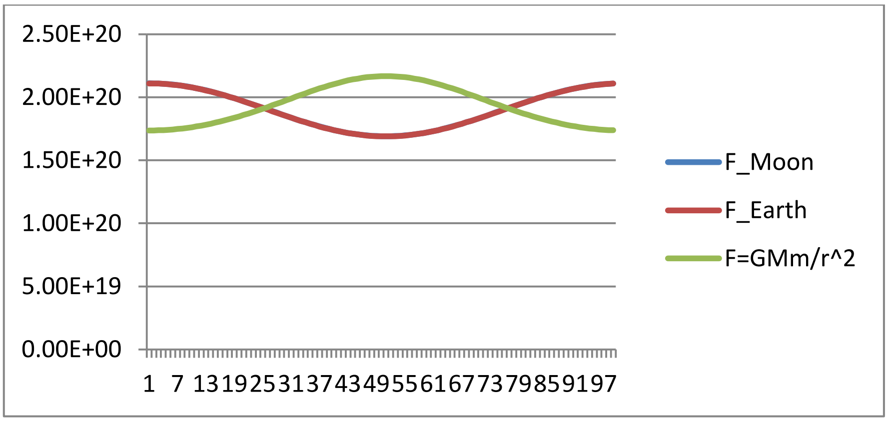


Fig. 2

Graph of accelerations in the Moon's orbit, Fig. 3.

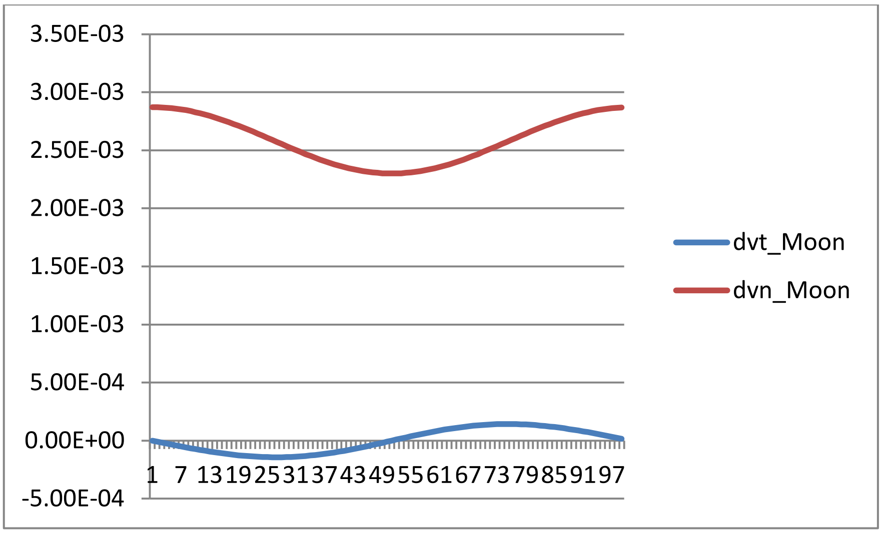


Fig. 3

Graph of accelerations in the orbit of the Earth's center, Fig. 4.

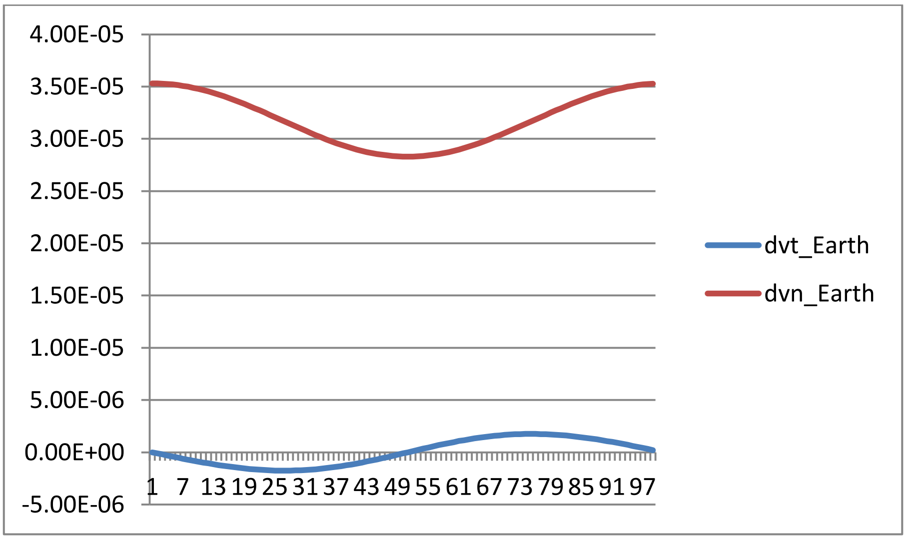


Fig. 4

dvt_Moon - tangential acceleration of the Moon, dvn_Moon - normal acceleration of the Moon.

dvt_Earth - tangential acceleration of the Earth, dvn_Earth - normal acceleration of the Earth.

The normal acceleration modules at perihelion are less than at aphelion, Fig. 3–4.

Modules of average forces:

$m_{Moon}, \dot{v}_{n\_Moon}$ – mass and normal acceleration of the Moon

$m_{Earth}, \dot{v}_{n\_Earth}$ – mass and normal acceleration of the Earth

m_{Moon} =7.3477e+22 kg, m_{Earth} =5.9726e+24 kg

G = 6.67418·10^{-11} m^3·s^{-2}·kg^{-1} – gravitational constant.

$$F_{Moon} = m_{Moon}\dot{v}_{n\_Moon} = 1.89934e + 20 \text{ кг·м·с}^{-2} \quad (2)$$

$$F_{Earth} = m_{Earth}\dot{v}_{n\_Earth} = 1.89934e + 20 \text{ кг·м·с}^{-2} \quad (3)$$

$$F = \frac{GMm}{r^2} = 1.93869e + 20 \text{ кг·м·с}^{-2} \quad (4)$$

$$|F| \approx |F_{Moon}| = |F_{Earth}| \quad (5)$$

$$\boldsymbol{F} \neq \boldsymbol{F}_{Moon} = \boldsymbol{F}_{Earth} \quad (6)$$

The force modules $|F| \approx |F_{Moon}|$ are approximately equal, with a difference of ≈ 2%.

$r = r_{Moon} + r_{Earth}$ – distance between the centers of the Moon and the Earth

r_{Moon} , r_{Earth} – radii from the barycenter to the center of the Moon and the Earth

As we can see from the formulas (2, 3), $F_{Moon} = F_{Earth}$ – Newton's third law in reverse.

The masses (m) and semi-major axes (a) of the Moon and Earth are taken from reference books.

$$\frac{a_1}{a_2} = 81.315; \ \frac{m_2}{m_1} = 81.2853$$

Proof of the equality of forces modulo $F_1 = F_2$

$$F_1 = F_2$$

$$F = m * \dot{v}$$

$$\dot{v} = \sqrt{\ddot{x}^2 + \ddot{y}^2} =$$

$$\frac{b^2\sqrt{\left(e^2 - 2e*cos(\varphi(t)) + 1\right)\left(e*cos(\varphi(t)) - 1\right)^2\ddot{\varphi}^2 + 4\left(e*cos(\varphi(t)) - 1\right)\dot{\varphi}^2\left(e^2 - \frac{3e*cos(\varphi(t))+1}{2}\right)e*\sin(\varphi(t))\ddot{\varphi} - 4\left(-cos(\varphi(t))^3e^3 + \left(e^4 - \frac{e^2}{4}\right)cos(\varphi(t))^2 + \left(e^3 + \frac{e}{2}\right)cos(\ldots}}{a\left(e*cos(\varphi(t)) - 1\right)^3}$$

(7)

We denote

$$Q\big(\varphi(t)\big) = \frac{\sqrt{(e^2-2e*cos(\varphi(t))+1)(e*cos(\varphi(t))-1)^2\ddot{\varphi}^2+4(e*cos(\varphi(t))-1)\dot{\varphi}^2\left(e^2-\frac{3e*cos(\varphi(t))+1}{2}\right)e*\sin(\varphi(t))\ddot{\varphi}-4\left(-cos(\varphi(t))^3e^3+\left(e^4-\frac{e^2}{4}\right)cos(\varphi(t))^2+\left(e^3+\frac{e}{2}\right)cos(\varphi(\cdots}}{(e*cos(\varphi(t))-1)^3}$$

(8),

then

$$F_1 = m_1 * \dot{v}_1 = m_1 * \frac{b_1^2}{a_1} Q\big(\varphi(t)\big),\ F_2 = m_2 * \dot{v}_2 = m_2 * \frac{b_2^2}{a_2} Q\big(\varphi(t)\big).$$

Since from (6) $m_1 * \frac{b_1^2}{a_1} = m_2 * \frac{b_2^2}{a_2}$, then $F_1 = F_2$.

The pulse of the Moon and Earth system

In classical mechanics, the total momentum of a system of material points is a vector quantity equal to the sum of the products of the masses of material points and their velocity.:

$$\vec{p} = \sum m_i \vec{v_i}$$

$|m_1 * \boldsymbol{v}_1| = |m_2 * \boldsymbol{v}_2|$, proof in [A2].

$$m_1 * v_1 = m_2 * v_2 \qquad (9)$$

When deriving formula (1), Appendix [A1], we obtained the formula for linear velocity:

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \frac{b^2*\dot{\varphi}*\sqrt{1+e^2-2e*\cos\varphi(t)}}{a(-1+e*\cos\varphi(t))^2} = \frac{r*\dot{\varphi}*\sqrt{1+e^2-2e*\cos\varphi(t)}}{(1-e*\cos\varphi(t))} \qquad (10)$$

$$v_1 = \frac{b_1^2*\dot{\varphi}*\sqrt{1+e^2-2e*\cos\varphi(t)}}{a_1(-1+e*\cos\varphi(t))^2} \qquad (11)$$

$$v_2 = \frac{b_2^2*\dot{\varphi}*\sqrt{1+e^2-2e*\cos\varphi(t)}}{a_2(-1+e*\cos\varphi(t))^2} \qquad (12)$$

Substitute (11, 12) into (9):

$$m_1 * \frac{b_1^2}{a_1} = m_2 * \frac{b_2^2}{a_2} \qquad (13)$$

$$\frac{m_2}{m_1} = \frac{a_2 b_1^2}{a_1 b_2^2} \qquad (14)$$

since $b = \sqrt{1-e^2}$, then $\frac{m_2}{m_1} = \frac{a_2 a_1^2(1-e^2)}{a_1 a_2^2(1-e^2)} = \frac{a_1}{a_2}$

$$\frac{m_2}{m_1} = \frac{a_1}{a_2} \qquad (15)$$

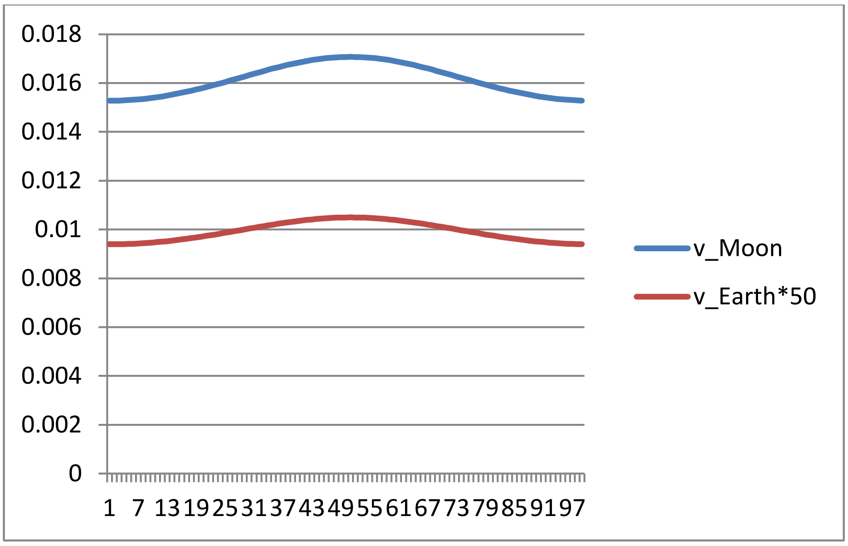
$$m_1 a_1 = m_2 a_2 \qquad (16)$$

$\vec{p} = m * \vec{v}$ – the direction of the momentum vector is the direction of the velocity vector. The linear rotational velocities of the Moon and the center of the Earth around the barycenter are shown in graph (5). From graphs (2, 5) we see that the directions of the momentum vectors coincide with the force vector calculated by the formula $F = G\frac{Mm}{r^2}$

Calculate the impulses:

$$p_{Moon} = 1.8621e + 21\mathrm{kg \cdot m \cdot s - 1} \quad (17)$$
$$p_{Earth} = 1.8578e + 21\mathrm{kg \cdot m \cdot s - 1} \quad (18)$$

Calculate the value of p using the formula $p = \frac{Mm}{r^2} = 2.90471e + 30$. Calculate the value of k using the formula $k = \frac{p}{p_{Moon}} = 4.08376e - 10$. Assign the k dimension $m^3 \cdot s^{-1} \cdot kg^{-1}$.



v_Moon is the speed of the Moon, v_Earth is the speed of the center of the Earth. For clarity, the v_Earth speed is multiplied by 50 on the graph.

Fig. 5

Reasons for the discrepancy between the force graphs in Fig. 2

Newton's law of universal gravitation states that "every object in the universe attracts every other object along a line connecting the centers of mass of the objects, proportional to the mass of each object, and inversely proportional to the square of the distance between the objects," [2].

a) Newton's law of universal gravitation assumes that the forces of interaction between objects are not equal to $F_{Moon} \neq F_{Earth}$.

b) It is also assumed that the forces are directed along the line connecting the centers of mass of the objects. However, when moving along a curve, the centripetal acceleration is directed toward the center of curvature of the trajectory, Fig. 6, 7, more details [1].

$e = 0$ we get a circle and $\frac{\ddot{x}}{\ddot{y}} = \frac{x}{y}$, a circle is a special case of an ellipse, Fig. 6.

In Figures 6 – 7, the red lines indicate the velocities, and the green ones indicate the accelerations.

The coordinates of the beginning of the velocity and acceleration vectors, the points of the original ellipse (*x*, *y*). The coordinates of the end of the velocity vector *(dx+x, dy+y).* The coordinates of the end of the acceleration vector *(ddx+x, ddy+y)*

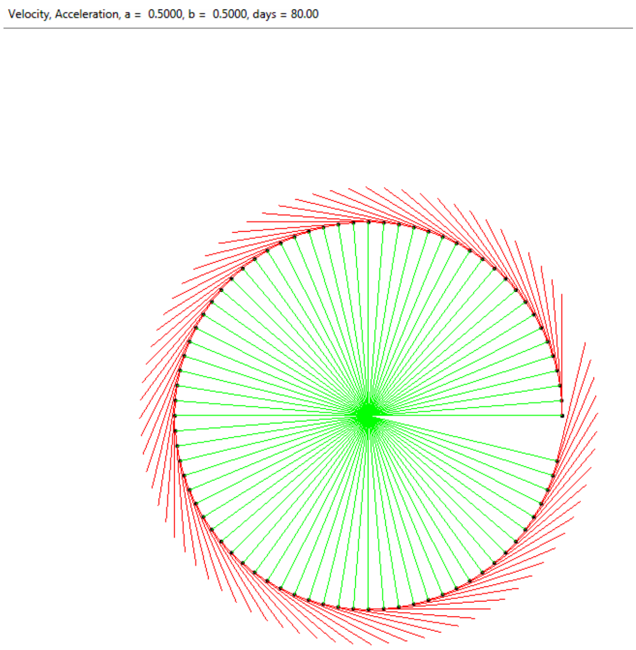


Figure 5

$e \neq 0$, then $\frac{\ddot{x}}{\ddot{y}} \neq \frac{x}{y}$, Figure 6

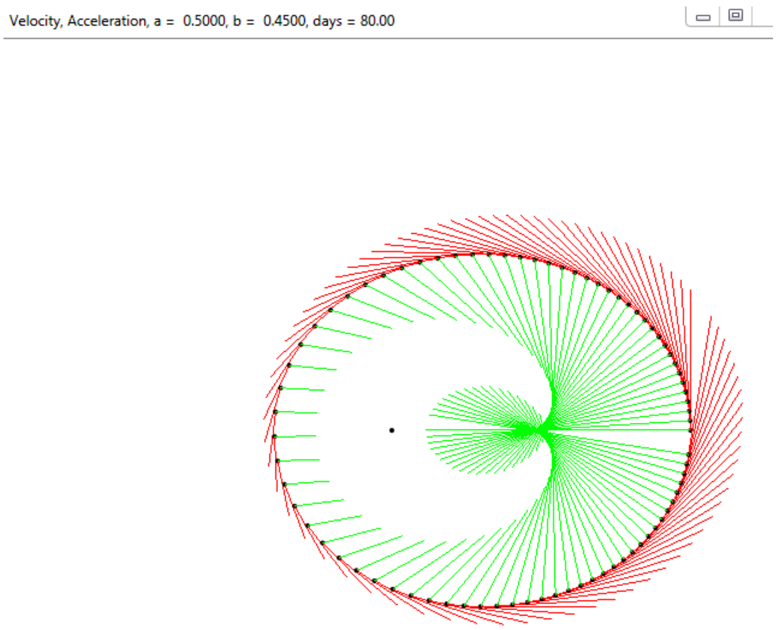


Figure 6

Links

1. Kinematics of the motion of a point along an ellipse,
https://www.academia.edu/94295697/Ellipse_kinematics

2. Newton's law of universal gravitation,
https://en.wikipedia.org/wiki/Newton%27s_law_of_universal_gravitation

Applications

A1. Derivation of formulas for velocity and acceleration using second-order curves:

There is a system of equations for a parametric pendulum (1)

The parameter is time (*t*).

$$\begin{cases} x = r(\varphi(t))\cdot cos(\varphi(t)) \\ \boldsymbol{y} = r(\varphi(t))\cdot sin(\varphi(t)) \end{cases} \tag{1.1}$$

Let's substitute the radius of the ellipse relative to the focus into system (1):

$$r\big(\varphi(t)\big) = \frac{b^2}{a\big(1-e*cos(\varphi(t))\big)} \tag{1.2}$$

$$\begin{cases} x = \dfrac{b^2}{a\big(1-e*cos(\varphi(t))\big)}\cdot cos\big(\varphi(t)\big) \\ y = \dfrac{b^2}{a\big(1-e*cos(\varphi(t))\big)}\cdot sin\big(\varphi(t)\big) \end{cases} \tag{1.3}$$

Let's differentiate twice. We'll get the coordinates of velocity and acceleration:

$$\dot{x} = \frac{d}{dt}\Big(r\big(\varphi(t)\big)cos\big(\varphi(t)\big)\Big) = -\frac{b^2*\dot{\varphi}*\sin(\varphi(t))}{a\big(e*\cos(\varphi(t))-1\big)^2} = \frac{r^2*\dot{\varphi}*\sin(\varphi(t))}{e*\cos(\varphi(t))-1} \tag{1.4}$$

$$\dot{y} = \frac{d}{dt}\left(\frac{p}{1-e*cos(\varphi(t))}sin\big(\varphi(t)\big)\right) = \frac{b^2*\dot{\varphi}*(-e+\cos(\varphi(t)))}{a\big(e*\cos(\varphi(t))-1\big)^2} = \frac{r^2*\dot{\varphi}*(-e+\dot{\cos}(\varphi(t)))}{1-e*\cos(\varphi(t))} \tag{1.5}$$

$$\ddot{x} = \frac{b^2\Big(\big(-e*cos(\varphi(t))*sin(\varphi(t))+sin(\varphi(t))\big)\ddot{\varphi}+\dot{\varphi}^2\big(e*cos(\varphi(t))^2-2e+cos(\varphi(t))\big)\Big)}{a\big(e*cos(\varphi(t))-1\big)^3} \tag{1.6}$$

$$\ddot{y} = \frac{-b^2\Big(\big(-cos(\varphi(t))\big)\big(e*cos(\varphi(t))-1\big)+e\big)\ddot{\varphi}+2\dot{\varphi}^2\Big(e^2-\frac{e*cos(\varphi(t))+1}{2}\Big)sin(\varphi(t))\Big)}{a\big(e*cos(\varphi(t))-1\big)^3} \tag{1.7}$$

speed $v = \sqrt{\dot{x}^2 + \dot{y}^2} = \frac{b^2*\dot{\varphi}*\sqrt{1+e^2-2e*\cos\varphi(t)}}{a(-1+e*\cos\varphi(t))^2} = \frac{r*\dot{\varphi}*\sqrt{1+e^2-2e*\cos\varphi(t)}}{(1-e*\cos\varphi(t))}$ (1.8)

acceleration $\dot{v} = \sqrt{\ddot{x}^2 + \ddot{y}^2} =$

$$b^2\left(\frac{\sqrt{\left(e^2-2e*cos(\varphi(t))+1\right)\left(e*cos(\varphi(t))-1\right)^2*\ddot{\varphi}^2}}{a\left(e*cos(\varphi(t))-1\right)^3}+\frac{\sqrt{4\left(e^2-\frac{3*e*cos(\varphi(t))+1}{2}\right)\dot{\varphi}^2\left(e*cos(\varphi(t))\sin(\varphi(t))-1\right)\ddot{\varphi}}}{a\left(e*cos(\varphi(t))-1\right)^3}-\frac{\sqrt{4\dot{\varphi}^4\left(-cos(\varphi(t))^3e^3+\left(e^4-\frac{e^2}{4}\right)cos(\varphi(t))^2+\left(e^3+\frac{e}{2}\right)cos(\varphi(t))-e^4-\frac{1}{4}\right)}}{a\left(e*cos(\varphi(t))-1\right)^3}\right) \qquad (1.9)$$

There is a system of equations for a parametric pendulum (1)

The parameter is time (*t*).

$$\begin{cases} x = r(\varphi(t))\cdot cos(\varphi(t)) \\ \boldsymbol{y} = r(\varphi(t))\cdot sin(\varphi(t)) \end{cases} \qquad (1.1)$$

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Let's differentiate twice. We'll get the coordinates of velocity and acceleration:

$$\dot{x} = \frac{d}{dt}\left(r\left(\varphi(t)\right)cos\left(\varphi(t)\right)\right) = -\frac{b^2*\dot{\varphi}*\sin\left(\varphi(t)\right)}{a\left(e*\cos\left(\varphi(t)\right)-1\right)^2} = \frac{r^2*\dot{\varphi}*\sin\left(\varphi(t)\right)}{e*\cos\left(\varphi(t)\right)-1} \qquad (1.4)$$

$$\dot{y} = \frac{d}{dt}\left(\frac{p}{1-e*cos\left(\varphi(t)\right)}sin\left(\varphi(t)\right)\right) = \frac{b^2*\dot{\varphi}*\left(-e+\cos\left(\varphi(t)\right)\right)}{a\left(e*\cos\left(\varphi(t)\right)-1\right)^2} = \frac{r^2*\dot{\varphi}*\left(-e+\cos\left(\varphi(t)\right)\right)}{1-e*\cos\left(\varphi(t)\right)} \qquad (1.5)$$

$$\ddot{x} = \frac{b^2\left(\left(-e*cos\left(\varphi(t)\right)*sin\left(\varphi(t)\right)+sin\left(\varphi(t)\right)\right)\ddot{\varphi}+\dot{\varphi}^2\left(e*cos\left(\varphi(t)\right)^2-2e+cos\left(\varphi(t)\right)\right)\right)}{a\left(e*cos\left(\varphi(t)\right)-1\right)^3} \qquad (1.6)$$

$$\ddot{y} = \frac{-b^2\left(\left(-cos\left(\varphi(t)\right)\left(e*cos\left(\varphi(t)\right)-1\right)+e\right)\ddot{\varphi}+2\dot{\varphi}^2\left(e^2-\frac{e*cos\left(\varphi(t)\right)+1}{2}\right)sin\left(\varphi(t)\right)\right)}{a\left(e*cos\left(\varphi(t)\right)-1\right)^3} \qquad (1.7)$$

speed $v = \sqrt{\dot{x}^2+\dot{y}^2} = \frac{b^2*\dot{\varphi}*\sqrt{1+e^2-2e*\cos\varphi(t)}}{a(-1+e*\cos\varphi(t))^2} = \frac{r*\dot{\varphi}*\sqrt{1+e^2-2e*\cos\varphi(t)}}{(1-e*\cos\varphi(t))}$ (1.8)

acceleration $\dot{v} = \sqrt{\ddot{x}^2+\ddot{y}^2} =$

$$b^2 \left(\frac{\sqrt{(e^2-2e*cos(\varphi(t))+1)(e*cos(\varphi(t))-1)^2*\ddot{\varphi}^2}}{a(e*cos(\varphi(t))-1)^3} + \frac{\sqrt{4\left(e^2-\frac{3*e*cos(\varphi(t))+1}{2}\right)\dot{\varphi}^2(e*cos(\varphi(t))\sin(\varphi(t))-1)\ddot{\varphi}}}{a(e*cos(\varphi(t))-1)^3} - \frac{\sqrt{4\dot{\varphi}^4\left(-cos(\varphi(t))^3 e^3+\left(e^4-\frac{e^2}{4}\right)cos(\varphi(t))^2+\left(e^3+\frac{e}{2}\right)cos(\varphi(t))-e^4-\frac{1}{4}\right)}}{a(e*cos(\varphi(t))-1)^3} \right) \quad (1.9)$$

A2. Proof of equality $|m_1 * \boldsymbol{v}_1| = |m_2 * \boldsymbol{v}_2|$, DeepSeek

A system of two material points, mass *m1* and *m2*. The points move in elliptical orbits with speeds *v1* and *v2* around a common focus. Prove the equality of pulses modulo, |*m1*v1*| = |*m2*v2*|.

Step 1: Angular Momentum in Elliptical Orbits

In a two-body system where two masses *m1*and *m2* orbit a common focus (e.g., their center of mass), the total angular momentum $\boldsymbol{L}$ of the system is conserved. The angular momentum of each mass is given by:

$\boldsymbol{L}_1 = \boldsymbol{r}_1 \times m_1 \boldsymbol{v}_1, \boldsymbol{L}_2 = \boldsymbol{r}_2 \times m_2 \boldsymbol{v}_2$

where $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ are the position vectors of m_1 and m_2, respectively, relative to the common focus, and $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are their velocities.

Since the system is isolated, the total angular momentum $\boldsymbol{L} = \boldsymbol{L}_1 + \boldsymbol{L}_2$ is conserved. For simplicity, assume the orbits lie in the same plane, so the cross products reduce to scalar magnitudes:

$L_1 = m_1 r_1 v_1 \sin(\Theta_1), L_2 = m_2 r_2 v_2 \sin(\Theta_2),$

where Θ_1 and Θ_2 are the angles between $\boldsymbol{r}_1$ and $\boldsymbol{v}_1$, and $\boldsymbol{r}_2$ and $\boldsymbol{v}_2$, respectively.

Step 2: Kepler's Second Law

Kepler's second law states that the line connecting the two masses sweeps out equal areas in equal times. This implies that the angular momentum per unit mass is constant for each orbit. Thus:

$\frac{L_1}{m_1} = \frac{L_2}{m_2}.$

Substituting the expressions for L_1 and L_2:

$r_1 v_1 \sin(\Theta_1) = r_2 v_2 \sin(\Theta_2).$

Step 3: Equal Areas and Equal Momenta

At any point in the elliptical orbit, the product $rv\sin(\Theta)$ is proportional to the angular momentum per unit mass. Since the angular momentum is conserved, this product is constant for each mass. Therefore:

$m_1 r_1 v_1 \sin(\Theta_1) = r_2 v_2 \sin(\Theta_2)$

If we consider the magnitudes of the linear momenta $|m_1 \boldsymbol{v}_1|$ and $|m_2 \boldsymbol{v}_2|$, and assume the angles Θ_1 and Θ_2 are such that $\sin(\Theta_1) = \sin(\Theta_2)$ (e.g., at corresponding points in the orbits), we can simplify the equation to:

$\sin(\Theta_1) = \sin(\Theta_2)$

Thus, the magnitudes of the linear momenta are equal:

$|m_1 \boldsymbol{v}_1| = |m_2 \boldsymbol{v}_2|$.

Conclusion

The equality $|m_1 \boldsymbol{v}_1| = |m_2 \boldsymbol{v}_2|$ arises from the conservation of angular momentum and the application of Kepler's second law in a two-body system with elliptical orbits. This result shows that the magnitudes of the linear momenta of the two masses are equal at corresponding points in their orbits.

A3. **Table 1.** Calculations of the motion of the Moon and Earth around the barycenter. The barycenter is located at the left focus. The motion begins from aphelion.

φ_i – angle, i = 0, 1, ..., 99.
r_{Moon} – radius from the center of the Moon to the barycenter, r_{Earth} – radius from the center of the Earth to the barycenter.

$\dot{v}_1, \dot{v}_2$ – corresponding normal accelerations.

m_{Moon}, m_{Earth} – masses of the Moon and the Earth.

Moon - Earth									
N	ϕ	r1	r2	v1	dvt_Moon	dvn_Moon	v2	dvt_Earth	dvn_Earth
0	0	0.002712	3.34E-05	0.015276	1.76E-19	0.002872	0.000188	2.16E-21	3.53E-05
1	0.056323	0.002712	3.34E-05	0.015278	-8.43E-06	0.002871	0.000188	-1.04E-07	3.53E-05
2	0.112667	0.002711	3.33E-05	0.015282	-1.68E-05	0.002869	0.000188	-2.07E-07	3.53E-05
3	0.169053	0.00271	3.33E-05	0.01529	-2.52E-05	0.002867	0.000188	-3.10E-07	3.53E-05
4	0.225502	0.002708	3.33E-05	0.0153	-3.35E-05	0.002863	0.000188	-4.11E-07	3.52E-05
5	0.282034	0.002706	3.33E-05	0.015314	-4.16E-05	0.002858	0.000188	-5.12E-07	3.51E-05
6	0.338671	0.002703	3.32E-05	0.01533	-4.97E-05	0.002852	0.000189	-6.10E-07	3.51E-05
7	0.395432	0.0027	3.32E-05	0.015349	-5.75E-05	0.002845	0.000189	-7.07E-07	3.50E-05
8	0.452339	0.002697	3.32E-05	0.015372	-6.52E-05	0.002837	0.000189	-8.01E-07	3.49E-05
9	0.509411	0.002692	3.31E-05	0.015396	-7.27E-05	0.002828	0.000189	-8.93E-07	3.48E-05
10	0.566669	0.002688	3.31E-05	0.015424	-7.99E-05	0.002818	0.00019	-9.82E-07	3.47E-05
11	0.624133	0.002683	3.30E-05	0.015454	-8.69E-05	0.002807	0.00019	-1.07E-06	3.45E-05
12	0.681821	0.002677	3.29E-05	0.015487	-9.35E-05	0.002796	0.00019	-1.15E-06	3.44E-05
13	0.739753	0.002671	3.29E-05	0.015522	-9.99E-05	0.002783	0.000191	-1.23E-06	3.42E-05
14	0.797947	0.002665	3.28E-05	0.01556	-0.00011	0.00277	0.000191	-1.30E-06	3.41E-05
15	0.856423	0.002659	3.27E-05	0.0156	-0.00011	0.002756	0.000192	-1.37E-06	3.39E-05
16	0.915197	0.002652	3.26E-05	0.015642	-0.00012	0.002742	0.000192	-1.43E-06	3.37E-05
17	0.974286	0.002644	3.25E-05	0.015687	-0.00012	0.002726	0.000193	-1.49E-06	3.35E-05
18	1.03371	0.002637	3.24E-05	0.015733	-0.00013	0.002711	0.000193	-1.55E-06	3.33E-05
19	1.09348	0.002629	3.23E-05	0.015781	-0.00013	0.002694	0.000194	-1.60E-06	3.31E-05
20	1.15361	0.002621	3.22E-05	0.015831	-0.00013	0.002678	0.000195	-1.64E-06	3.29E-05
21	1.21412	0.002613	3.21E-05	0.015882	-0.00014	0.00266	0.000195	-1.67E-06	3.27E-05
22	1.27502	0.002604	3.20E-05	0.015935	-0.00014	0.002643	0.000196	-1.70E-06	3.25E-05
23	1.33632	0.002596	3.19E-05	0.015988	-0.00014	0.002625	0.000197	-1.73E-06	3.23E-05

24	1.39804	0.002587	3.18E-05	0.016043	-0.00014	0.002608	0.000197	-1.74E-06	3.21E-05
25	1.46018	0.002578	3.17E-05	0.016098	-0.00014	0.00259	0.000198	-1.75E-06	3.18E-05
26	1.52275	0.002569	3.16E-05	0.016154	-0.00014	0.002572	0.000199	-1.76E-06	3.16E-05
27	1.58575	0.00256	3.15E-05	0.016211	-0.00014	0.002554	0.000199	-1.75E-06	3.14E-05
28	1.6492	0.002551	3.14E-05	0.016267	-0.00014	0.002536	0.0002	-1.74E-06	3.12E-05
29	1.7131	0.002542	3.13E-05	0.016324	-0.00014	0.002519	0.000201	-1.72E-06	3.10E-05
30	1.77744	0.002533	3.12E-05	0.01638	-0.00014	0.002501	0.000201	-1.70E-06	3.08E-05
31	1.84223	0.002525	3.10E-05	0.016436	-0.00014	0.002484	0.000202	-1.66E-06	3.06E-05
32	1.90746	0.002516	3.09E-05	0.01649	-0.00013	0.002468	0.000203	-1.62E-06	3.03E-05
33	1.97314	0.002508	3.08E-05	0.016544	-0.00013	0.002451	0.000203	-1.58E-06	3.01E-05
34	2.03924	0.0025	3.07E-05	0.016597	-0.00012	0.002436	0.000204	-1.52E-06	3.00E-05
35	2.10578	0.002492	3.06E-05	0.016648	-0.00012	0.002421	0.000205	-1.46E-06	2.98E-05
36	2.17272	0.002484	3.06E-05	0.016697	-0.00011	0.002406	0.000205	-1.40E-06	2.96E-05
37	2.24006	0.002477	3.05E-05	0.016744	-0.00011	0.002392	0.000206	-1.32E-06	2.94E-05
38	2.30779	0.00247	3.04E-05	0.016788	-0.0001	0.00238	0.000206	-1.25E-06	2.93E-05
39	2.37588	0.002464	3.03E-05	0.01683	-9.47E-05	0.002367	0.000207	-1.16E-06	2.91E-05
40	2.44432	0.002458	3.02E-05	0.01687	-8.75E-05	0.002356	0.000207	-1.07E-06	2.90E-05
41	2.51307	0.002452	3.02E-05	0.016906	-7.99E-05	0.002346	0.000208	-9.81E-07	2.89E-05
42	2.58213	0.002447	3.01E-05	0.016939	-7.19E-05	0.002336	0.000208	-8.84E-07	2.87E-05
43	2.65145	0.002443	3.00E-05	0.016969	-6.36E-05	0.002328	0.000209	-7.82E-07	2.86E-05
44	2.72101	0.002439	3.00E-05	0.016995	-5.51E-05	0.002321	0.000209	-6.77E-07	2.85E-05
45	2.79078	0.002435	3.00E-05	0.017017	-4.63E-05	0.002315	0.000209	-5.69E-07	2.85E-05
46	2.86072	0.002433	2.99E-05	0.017036	-3.73E-05	0.002309	0.000209	-4.58E-07	2.84E-05
47	2.93081	0.00243	2.99E-05	0.01705	-2.81E-05	0.002305	0.00021	-3.45E-07	2.84E-05
48	3.00101	0.002429	2.99E-05	0.017061	-1.88E-05	0.002302	0.00021	-2.31E-07	2.83E-05
49	3.07128	0.002428	2.99E-05	0.017067	-9.43E-06	0.002301	0.00021	-1.16E-07	2.83E-05
50	3.14159	0.002428	2.99E-05	0.017069	-4.69E-10	0.0023	0.00021	-1.82E-12	2.83E-05
51	3.2119	0.002428	2.99E-05	0.017067	9.43E-06	0.002301	0.00021	1.16E-07	2.83E-05
52	3.28217	0.002429	2.99E-05	0.017061	1.88E-05	0.002302	0.00021	2.31E-07	2.83E-05
53	3.35236	0.00243	2.99E-05	0.01705	2.81E-05	0.002305	0.00021	3.45E-07	2.84E-05
54	3.42245	0.002433	2.99E-05	0.017036	3.73E-05	0.002309	0.000209	4.58E-07	2.84E-05
55	3.4924	0.002435	3.00E-05	0.017017	4.63E-05	0.002315	0.000209	5.69E-07	2.85E-05
56	3.56217	0.002439	3.00E-05	0.016995	5.51E-05	0.002321	0.000209	6.77E-07	2.85E-05
57	3.63173	0.002443	3.00E-05	0.016969	6.36E-05	0.002328	0.000209	7.82E-07	2.86E-05
58	3.70105	0.002447	3.01E-05	0.016939	7.19E-05	0.002336	0.000208	8.84E-07	2.87E-05
59	3.77011	0.002452	3.02E-05	0.016906	7.99E-05	0.002346	0.000208	9.81E-07	2.89E-05
60	3.83886	0.002458	3.02E-05	0.01687	8.75E-05	0.002356	0.000207	1.07E-06	2.90E-05
61	3.9073	0.002464	3.03E-05	0.01683	9.47E-05	0.002367	0.000207	1.16E-06	2.91E-05
62	3.97539	0.00247	3.04E-05	0.016788	0.000101	0.00238	0.000206	1.25E-06	2.93E-05
63	4.04312	0.002477	3.05E-05	0.016744	0.000108	0.002392	0.000206	1.32E-06	2.94E-05
64	4.11046	0.002484	3.06E-05	0.016697	0.000114	0.002406	0.000205	1.40E-06	2.96E-05
65	4.1774	0.002492	3.06E-05	0.016648	0.000119	0.002421	0.000205	1.46E-06	2.98E-05
66	4.24393	0.0025	3.07E-05	0.016597	0.000124	0.002436	0.000204	1.52E-06	3.00E-05
67	4.31004	0.002508	3.08E-05	0.016544	0.000128	0.002451	0.000203	1.58E-06	3.01E-05
68	4.37572	0.002516	3.09E-05	0.01649	0.000132	0.002468	0.000203	1.62E-06	3.03E-05
69	4.44095	0.002525	3.10E-05	0.016436	0.000135	0.002484	0.000202	1.66E-06	3.06E-05
70	4.50574	0.002533	3.12E-05	0.01638	0.000138	0.002501	0.000201	1.70E-06	3.08E-05

71	4.57008	0.002542	3.13E-05	0.016324	0.00014	0.002519	0.000201	1.72E-06	3.10E-05
72	4.63398	0.002551	3.14E-05	0.016267	0.000142	0.002536	0.0002	1.74E-06	3.12E-05
73	4.69742	0.00256	3.15E-05	0.016211	0.000143	0.002554	0.000199	1.75E-06	3.14E-05
74	4.76043	0.002569	3.16E-05	0.016154	0.000143	0.002572	0.000199	1.76E-06	3.16E-05
75	4.823	0.002578	3.17E-05	0.016098	0.000143	0.00259	0.000198	1.75E-06	3.18E-05
76	4.88514	0.002587	3.18E-05	0.016043	0.000142	0.002608	0.000197	1.74E-06	3.21E-05
77	4.94685	0.002596	3.19E-05	0.015988	0.000141	0.002625	0.000197	1.73E-06	3.23E-05
78	5.00816	0.002604	3.20E-05	0.015935	0.000139	0.002643	0.000196	1.70E-06	3.25E-05
79	5.06906	0.002613	3.21E-05	0.015882	0.000136	0.00266	0.000195	1.67E-06	3.27E-05
80	5.12957	0.002621	3.22E-05	0.015831	0.000133	0.002678	0.000195	1.64E-06	3.29E-05
81	5.1897	0.002629	3.23E-05	0.015781	0.00013	0.002694	0.000194	1.60E-06	3.31E-05
82	5.24947	0.002637	3.24E-05	0.015733	0.000126	0.002711	0.000193	1.55E-06	3.33E-05
83	5.30889	0.002644	3.25E-05	0.015687	0.000122	0.002726	0.000193	1.49E-06	3.35E-05
84	5.36798	0.002652	3.26E-05	0.015642	0.000117	0.002742	0.000192	1.43E-06	3.37E-05
85	5.42676	0.002659	3.27E-05	0.0156	0.000111	0.002756	0.000192	1.37E-06	3.39E-05
86	5.48523	0.002665	3.28E-05	0.01556	0.000106	0.00277	0.000191	1.30E-06	3.41E-05
87	5.54343	0.002671	3.29E-05	0.015522	9.99E-05	0.002783	0.000191	1.23E-06	3.42E-05
88	5.60136	0.002677	3.29E-05	0.015487	9.35E-05	0.002796	0.00019	1.15E-06	3.44E-05
89	5.65905	0.002683	3.30E-05	0.015454	8.69E-05	0.002807	0.00019	1.07E-06	3.45E-05
90	5.71651	0.002688	3.31E-05	0.015424	7.99E-05	0.002818	0.00019	9.82E-07	3.47E-05
91	5.77377	0.002692	3.31E-05	0.015396	7.27E-05	0.002828	0.000189	8.93E-07	3.48E-05
92	5.83084	0.002697	3.32E-05	0.015371	6.52E-05	0.002837	0.000189	8.01E-07	3.49E-05
93	5.88775	0.0027	3.32E-05	0.015349	5.75E-05	0.002845	0.000189	7.07E-07	3.50E-05
94	5.94451	0.002703	3.32E-05	0.01533	4.97E-05	0.002852	0.000189	6.10E-07	3.51E-05
95	6.00114	0.002706	3.33E-05	0.015314	4.16E-05	0.002858	0.000188	5.12E-07	3.51E-05
96	6.05768	0.002708	3.33E-05	0.0153	3.35E-05	0.002863	0.000188	4.11E-07	3.52E-05
97	6.11412	0.00271	3.33E-05	0.01529	2.52E-05	0.002867	0.000188	3.10E-07	3.53E-05
98	6.17051	0.002711	3.33E-05	0.015282	1.68E-05	0.002869	0.000188	2.07E-07	3.53E-05