Solution to Gravity Divergence, Gravity Renormalization, and Physical Origin of Planck-Scale Cut-off

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Abstract

In contrast to the standard Effective Field Theory (EFT), which relies on an infinite series of unknown coefficients ($c_1, c_2, \ldots$) to parameterize divergences, this paper demonstrates that gravitational self-energy provides the physical mechanism for a self-renormalizing theory, where both the divergences and the unknown coefficients required to absorb them (features inherent to the standard EFT model) are naturally eliminated. Based on the principle that the gravitational source is the effective mass (M_{eff}), which includes its own self-energy, we derive a running coupling $G(k)$, that not only reproduces the canonical low-energy quantum corrections of EFT but also vanishes at a critical scale, $R_{gs-GR-1st} \approx 1.16\frac{G_N M_{fr}}{c^2} \approx 0.58R_S$. This self-renormalization mechanism eliminates divergences at their source, rendering higher-order counter-terms unnecessary and nullifying all classical and quantum gravitational interactions at this critical scale. This framework provides physical origins for two fundamental concepts in physics. First, the repulsive force that emerges for radii smaller than $R_{gs-GR-1st}$ (where $G(k) < 0$) offers a natural resolution to the black hole singularity problem. Second, the Planck-scale cutoff ($\Lambda \sim M_P c^2$) is identified as a physical boundary where the negative gravitational self-energy of a quantum fluctuation precisely balances its positive mass-energy, yielding a total energy $E_T \approx 0$. This mechanism dynamically prevents the formation of negative energy states. In conclusion, this work demonstrates that the single, fundamental principle of gravitational self-energy (or binding energy) offers a unified framework to consistently describe gravity from astrophysical to Planck scales. It provides a coherent solution for the problems of gravitational divergence, renormalization, singularities, and the physical origin of the Planck cutoff, while also offering a new perspective on cosmological phenomena such as cosmic acceleration.

1. Introduction

Gravity is basically given by the Einstein-Hilbert action, where G is Newton's constant and R is the scalar curvature derived from the Riemann curvature tensor.

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}R \tag{1}$$

The quantization of the Einstein-Hilbert action presents profound theoretical challenges, primarily stemming from two interconnected issues. First, the gravitational coupling constant, Newton's constant G_N, possesses a negative mass dimension ($[mass]^{-2}$) [1], which causes perturbative quantum corrections to diverge uncontrollably at high energies (the ultraviolet, or UV, regime). Second, this leads to the theory's non-renormalizability; beginning at the two-loop level, divergences appear that cannot be absorbed by a finite number of counter-terms, rendering the theory predictively powerless at the Planck scale and beyond.

Faced with these obstacles, several major research programs have sought to formulate a consistent theory of quantum gravity. One prominent approach is Asymptotic Safety, proposed by Weinberg [2] [3] [4], which

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postulates that gravity might be fundamentally non-perturbative. This hypothesis suggests that the renormalization group flow of the gravitational coupling could possess a non-trivial fixed point at high energies, taming the divergences and making the theory UV-complete.

An alternative, highly successful paradigm is the Effective Field Theory (EFT) approach [5]. EFT treats general relativity as a perfectly valid, predictive quantum theory at low energies. Its core philosophy is to systematically separate reliable low-energy quantum predictions from the unknown physics of the UV regime.

The EFT framework accomplishes this by parameterizing our ignorance of UV physics into an infinite series of higher-derivative terms with undetermined coefficients, such as c_1R^2 and $c_2R_{\mu\nu}R^{\mu\nu}$, which absorb the divergences arising from loop calculations [5]. While this method yields robust low-energy predictions—most notably the leading quantum correction to the Newtonian potential—it does not solve the fundamental problem of non-renormalizability by design. Instead, it sidesteps the issue, leaving the ultimate high-energy behavior of gravity an open question.

In this paper, we propose a different path that offers a physical resolution to these foundational issues. Instead of postulating a fixed point or parameterizing UV physics, we argue that the solution is already embedded within general relativity itself, through the fundamental principle of gravitational self-energy. By incorporating this energy into the definition of the gravitational source ($M \to M_{eff}$), we derive a running gravitational coupling, $G(k)$, that naturally vanishes at a critical scale, $R_{gs-GR-1st}$. This behavior leads to a trivial (Gaussian) fixed point ($G \to 0$), offering a powerful mechanism for gravity's self-renormalization.

This framework provides a unique synthesis. At low energies, it is fully consistent with the EFT approach, successfully reproducing its canonical quantum corrections. However, at high energies, it provides the very physics that EFT leaves undetermined. The vanishing of the coupling, $\kappa(k) = \sqrt{32\pi G(k)}$, at the critical scale naturally quenches all interactions, eliminating divergences at their source and rendering the infinite tower of counter-terms like c_1 and c_2 unnecessary.

The scope of our model extends beyond the divergence problem. The central idea, which originated from resolving the black hole singularity problem [6] (Chapter 2), provides a unified foundation for several long-standing puzzles. We will demonstrate that this single principle not only resolves the issues of singularities and renormalization (Chapter 4) but also establishes a physical origin for the Planck-scale cutoff in quantum field theory (Chapter 4.6). Finally, we will show how this model is formally integrated with the standard EFT framework, creating a more complete and powerful description of quantum gravity across all scales (Chapter 5).

This investigation begins by revisiting the solution to the black hole singularity problem, the original genesis of this work's central idea, before extending this physical principle to address the fundamental challenge of gravitational renormalization.

2. Solution of the singularity problem of a black hole [2]

2.1. Mass defect effect due to gravitational binding energy or gravitational potential energy

When two masses m are separated by r, the total energy of the system is

$$E_T = 2mc^2 - \frac{Gmm}{r} \tag{2}$$

If we introduce the negative equivalent mass $-m_{gp}$ for the gravitational potential energy,

$$-\frac{Gmm}{r} = -m_{gp}c^2 \tag{3}$$

[2] Chapter 2 is almost the same as the contents of the previous paper [6]. And, some research contents have been added. It is cited to understand the mass defect effect due to gravitational binding energy or gravitational potential energy, and the negative equivalent mass effect.

$$E_T = 2mc^2 - \frac{Gmm}{r} = 2mc^2 - m_{gp}c^2 = (2m - m_{gp})c^2 = m^*c^2 \tag{4}$$

The gravity of a composite particle composed of two objects acting on a mass m_3 that is relatively far away is

$$F = -\frac{Gm^*m_3}{R^2} = -\frac{G(2m - m_{gp})m_3}{R^2} = -\frac{G(2m)m_3}{R^2} - \frac{G(-m_{gp})m_3}{R^2} \tag{5}$$

That is, **when considering the gravitational action of a bound system, not only the mass in its free state but also the binding energy term ($-m_{gp}$) should be considered.** The total mass or equivalent mass m^* of the system is less than the mass of 2m when the two objects were in a free state. The bound objects experience a mass loss (defect) due to the gravitational binding energy. This is equivalent to having a negative equivalent mass in the system.

In gravitationally bound systems, changes in configuration (e.g., orbital reduction) lead to a decrease in total energy and effective mass due to energy radiation, as seen in celestial mechanics [7].

2.2. Gravitational self-energy or total gravitational potential energy of an object

The concept of gravitational self-energy (U_{gs}) is the total of gravitational potential energy (U_{gp}) possessed by a certain object M itself. Since a certain object M itself is a binding state of infinitesimal mass dMs, it involves the existence of gravitational potential energy among these dMs and is the value of adding up these. $M = \sum dM$. The gravitational self-energy is equal to the minus sign of the gravitational binding energy (U_{gb}). Only the sign is different because it defines the gravitational binding energy as the energy that must be supplied to the system to bring the bound object into a free state.

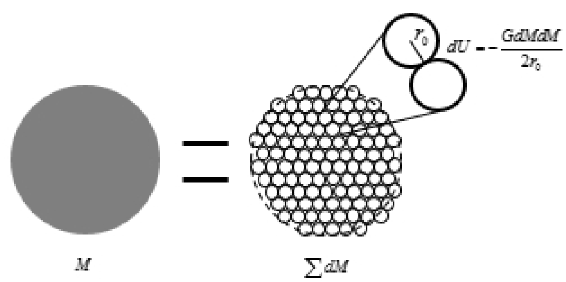

Figure 1: Since all mass M is a set of infinitesimal mass dMs and each dM is gravitational source, too, there exists gravitational potential energy among each of dMs. Generally, mass of an object measured from its outside corresponds to the value of dividing the total of all energy into c^2.

In the case of a spherical uniform distribution, total gravitational potential energy or gravitational binding energy ($-U_{gp}$) is

$$U = -\frac{3}{5}\frac{GM^2}{R} \tag{6}$$

In general, the above notation is used for the gravitational self-energy (potential energy) of an object, but in this paper, there are various mass terms such as the mass M, the free state mass M_{fr}, and the effective mass M_{eff}, and the radius R of the mass distribution can also be confused with the distance R. Therefore, in order to make the concepts of mass and distance included in the equation more clear, this paper plans to use the following notation.

$$U_{gs-NM} = -\frac{3}{5}\frac{G_N {M_{fr}}^2}{R_m} \tag{7}$$

Here, U_{gs-NM} means the **g**ravitational **s**elf-energy obtained from **N**ewtonian **M**echanics, G_N is Newton's gravitational constant, M_{fr} is the mass when the masses constituting the object are in a free state, and R_m means the radius of the mass or energy distribution.

$$U_{gs-NM-BH}(R=R_S) = -\frac{3}{5}\frac{G_N {M_{fr}}^2}{R_m} \approx -\frac{3}{5}\frac{G_N {M_{fr}}^2}{(\frac{2G_N M_{fr}}{c^2})} = -0.3 M_{fr}c^2 \tag{8}$$

Strictly speaking, the mass M of a black hole is not the mass M_{fr} in the free state, but the equivalent mass (or effective mass) including the binding energy. Here, M_{fr} is used for simple estimation.

In the general case, the value of gravitational self-energy is small enough to be negligible, compared to mass energy Mc^2. So generally, there was no need to consider gravitational self-energy. However the smaller R_m becomes, the higher the absolute value of U_{gs-NM}. For this reason, we can see that U_{gs-NM} is likely to offset the mass energy in a certain critical radius.

Thus, **looking for the size in which gravitational self-energy becomes equal to mass energy by comparing both,**

$$U_{gs-NM} = |-\frac{3}{5}\frac{G_N {M_{fr}}^2}{R_{gs}}| = M_{fr}c^2 \tag{9}$$

$$R_{gs-NM} = \frac{3}{5}\frac{G_N M_{fr}}{c^2} = \frac{3}{10}(\frac{2G_N M_{fr}}{c^2}) = 0.3R_S \tag{10}$$

R_S is the Schwarzschild radius based on the free state mass M_{fr}. This equation means that if mass M_{fr} is uniformly distributed within the radius R_{gs-NM}, negative gravitational potential energy for such an object equals positive mass energy in size. So, in case of such an object, positive mass energy and negative gravitational potential energy can be completely offset while total energy is zero. Since total energy of such an object is 0, gravity exercised on another object outside is also 0.

Comparing R_{gs-NM} with R_S, the radius of Schwarzschild black hole,

In the rough estimate above, since the gravitational potential energy at the event horizon is $U_{gs-NM} \approx -0.3M_{fr}c^2$, the mass energy of the black hole will be approximately $E_{BH} \approx 0.7M_{fr}c^2$.

$$R_S{}' = \frac{2GM}{c^2} \approx \frac{2G_N(\frac{7}{10}M_{fr})}{c^2} = \frac{7}{5}\frac{G_N M_{fr}}{c^2} \tag{11}$$

$$R_{gs-NM} = \frac{3}{5}\frac{G_N M_{fr}}{c^2} = \frac{3}{7}(\frac{7G_N M_{fr}}{5c^2}) \approx \frac{3}{7}R_S{}' \approx 0.43R_S{}' \tag{12}$$

This means that there exists the point where negative gravitational potential energy becomes equal to positive mass energy within the radius of black hole, and that, supposing a uniform distribution, the value exists approximately at the point $0.43R_S{}'$.

Even if we apply the kinetic energy and virial theorem, the radius only decreases as negative energy cancels out positive energy, but the core claim that "there is a region that cannot be compressed any further due to negative gravitational potential energy" remains unchanged. Although potential energy changes to kinetic energy, in order to achieve a stable bonded state, a part of the kinetic energy must be released to the outside of the system.

Considering the Virial Theorem ($K = -U/2$),

$$R_{gs-NM-vir} = \frac{1}{2}R_{gs-NM} \tag{13}$$

2.3. No singularity at the center of a black hole

The total energy of the system, including the gravitational potential energy or binding energy, is

$$E_T(R_m) = \sum_i m_i c^2 + \sum_{i<j} -\frac{Gm_i m_j}{r_{ij}} = M_{fr}c^2 - \frac{3}{5}\frac{G_N {M_{fr}}^2}{R_m} \tag{14}$$

Let's gradually reduce R_m from when R_m is infinite.

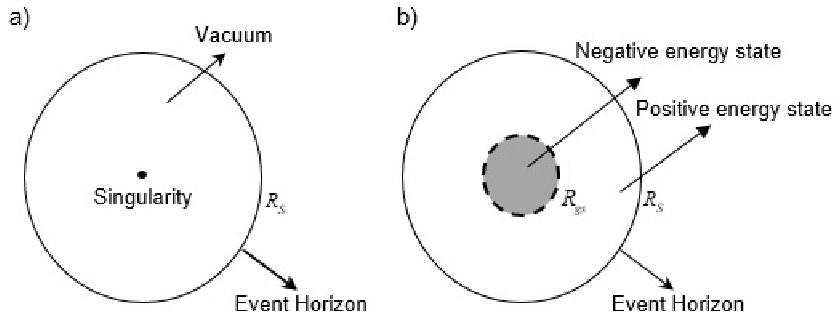

Figure 2: The internal structure of a black hole based on the radius of the mass (or energy) distribution. a) Existing Model. b) New Model. If R_m is less than R_{gs-NM} (or $R_{gs-NM-vir}$), this region becomes negative energy(mass) state. There is a repulsive gravitational effect between the negative masses, which causes it to expand again. This region (within R_{gs-NM} (or $R_{gs-NM-vir}$)) exercises anti-gravity on all particles entering this area, and accordingly prevents all masses from gathering to $r = 0$. If, over time, the black hole stabilizes, the black hole does not have a singularity in the center, but it has a zero (total) energy zone. Since there is a repulsive gravitational effect between negative energies (masses), the mass distribution expands, and when the mass distribution expands, the magnitude of the negative gravitational potential energy decreases, so it enters the positive energy state again. When the system (mass distribution) becomes a positive energy state, gravitational contraction will exist again. In this way, gravitational contraction and expansion will be repeated until the total energy of the system becomes 0, and finally it will stabilize at a state where the total energy is 0. The maximum size of the Zero Energy Zone is R_{gs-NM}.

This is assuming that it is stationary after the orbital transition. If there is kinetic energy due to rotation in the orbit, we can reflect only half of the negative gravitational potential energy term by using the virial theorem. $K = -\frac{1}{2}U$

$$E_T(R_m = \infty) = M_{fr}c^2 - \frac{3}{5}\frac{G_N {M_{fr}}^2}{R_m} = M_{fr}c^2 \tag{15}$$

$$E_T(R_m = R_S) = M_{fr}c^2 - \frac{3}{5}\frac{G_N {M_{fr}}^2}{R_m} \approx M_{fr}c^2 - \frac{3}{5}\frac{G_N {M_{fr}}^2}{(\frac{2G_N M_{fr}}{c^2})} = M_{fr}c^2 - \frac{3}{10}M_{fr}c^2 = 0.7M_{fr}c^2 \tag{16}$$

$$E_T(R_m = R_{gs-NM}) = M_{fr}c^2 - \frac{3}{5}\frac{G_N {M_{fr}}^2}{R_m} = M_{fr}c^2 - \frac{3}{5}\frac{G_N {M_{fr}}^2}{(\frac{3}{5}\frac{G_N M_{fr}}{c^2})} = M_{fr}c^2 - M_{fr}c^2 = 0 \tag{17}$$

$$E_T(R_m = \frac{1}{10}R_{gs-NM} < R_{gs-NM}) = M_{fr}c^2 - \frac{3}{5}\frac{G_N {M_{fr}}^2}{R_m} = M_{fr}c^2 - 10M_{fr}c^2 = -9M_{fr}c^2 \tag{18}$$

From the equation above, even if some particle comes into the radius of black hole, it is not a fact that it contracts itself infinitely to the point $R = 0$. From the point R_{gs-NM} (or $R_{gs-NM-vir}$), gravity is 0, and when it enters into the area of R_{gs-NM} (or $R_{gs-NM-vir}$), total energy within R_{gs-NM} (or $R_{gs-NM-vir}$) region corresponds to negative values enabling anti-gravity to exist. This R_{gs-NM} (or $R_{gs-NM-vir}$) region comes to exert repulsive effects of gravity on the particles outside of it, therefore it interrupting the formation of singularity at the near the area $R = 0$.

However, it still can perform the function as black hole because the emitted energy will exist in a region larger than $r > R_{gs-NM}$ (or $R_{gs-NM-vir}$). Since the emitted energy cannot escape the black hole, it is distributed in the region R_{gs-NM} (or $R_{gs-NM-vir}$) $< r < R_S$. Since the total energy of the entire range ($0 \le r < R_S$) inside the black hole is positive, it functions as a black hole.

If you have only the concept of positive energy, please refer to the following explanation.
If, $R_m = R_{gs-NM}$, the total energy of the system, including the gravitational potential energy, is

$$E_T(R_m = R_{gs-NM}) = M_{fr}c^2 - \frac{3}{5}\frac{G_N {M_{fr}}^2}{R_m} = M_{fr}c^2 - \frac{3}{5}\frac{G_N {M_{fr}}^2}{(\frac{3}{5}\frac{G_N M_{fr}}{c^2})} = M_{fr}c^2 - M_{fr}c^2 = 0 \tag{19}$$

From the point of view of mass defect, $r = R_{gs-NM}$ (or $R_{gs-NM-vir}$) is the point where the total energy of the system is zero. For the system to compress more than this point, there must be an positive energy release from the system. However, since the total energy of the system is zero, there is no positive energy that the system can release. Therefore, the system cannot be more compressed than $r = R_{gs-NM}$ (or $R_{gs-NM-vir}$). So black hole doesn't have singularity.

2.4. The gravitational singularity can be solved by gravity, not by quantum mechanics

In case of the smallest black hole with three times the solar mass [8], $R_S = 9km$. R_{gs-NM} of this object is as far as $3.87km$. In other words, **even in a black hole with smallest size that is made by the contraction of a star, the distribution of internal mass can't be reduced to at least radius 3.87km($R_{gs-NM-vir} = 1.94km$).** Even for black holes of varying sizes, from supermassive to stellar-mass, the critical radius R_{gs-NM} prevents singularity formation before quantum scales are reached.

Before reaching quantum mechanical scales, the singularity problem is solved by gravity itself.

2.5. The minimal size of existence

[**Existence = the sum of infinitesimal existences composing an existence**]

A single mass M for some object means that it can be expressed as $M = \sum dM$ and, for energy, $E = \sum dE$. The same goes for elementary particles, which can be considered a set of dMs, the infinitesimal mass.

R_{gs-NM} equation means that if masses are uniformly distributed within the radius R_{gs-NM}, the size of negative binding energy becomes equal to that of mass energy. This can be the same that the rest mass, which used to be free for the mass defect effect caused by binding energy, has all disappeared. This means the total energy value representing "some existence" coming to 0 and "extinction of the existence". Therefore, R_{gs-NM} is considered to act as "the minimal radius" or "a bottom line" of existence with some positive energy.

Gravitational self-energy can provide the concept of minimal length or minimal radius, one of the reasons for introducing string theory.

$$l_{\min} \approx R_{\min} \geq R_{gs-GR-1st} \tag{20}$$

Looking at the $R_{gs-NM} \approx \frac{3}{5}\frac{G_N M_{fr}}{c^2}$ obtained from Newtonian mechanics and the $R_{gs-GR-1st} \approx 1.16\frac{G_N M_{fr}}{c^2}$ value obtained from the first approximation of GR, we can see that **"the minimum length or minimum radius is proportional to the mass M or energy E, which are fundamental physical quantities of existence."**

This resolution of the singularity problem via gravitational self-energy sets the foundation for addressing gravitational divergences at high energies, as discussed in Section 4.

3. Extension of general relativity and new solution [3]

In all existing solutions, the mass term M must be replaced by $M_{fr} - M_{binding} = M_{fr} - \frac{|U_{binding}|}{c^2}$

[3]Chapter 3 is almost the same as the contents of the previous paper [6]. And, some research contents have been added. It is cited to understand the mass defect effect due to gravitational binding energy or gravitational potential energy, and the negative equivalent mass effect.

As discussed in Chapter 2, binding energy becomes significant in strong gravitational fields, necessitating a redefinition of mass as $M_{fr} - M_{binding}$ in solutions. We can solve the problem of singularity by separating the equivalent mass term $(-M_{binding})$ of gravitational potential energy (gravitational self-energy) from mass and including it in the solutions of field equation.

$M \to (M_{fr}) + (-M_{binding})$, In all existing solutions (Schwarzschild, Kerr, Reissner-Nordström, ...), the mass term M must be replaced by $(M_{fr} - M_{binding})$.

For example, Schwarzschild solution is,

$$ds^2 = -(1 - \frac{2GM}{c^2 r})c^2dt^2 + \frac{1}{(1 - \frac{2GM}{c^2 r})}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{21}$$

Schwarzschild-Choi solution is

$$ds^2 = -(1 - \frac{2G(M_{fr} - M_{binding})}{c^2 r})c^2dt^2 + \frac{1}{(1 - \frac{2G(M_{fr} - M_{binding})}{c^2 r})}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{22}$$

$$M_{eff} = M_{fr} - M_{binding} = M_{fr} - \frac{|U_{binding}|}{c^2} \tag{23}$$

In the case of assuming Newtonian mechanics and a spherical uniform distribution,

$$-M_{binding} = -\frac{|U_{binding}|}{c^2} = -\frac{3}{5}\frac{G_N {M_{fr}}^2}{R_m c^2} \tag{24}$$

In the case of general relativity,

$$U_{binding} = c^2 \int_0^R 4\pi r^2 \rho(r) \left[1 - \left(1 - \frac{2GM(r)}{rc^2}\right)^{-\frac{1}{2}}\right] dr \tag{25}$$

$$-M_{binding} = -M_{gs-GR} = -\frac{|U_{binding}|}{c^2} \tag{26}$$

$-M_{binding}$ is the equivalent mass of the negative binding energy, and $-M_{gs-GR}$ is the equivalent mass of the gravitational self-energy. Since the analytical function solution of above equation cannot be obtained, the solution must be obtained through approximation or numerical analysis. In this process, the rotation and virial theorem of the black hole must be taken into account.

In Chapter 4, from the first term approximation of the gravitational binding energy, $R_{gs-GR-1st} \approx 1.16\frac{G_N M_{fr}}{c^2} \approx 0.58R_S$ value is presented, and the $R_{gs-GR-vir} \approx 1.02\frac{G_N M_{fr}}{c^2} \approx 0.51R_S$ value obtained using the rotation and virial theorem is presented.

1) If $M_{fr} \gg |-M_{gs-GR}|$, in other words if $r \gg R_S$, we get the Schwarzschild solution.

2) If $M_{fr} = |-M_{gs-GR}|$, It has a flat space-time.

3) If $M_{fr} \ll |-M_{gs-GR}|$, in other words if $0 \leq r \ll R_{gs-GR}$, Here, R_{gs-GR} is the radius obtained through GR when the negative gravitational self-energy (binding energy) becomes equal in size to the positive mass energy.

$$ds^2 \simeq -(1 + \frac{2GM_{gs-GR}}{c^2 r})c^2dt^2 + \frac{1}{(1 + \frac{2GM_{gs-GR}}{c^2 r})}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{27}$$

In the domain of $0 \leq r \ll R_{gs-GR}$,

The area of within R_{gs-GR} has gravitational potential energy of negative value, which is larger than mass energy of positive value. Negative mass has gravitational effect which is repulsive to each other [9]. So, we can assume that $-M_{gs-GR}$ is almost evenly distributed.

4. Effective renormalization of gravity

4.1. Asymptotic Safety Method

Since Newton's constant G_N has a negative mass dimension ($[G_N] = -2$ in 4 dimensions), it is difficult to renormalize because high-order infinities appear during perturbation expansion. However, the Asymptotic Safety method is the idea that even in theories such as quantum gravity, which are difficult to renormalize using traditional perturbation methods, a theory that can be predicted at UV (ultra-high energy) can be constructed using a nonperturbative method [2] [3] [4].

Generally, the RG (Renormalization Group) flow for coupling g_i is expressed as follows:
Beta function equation

$$\beta_i(g) = \frac{dg_i}{d \ln k} \tag{28}$$

The conventional beta function form of $G(k)$ in nonperturbative RG flow

$$\frac{dG(k)}{d \ln k} = \beta(G) = (d-2)G - cG^2 \tag{29}$$

d: spacetime dimension (usually d = 4 is assumed)
c: quantum correction factor, which varies depending on the details of the theory.
When solving the RG flow equation, the general solution of $G(k)$ is expressed as follows.

$$G(k) = \frac{G_0}{1 + cG_0 \ln(k/k_0)} \tag{30}$$

G_0: Initial Newton constant (value at low energy, usually known as G_N)
k_0 : Initial energy scale

4.2. Find $G(k)$ or $M_{eff}(k)$

Usually, when applying RG flow, $G(k)$ is used as follows.

$$F = -\frac{G(k)Mm}{r^2} \tag{31}$$

$G(k)$ is a function that varies with distance or energy, and k basically means energy scale (or momentum scale). $k \sim p \sim \frac{E}{c}$

In Newtonian gravity, the mass M of an object is not simply its free state mass, but rather the equivalent mass that includes all forms of energy associated with the object-such as rest mass energy, binding energy, kinetic energy, and potential energy. Similarly, in general relativity, the energy-momentum tensor $T_{\mu\nu}$ represents the equivalent mass-energy, encompassing all energy contributions present in the system [10].

Therefore, when using the mass term in gravitational analyses, it is essential to recognize that the physically relevant quantity is not the free mass but the equivalent mass M_{eff}, which incorporates various energy components. While M_{eff} can be complex due to the inclusion of multiple energy terms, for practical analysis, we often focus on the minimal physical quantities that must exist whenever mass or energy is present. For example, in the case of electromagnetic energy, the presence of charge is required; if there is no charge, electromagnetic energy does not need to be considered.

However, for any nonzero mass or energy, there always exists a minimal physical quantity that must be included: the gravitational binding energy or gravitational self-energy. This gravitational self-energy arises inherently from the existence of mass or energy and, being negative, has a unique and essential role.

Thus, when considering mass or energy distributions, the minimal form of the effective mass can be expressed as

$$M_{eff} = M_{fr} - M_{binding} = M_{fr} - \frac{|U_{binding}|}{c^2} \tag{32}$$

where $M_{binding}$ denotes the equivalent mass of the gravitational binding energy or gravitational self-energy.

Existing researchers are having difficulties while focusing on $G(k)$, but let's think a little differently,

$$F = -\frac{G_N M_{eff} m}{r^2} = -\frac{(G_N \frac{M_{eff}}{M_{fr}}) M_{fr} m}{r^2} = -\frac{G(k) M_{fr} m}{r^2} \tag{33}$$

$$G(k) = \frac{G_N M_{eff}}{M_{fr}} = G_N (1 - \frac{|U_{binding}|}{M_{fr} c^2}) \tag{34}$$

In the framework of classical Newtonian mechanics and general relativity, the effective mass M_{eff} is inherently incorporated, and from this M_{eff}, a new gravitational coupling constant $G(k)$ naturally emerges.

Previously, when solving the singularity problem of black holes, we were able to know that the mass M changes by including binding energy or gravitational potential energy. This is a method that utilizes that.

For a simple calculation, assuming a spherical uniform distribution

$$M_{eff} = M_{fr} - M_{binding} = M_{fr} - \frac{3}{5} \frac{G_N {M_{fr}}^2}{R_m c^2} \tag{35}$$

$$M_{eff}(k) = (1 - \frac{3}{5} \frac{G_N M_{fr}}{R_m c^2}) M_{fr} = (1 - \frac{3}{5} \frac{G_N \frac{E}{c^2}}{R_m c^2}) M_{fr} = (1 - \frac{3 G_N}{5 R_m c^3} k) M_{fr} \tag{36}$$

This can be reorganized and expressed in the form of $G(k)$.

$$F = -\frac{G(k) M_{fr} m}{r^2} = -\frac{G_N M_{eff} m}{r^2} = -\frac{G_N (1 - \frac{3G_N}{5 R_m c^3} k) M_{fr} m}{r^2} = -(1 - \frac{3 G_N}{5 R_m c^3} k) G_N \frac{M_{fr} m}{r^2} \tag{37}$$

$$G(k) = (1 - \frac{3 G_N}{5 R_m c^3} k) G_N = (1 - \frac{3 G_N M_{fr}}{5 R_m c^2}) G_N = (1 - \frac{R_{gs-NM}}{R_m}) G_N \tag{38}$$

$$R_{gs-NM} = \frac{3}{5} \frac{G_N M_{fr}}{c^2} \tag{39}$$

If $B \equiv \frac{3G_N}{5 R_m c^3}$ is defined,

$$G(k) = (1 - \frac{3 G_N}{5 R_m c^3} k) G_N = (1 - Bk) G_N \tag{40}$$

If, $k^* = \frac{1}{B} = \frac{5 R_m c^3}{3 G_N}$ or $R_m = R_{gs-NM}$, $G(k^*) = 0$

$G(k^*) = 0$ means that at that particular energy scale (for example, in the UV regime) the effective gravitational coupling vanishes. In other words, rather than diverging to infinity at high energies, the gravitational interaction actually disappears at that scale.

We want $\lim_{r \to 0} \frac{M_{eff}}{r^2} = 0$ to eliminate divergence. That is, M_{eff} must decrease faster than r^2.

In the previous analysis, $R_{gs-NM} = \frac{3}{5} \frac{G_N M_{fr}}{c^2} \approx \frac{3}{10} R_S$

At $R_{gs-NM} = \frac{3}{5} \frac{G_N M_{fr}}{c^2}$ before r reaches 0, M_{eff} goes to 0. Therefore, we can solve the gravitational divergence problem. Also, in the low energy limit, $G(k) \to G_N$. And, in the M_{eff} equation, when $r \gg R_S$, we can see that it is consistent with the Newton equation.

4.3. New beta function

$$G(k) = (1 - \frac{3 G_N}{5 R_m c^3} k) G_N = (1 - Bk) G_N \tag{41}$$

Differentiating both sides with respect to $\ln k$:

$$\frac{dG(k)}{d \ln k} = \frac{d}{d \ln k} (1 - Bk) G_N = G_N (-B) \frac{dk}{d \ln k} \tag{42}$$

$$\frac{dk}{d\ln k} = k \tag{43}$$

$$\beta(G) = -BG_N k \tag{44}$$

At a specific $k^* = \frac{1}{B} = \frac{5R_m c^3}{3G_N}$, $G(k^*) = 0$, the value of the beta function is

$$\beta(G)|_{k=\frac{1}{B}} = -BG_N k = -G_N \tag{45}$$

Therefore, the new $\beta(G)$ is, if we adjust the existing equation,

$$\beta(G) = (d-2)G(k) - cG(k)^2(1 - \frac{G(k)}{G_N}) - G_N \tag{46}$$

Looking at this equation, if $k = k^* = \frac{1}{B} = \frac{5R_m c^3}{3G_N}$ or $R_m = R_{gs-NM}$, $G(k) = 0$, and we get $\beta(0) = -G_N$, which is consistent with the previous result.

In the Asymptotic Safety method, when the energy goes to infinity ($k \to \infty$), we find a Non-Gaussian Fixed Point (NGFP) where the coupling constants have a specific finite value. However, in this model, G(k) does not simply converge to a finite value, but there is a point where $G(k) \to 0$ at a specific scale $R_m = R_{gs-NM}$. This solves the divergence problem of gravity in a new way.

Also, when $k > \frac{1}{B} = \frac{5Rc^3}{3G_N}$ or $R_m < R_{gs-NM}$, we get $G(k) < 0$, a repulsive force occurs. This repulsive force prevents gravitational collapse, so that a singularity is not formed at the center of the black hole.

And, in the existing model, a quantum correction term was added to produce the Non-Gaussian Fixed Point (NGFP) and repulsive effects. However, in this model, if $k > \frac{1}{B}$, antigravity is generated, solving the singularity problem. Therefore, there is no need to introduce a quantum correction term.

Therefore, If the quantum correction term is deleted, the beta function becomes a simpler form.

$$\beta(G) = (d-2)G(k) - G_N \tag{47}$$

To find a fixed point, if $d = 4$, $\beta(G) = 0$

$$\beta(G) = 2G(k) - G_N = 2(1 - \frac{R_{gs-NM}}{R_m})G_N - G_N = (1 - \frac{2R_{gs-NM}}{R_m})G_N = 0 \tag{48}$$

Fixed point is

$$R_m = 2R_{gs-NM} = \frac{6}{5}\frac{G_N M_{fr}}{c^2}$$

4.4. Relativistic correction to gravitational binding energy and the running gravitational coupling[4]

[4] Here I am using several gravitational binding energy or gravitational potential energy functions. This may not be a completely accurate value. However, the core argument remains the same: "We must consider gravitational binding energy (or self-energy), and considering gravitational binding energy (or gravitational self-energy) will solve the problem of gravity divergence." And, we use approximations in many fields. If you can find a better binding energy function or gravitational self-energy function, you can use that.

Also, if you want to consider the energy of the gravitational field, not the gravitational potential energy, then here is the possibility: According to Shell Theorem and Birkhoff's Theorem, in a spherically symmetric system, the gravitational effect at a given radius is determined only by the mass or energy content surrounded within that radius, and contributions from outside the shell do not affect the internal dynamics. Although the energy of the gravitational field is generally considered to be a global quantity and is difficult to localize in general relativity, the application of these theorems allows us to treat the gravitational field energy as a localized contribution within the shell (That is, the energy of the gravitational field is considered only in the $0 \leq r \leq R_m$ part). This approach is justified by including only the energy density, pressure, and other physical quantities inside the shell in deriving cosmological equations such as the Friedmann equation.

In previous sections, the gravitational binding (self) energy of a uniform sphere was described using the Newtonian mechanics,

$$U_{gs-NM} = -\frac{3}{5}\frac{G_N {M_{fr}}^2}{R_m} \tag{49}$$

which is accurate for weak gravitational fields ($\frac{G_N M_{fr}}{R_m c^2} \ll 1$).

However, in regimes approaching the Planck scale or inside black holes, general relativistic effects become significant.

4.4.1. Post-Newtonian binding energy [10]

$$U_{gs-PN} = -\int_0^R \frac{Gm(r)}{r} \frac{dm(r)}{\left[1 - \frac{2Gm(r)}{rc^2}\right]^{\frac{1}{2}}} \tag{50}$$

$$U_{gs-PN} \approx -\frac{3GM^2}{5R}\left(1 + \frac{5}{7}\frac{GM}{Rc^2} + \frac{5}{6}(\frac{GM}{Rc^2})^2 + \cdots\right) \tag{51}$$

If we approximate only the first term and change it to the notation of this paper

$$U_{gs-PN} \approx -\frac{3G_N {M_{fr}}^2}{5R_m}\left(1 + \frac{5}{7}\frac{G_N M_{fr}}{R_m c^2}\right) \tag{52}$$

$$G(k) = G_N\left[1 - \frac{3G_N M_{fr}}{5R_m c^2}\left(1 + \frac{5}{7}\frac{G_N M_{fr}}{R_m c^2}\right)\right] = G_N\left[1 - \frac{3G_N}{5R_m c^3}k\left(1 + \frac{5}{7}\frac{G_N}{R_m c^3}k\right)\right] \tag{53}$$

If we look for the R_{gs-PN} value that makes $G(k) = 0$,

$$R_{gs-PN} \approx 1.02(\frac{G_N M_{fr}}{c^2}) = 0.51 R_S \tag{54}$$

4.4.2. Relativistic binding energy (using a first-order approximation) [10] [11]

$$U_{gs-GR} = c^2\int_0^R 4\pi r^2\rho(r)\left[1 - \left(1 - \frac{2GM(r)}{rc^2}\right)^{-\frac{1}{2}}\right]dr \tag{55}$$

$$U_{gs-GR} \approx c^2\int_0^R 4\pi r^2\rho(r)\left[-\frac{GM(r)}{rc^2} - \frac{3}{2}\frac{G^2M(r)^2}{r^2c^4} - \frac{5}{2}\frac{G^3M(r)^3}{r^3c^6}\cdots\right]dr \tag{56}$$

$$U_{gs-GR} \approx -\frac{3GM^2}{5R}\left(1 + \frac{15}{14}\frac{GM}{Rc^2} + \frac{25}{18}(\frac{GM}{Rc^2})^2\right) \tag{57}$$

If we approximate only the first term and change it to the notation of this paper

$$U_{gs-GR-1st} \approx -\frac{3G_N {M_{fr}}^2}{5R_m}\left(1 + \frac{15}{14}\frac{G_N M_{fr}}{R_m c^2}\right) \tag{58}$$

Accordingly, the effective mass incorporating relativistic gravitational self-energy becomes

$$M_{eff} = M_{fr} - \frac{|U_{gs-GR}|}{c^2} \approx M_{fr} - \frac{3G_N M_{fr}}{5R_m c^2}\left(1 + \frac{15}{14}\frac{G_N M_{fr}}{R_m c^2}\right) \tag{59}$$

The running gravitational coupling constant $G(k)$ is derived based on the effective mass M_{eff} incorporating the general relativistic (GR) correction to the gravitational self-energy. We define $G(k)$ as proportional to the ratio of effective mass to free mass.

$$G(k) = G_N\left[1 - \frac{3G_N M_{fr}}{5R_m c^2}\left(1 + \frac{15}{14}\frac{G_N M_{fr}}{R_m c^2}\right)\right] = G_N\left[1 - \frac{3G_N}{5R_m c^3}k\left(1 + \frac{15}{14}\frac{G_N}{R_m c^3}k\right)\right] \tag{60}$$

Using the critical radius $R_{gs-NM} = \frac{3}{5}\frac{G_N M_{fr}}{c^2}$, derived from the Newtonian approximation where the total energy $E_T = 0$, the running gravitational coupling constant $G(k)$ can be expressed as:

$$G(k) = G_N \left[1 - \frac{R_{gs-NM}}{R_m}\left(1 + \frac{25}{14}\frac{R_{gs-NM}}{R_m}\right)\right] \tag{61}$$

This expression incorporates both the first-order (Newtonian) and second-order (general relativistic correction) terms. At approximately $R_m \approx R_{gs-NM}$, the gravitational coupling $G(k)$ approaches zero under the first-order approximation, indicating the vanishing of gravitational interaction. For smaller R_m, $G(k)$ becomes negative, suggesting a repulsive force.

Using the Schwarzschild radius $R_S = \frac{2G_N M_{fr}}{c^2}$, which characterizes the event horizon of a black hole for mass M_{fr}(based on free state mass), the running gravitational coupling constant $G(k)$ can beexpressed as:

$$G(k) = G_N \left[1 - \frac{3R_S}{10R_m}\left(1 + \frac{15}{28}\frac{R_S}{R_m}\right)\right] \tag{62}$$

This form incorporates both the Newtonian term and the general relativistic correction. At approximately $R_m \approx 0.3R_S$, which aligns with the critical radius under first-order approximation, $G(k) \approx 0$, indicating the vanishing of gravitational interaction. For smaller R_m, $G(k) < 0$, suggesting a repulsive gravitational force that prevents singularity formation.

If we look for the $R_{gs-GR-1st}$ value that makes $G(k) = 0$,

$$R_{gs-GR-1st} \approx 1.93R_{gs-NM} \approx 1.16\frac{G_N M_{fr}}{c^2} \approx 0.58R_S \tag{63}$$

This result indicates that the GR correction increases the magnitude of the gravitational self-energy, resulting in a critical radius approximately 1.93 times larger than the Newtonian approximation and about half of the Schwarzschild radius R_S. At this critical radius $R_{gs-GR-1st}$, the effective mass approaches zero, leading to $G(k) = 0$, which signifies the vanishing of gravitational interaction and resolves divergences at high energy scales. If the radius is less than $R_{gs-GR-1st}$, $G(k) < 0$, suggesting a repulsive gravitational force that prevents singularity formation in black holes.

[**Implications and physical interpretation**]

The relativistic corrections make the critical radius at which the total energy disappears approximately 1.93 times larger.

For $R_m \gg R_{gs-GR-1st} \approx 0.58R_S$, the gravitational self-energy term is negligible, and the running gravitational coupling $G(k)$ returns to the gravitational coupling constant G_N.

As the radius approaches the critical value $R_m = R_{gs-GR-1st} \approx 0.58R_S$, the coupling G(k) smoothly goes to zero, ensuring that gravitational self-energy does not diverge. Remarkably, this mechanism allows gravity to undergo self-renormalization, naturally circumventing the issue of infinite divergences without invoking quantum modifications.

For $R_m < R_{gs-GR-1st} \approx 0.58R_S$, the gravitational coupling becomes negative ($G(k) < 0$), indicating a repulsive or antigravitational regime. This provides a natural mechanism preventing further gravitational collapse and singularity formation, consistent with the arguments in Section 2.

In summary, replacing the Newtonian gravitational self-energy term with the relativistically corrected form throughout our framework leads to an effective running gravitational coupling that remains finite-indeed, vanishes-at high energies or small length scales, thus providing a robust solution to the problem of gravitational divergences without recourse to quantum corrections. This strengthens the key claim of this work: that gravity can renormalize itself by including the full (relativistic) gravitational binding energy.

This approximation, leading to a critical radius of $R_{gs-GR-1st} \approx 0.58R_S$, provides a clear analytical form for $G(k)$ and captures the essential physics of self-renormalization. It demonstrates the core principle that a critical radius exists where gravitational interactions vanish.

In the following section, we will conduct a more rigorous analysis using the full integral form of the binding energy and consider additional physical effects, such as rotation, to verify that this conclusion holds under more realistic conditions.

4.4.3. Relativistic binding energy and when considering the rotation of the mass distribution

In this study, the point where $G(k) = 0$ was determined using the first-term approximation in Equation (58), rather than by directly solving the full Equation (55). A direct numerical analysis of Equation (55), reveals the zero of G(k) to be at:

$$R_{gs-GR-full} \approx 2.039 \frac{G_N M_{fr}}{c^2} \approx 1.02 R_S \tag{64}$$

However, the calculated value of $R_{gs-GR-full}$ in the equation above is larger than the Schwarzschild radius R_S. A direct interpretation of this result could lead to the assertion that black holes cannot exist, as the negative gravitational binding energy would completely offset the positive mass-energy before the object collapses to form a black hole. This analysis, however, relies on several simplifying assumptions. For this calculation, we have assumed a uniform, spherical, and non-rotating mass distribution. In a realistic gravitational collapse, most celestial bodies possess rotation. In such cases, the Virial Theorem must be taken into account. In addition, it becomes more complicated when considering differential rotation.

The Virial Theorem suggests that roughly half of the change in binding energy is converted into kinetic energy. This implies that the entirety of the binding energy change does not contribute to the reduction of the system's mass-energy. Therefore, for a rotating mass distribution, the Virial Theorem must be incorporated. When this is applied, the point where $G(k) = 0$ could be reduced by approximately half.

$$R_{gs-GR-vir} \approx \frac{1}{2} R_{gs-GR-full} \approx 0.51 R_S \tag{65}$$

This value is an estimate, as the dynamics of a realistic rotating collapse involve complex processes, such as differential rotation, which could alter the precise factor.

Thus, when considering rotation, a point where $G(k) = 0$ can still exist inside the event horizon, which resolves the apparent conflict with the observed existence of black holes. Furthermore, it is important to note that other factors may also play a role in the process of gravitational collapse.

The location of the point where $G(k) = 0$ may vary depending on the chosen gravitational binding energy function or approach method. However, this does not affect the core claim that there exists a point where $G(k) = 0$ when the gravitational binding energy is taken into account. Therefore, the core principle of solving the gravitational divergence problem remains the same.

Throughout this analysis, we have derived several critical radii based on models of increasing complexity

1) Newtonian model ($R_{gs-NM} \approx 0.30 R_S$): Establishes the foundational concept of a critical radius where total energy vanishes.

2) Post-Newtonian approximation ($R_{gs-PN} \approx 0.51 R_S$): The first relativistic correction.

3) First-order GR approximation ($R_{gs-GR-1st} \approx 0.58 R_S$): The central analytical model of this paper, providing an explicit form for G(k) and demonstrating the core principle of self-renormalization.

4) Full GR, non-rotating model ($R_{gs-GR-full} \approx 1.02 R_S$): A numerically exact calculation that reveals the paradox inherent in simplified relativistic models (i.e., neglecting rotation).

5) Full GR, rotating model ($R_{gs-GR-vir} \approx 0.51 R_S$): The most physically realistic scenario, which resolves the paradox and robustly confirms that the $G(k) = 0$ point exists well within the event horizon.

In the following analysis, the analysis is based on the first term approximation of general relativity. First, for the overall integral of U_{gs-GR}, it is difficult to present the $G(k)$ function because there is no analytic functional solution. Second, the value of $R_{gs-GR-vir} \approx 0.51 R_S$, which applies the rotation and virial theorem, is almost the same as that of $R_{gs-GR-1st} \approx 0.58 R_S$. Third, what is important in this paper is not the exact value of R_{gs}, but the fact that there exists a radius R_{gs} where the negative gravitational self-energy and the positive mass energy are equal.

4.4.4. Determination of momentum or energy scale k at which G(k)=0

In the context of the running gravitational coupling constant $G(k)$, the parameter k represents the momentum scale, which carries the dimension of momentum and is related to both momentum P and energy via $k \sim P \sim E/c$. This relationship reflects the characteristic energy or momentum scale associated with the physical system under consideration, often tied to the inverse of the spatial extent of the mass or energy distribution (R_m), as $k \sim c/R_m$.

We solve for the critical momentum scale k at which $G(k) = 0$, indicating the vanishing of gravitational coupling, using the expression:

$$G(k) = G_N \left[1 - \frac{3G_N}{5R_m c^3} k \left(1 + \frac{15}{14} \frac{G_N}{R_m c^3} k\right)\right] = 0 \tag{66}$$

Solving the above equation for k, we obtain the positive root corresponding to a physically meaningful scale:

$$k \approx 0.865 \frac{R_m c^3}{G_N} \tag{67}$$

For example, if R_m is on the Planck scale,

$$k \approx 0.865(\frac{R_m c^3}{G_N}) = 0.865(\frac{l_P c^3}{G_N}) = 0.865(\sqrt{\frac{\hbar c}{G_N}} c) \approx M_P c \tag{68}$$

This value of k represents the momentum or energy scale at which $G(k) = 0$, signifying the point where gravitational coupling vanishes. The result depends on the radius R_m, suggesting that the critical scale varies with the size of the mass or energy distribution. For instance, if R_m is on the Planck scale, i.e., $R_m \sim l_P = \sqrt{\frac{\hbar G_N}{c^3}}$, the critical k corresponds to a momentum scale near the Planck momentum, $k \sim M_P c$, where $M_P = \sqrt{\frac{\hbar c}{G_N}}$ is the Planck mass. This alignment with the Planck scale further supports the notion that gravitational interactions are suppressed at ultra-high energies, providing a natural mechanism to eliminate divergences without quantum corrections.

4.5. Solving the problem of gravitational divergence at high energy: Gravity's Self-Renormalization Mechanism

At low energy scales ($E \ll M_P c^2, \Delta t \gg t_P$), the divergence problem in gravity is addressed through effective field theory (EFT) [12] [13]. However, at high energy scales ($E \sim M_P c^2, \Delta t \sim t_P$), EFT breaks down due to non-renormalizable divergences, leaving the divergence problem unresolved [13].

Since the mass M is an equivalent mass including the binding energy, this study proposes the running coupling constant $G(k)$ that reflects the gravitational binding energy. **At the Planck scale ($R_m \sim R_{gs-GR-1st} \approx 1.16(\frac{G_N M_{fr}}{c^2}) \approx l_P$), $G(k) = 0$ eliminates divergences, and on higher energy scales than Planck's ($R_m < R_{gs-GR-ast}$), a repulsion occurs as $G(k) < 0$, solving the divergence problem in the entire energy range.**This implies that gravity achieves self-renormalization without the need for quantum corrections.

With the relativistic correction, the running gravitational constant $G(k)$ can be equivalently written as:

$$G(k) = G_N \left[1 - \frac{3G_N M_{fr}}{5R_m c^2} \left(1 + \frac{15}{14} \frac{G_N M_{fr}}{R_m c^2}\right)\right] = G_N \left[1 - \frac{3G_N}{5R_m c^3} k \left(1 + \frac{15}{14} \frac{G_N}{R_m c^3} k\right)\right] \tag{69}$$

where $k \sim P \sim E/c$

If $R_m > R_{gs-GR-1st} \approx l_P$, $G(k) > 0$, yielding an attractive force.
If $R_m = R_{gs-GR-1st} \approx l_P$, $G(k) = 0$, the gravitational coupling vanishes. Gravity is also zero.
If $R_m < R_{gs-GR-1st} \approx l_P$, $G(k) < 0$, yielding a repulsive force or antigravity.

This repulsive force prevents gravitational collapse and prevents the formation of a singularity at the center of the black hole. Since the point where $R_m < R_{gs-GR-1st}$ exists inside the event horizon of the black hole, it solves the singularity problem without colliding with observations.

4.5.1. At Planck scale

If, $M \sim M_P = \sqrt{\frac{\hbar c}{G_N}}$

$$R_{gs-GR-1st} \approx 1.16(\frac{G_N M_P}{c^2}) = 1.16\sqrt{\frac{\hbar G_N}{c^3}} = 1.16 l_P \tag{70}$$

This means that $R_{gs-GR-1st}$, where $G(k) = 0$, i.e. gravity is zero, is the same size as the Planck scale.
At $R_m = R_{gs-GR-1st}$,
$G(k) = 0 \Rightarrow \Pi^{div} \sim \frac{G(k)}{\varepsilon} R^2 = 0$
This means that divergence is eliminated at the Planck scale.

4.5.2. At high energy scales larger than the Planck scale

If $R_m < R_{gs-GR-1st} \approx l_P$ (That is, roughly $E > M_P c^2$)

$$G(k) = G_N \left[1 - \frac{3G_N M_{fr}}{5R_m c^2}\left(1 + \frac{15}{14}\frac{G_N M_{fr}}{R_m c^2}\right)\right] < 0 \tag{71}$$

In energy regimes beyond the Planck scale ($R_m < R_{gs-GP-1st}$), where $G(k) < 0$, the gravitational coupling becomes negative, inducing a repulsive force or antigravity effect. This anti-gravitational effect prevents gravitational collapse and singularity formation while maintaining uniform density properties, thus mitigating UV divergences across the entire energy spectrum by ensuring that curvature terms remain finite. While this repulsive force is a novel prediction of our model and may be regarded as unverified due to the lack of direct experimental evidence for antigravity in other research frameworks, we propose that this mechanism is not only a theoretical construct for resolving gravitational divergences but also manifests in observable cosmological phenomena.

Specifically, we argue that the accelerated expansion of the observable universe provides indirect evidence of antigravity effects related to $G(k) < 0$. The observable universe, with a radius of approximately 46.5 billion light-years, has a total mass-energy that would correspond to an event horizon of roughly 475.3 billion light-years [5]. if calculated based on standard general relativity. However, the critical radius $R_{gs-GR-1st}$, where negative gravitational self-energy balances positive mass-energy, is estimated at approximately 275.7 billion light-years ($R_{gs-GR-1st} = 0.58 R_S$) for the observable universe's mass-energy content [6]. Since the current radius of the observable universe ($R_m \approx 46.5$ billion light-years) is less than $R_{gs-GR-1st} \approx 275.7$ billion light-years, the universe itself resides in a regime where $R_m < R_{gs-GR-1st}$, implying $G(k) < 0$. Consequently, there exists a repulsive gravitational effect that promotes the accelerated expansion of the observable universe, which is the cause of the accelerated expansion and the source of dark energy [14]. This contrasts with conventional dark energy models by attributing cosmic acceleration to a fundamental gravitational mechanism rather than an additional energy component.

This interpretation suggests that the mechanism introduced to resolve gravitational divergences at high energies is actively at play on cosmological scales, providing a unified explanation for both the theoretical issue of UV divergences and the empirical phenomenon of cosmic acceleration. Thus, the anti-gravitational effects predicted by our model are not simply unverified claims, but is potentially verifiable by comparing the observed dark energy value with the dark energy value calculated by this model. Direct experimental verification of antigravity remains challenging, but it can also be verified by calculating the R_{gs}(inflection point) of the observable universe and comparing it to the inflection point of the observed acceleration expansion [14].

While the concept of negative mass and energy states associated with $G(k) < 0$ may face skepticism due to historical biases in mainstream physics favoring positive energy conditions (e.g., Strong and Weak Energy Conditions), such conditions are not fundamental laws but rather analytical tools used for categorizing systems and simplifying validations. In physics, the ultimate standard for judgment is not human perception but the reality of nature and the universe itself. The discovery of the universe's accelerated expansion, driven by dark energy with negative pressure [15] [16], illustrates that deviations from positive energy conditions do not result in physical inconsistencies.

[5] $R_m = 46.5BLY$, $\rho = \rho_c = 8.50 \times 10^{-27} kgm^{-3}$, $R_S = 2G_N M_{fr}/c^2$, $R_{gs-GR-1st} = 0.58 R_S$

Similarly, in our model, the negative mass state that generates a repulsive gravitational effect does not breach causality, as it does not entail superluminal propagation [9]. Superluminal propagation is linked to tachyons with imaginary mass, not negative mass.

The objections frequently raised against negative mass, such as the vacuum instability problem, runaway motion, and perpetual motion issues, arise from misconceptions about the characteristics of negative mass, and these assertions are flawed [17].

4.5.3. Resolution of the two-loop divergence in perturbative quantum gravity via the effective mass framework

In perturbative quantum gravity, the Einstein-Hilbert action is expanded around flat spacetime using a small perturbation $h_{\mu\nu}$, with the gravitational field expressed as $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$, where $\kappa = \sqrt{32\pi G(k)}$ and G_N is Newton's constant. Through this expansion, interaction terms such as $L^(3)$, $L^(4)$, etc., emerge, and Feynman diagrams with graviton loops can be computed accordingly.

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \tag{72}$$

$$S = \int d^4x (L^{(2)} + \kappa L^{(3)} + \kappa^2 L^{(4)} + \cdots) \tag{73}$$

$$\kappa = \sqrt{32\pi G_N} \tag{74}$$

where $L^{(2)}$ represents the free graviton Lagrangian, and $L^{(3)}, L^{(4)}, \ldots$ denote higher-order interaction terms (e.g. 3-point and 4-point graviton interactions). At the 2-loop level, Goroff and Sagnotti (1986) [13] demonstrated that the perturbative quantization of gravity leads to a divergence term of the form:

$$\Gamma^{(2)}_{div} \propto \kappa^4 R^3 \tag{75}$$

This divergence is non-renormalizable, as it introduces terms not present in the original Einstein-Hilbert action, thus requiring an infinite number of counterterms and destroying the predictive power of the theory.

However, this divergence occurs by treating the mass M involved in gravitational interactions as a constant quantity. The concept of invariant mass pertains to the rest mass remaining unchanged under coordinate transformations; this does not imply that the rest mass of a system is intrinsically immutable. For instance, a hydrogen atom possesses different rest masses corresponding to the varying energy levels of its electrons. Both Newtonian gravity and general relativity dictate that the physically relevant source term is the equivalent mass, which includes not only rest mass energy but also binding energy, kinetic energy, and potential energy. When gravitational binding energy is included, the total energy of a system is reduced, yielding an effective mass.

Based on this, a running gravitational coupling $G(k)$ can be derived:

$$G(k) = G_N \left[1 - \frac{3G_N M_{fr}}{5R_m c^2}\left(1 + \frac{15}{14}\frac{G_N M_{fr}}{R_m c^2}\right)\right] = G_N \left[1 - \frac{3G_N}{5R_m c^3}k\left(1 + \frac{15}{14}\frac{G_N}{R_m c^3}k\right)\right] \tag{76}$$

At this point $R_m = R_{gs-GR-1st} \approx 1.16(\frac{G_N M_{fr}}{c^2})$, $G(k) = 0$, implying that the gravitational interaction vanishes.

Since the perturbative expansion uses $\kappa = \sqrt{32\pi G(k)}$, it follows that:

$\kappa(k) = \sqrt{32\pi G(k)} \to 0$ as $R_m \to R_{gs-GR-1st}$

Building upon the resolution of the 2-loop divergence identified by Goroff and Sagnotti (1986), our model extends to address divergences across all loop orders in perturbative gravity through the running gravitational coupling constant $G(k)$. At the Planck scale ($R_m = R_{gs-GR-1st}$), $G(k) = 0$, nullifying the coupling parameter $\kappa(k) = \sqrt{32\pi G(k)}$. If $G(k) \to 0$, $\kappa \to 0$.

As a result, all interaction terms involving κ, including the divergent 2-loop terms proportional to $\kappa^4 R^3$, vanish at this scale. This naturally eliminates the divergence without requiring quantum corrections, rendering the theory effectively finite at high energies. Here, R^3 refers to the third-order term of the Riemann curvature tensor, specifically of the form $R^{\rho\sigma}_{\mu\nu} R^{\lambda\tau}_{\rho\sigma} R^{\mu\nu}_{\lambda\tau}$, which arises in the 2-loop divergence as computed by Goroff and Sagnotti (1986) [13]. **This mechanism effectively removes divergences, such as the 2-loop R^3 term, as well as higher-order divergences (e.g., $R^4, R^5, \ldots$) at 3-loop and beyond**, which are characteristic of gravity's non-renormalizability.

In addition, in the energy regime above the Planck scale ($R_m < R_{gs-GR-1st} \approx l_P$), $G(k) < 0$, and the corresponding energy distribution becomes a negative mass and negative energy state in the presence of an anti-gravitational effect. This anti-gravitational effect prevents gravitational collapse and singularity formation while maintaining uniform density properties, thus mitigating UV divergences across the entire energy spectrum by ensuring that curvature terms remain finite.

However, due to the repulsive gravitational effect between negative masses, the mass distribution expands over time, passing through the point where $G(k) = 0$ due to the expansion speed, and reaching a state where $G(k) > 0$. This occurs because the gravitational self-energy decreases as the radius R_m of the mass distribution increases, whereas the mass-energy remains constant at Mc^2. When $G(k) > 0$, the state of attractive gravity acts, causing the mass distribution to contract again. As this process repeats, the mass and energy distributions eventually stabilize at $G(k) = 0$ [6], with no net force acting on them.

Unlike traditional renormalization approaches that attempt to absorb divergences via counterterms, this method circumvents the issue by nullifying the gravitational coupling at high energies, thus providing a resolution to the divergence problem across all energy scales. This effect arises because there exists a scale at which negative gravitational self-energy equals positive mass-energy.

Furthermore, in the low-energy (infrared) regime where $R_m \gg R_{gs-GR-1st} \approx 1.16(\frac{G_N M_{fr}}{c^2})$, we find $G(k) \approx G_N$. In this domain, gravitational interactions behave classically, and gravitational self-energy is negligible. Although divergences formally persist, they are well-controlled within the effective field theory (EFT) framework and do not affect physical observables.

Thus, by treating mass M as the equivalent mass M_{eff} and deriving the scale-dependent coupling $G(k)$, we introduce a self-consistent mechanism that suppresses ultraviolet divergences dynamically, without invoking additional fields or symmetry principles. This approach provides a viable resolution to the gravitational divergence problem and aligns naturally with both general relativity and renormalization group flow.

Einstein-Hilbert action is

$$S = \int dx^4 \frac{\sqrt{-g}}{16\pi G(k)} R \tag{77}$$

4.6. The physical origin of the cut-off energy at the Planck scale

In quantum field theory (QFT), the cut-off energy Λ or cut-off momentum is introduced to address the infinite divergence problem inherent in loop integrals, a cornerstone of the renormalization process [18]. However, this cut-off has traditionally been viewed as a mathematical convenience, with its physical origin or justification remaining poorly understood [18].

This work proposes that Λ represents a physical boundary determined by the scale where the sum of positive mass-energy and negative gravitational self-energy equals zero, preventing negative energy states at the Planck scale. This mechanism, rooted in the negative gravitational self-energy of positive mass or energy, provides a physical explanation for the Planck-scale cut-off.

4.6.1. $G(k) = 0$ and Planck scale

At $R_m = R_{gs-GR-1st} \approx 1.16(\frac{G_N M_{fr}}{c^2})$, the running coupling constant $G(k) = 0$,

For a mass $M_{fr} \sim M_P = \sqrt{\frac{\hbar c}{G_N}}$, the characteristic radius is:

$$R_{gs-GR-1st} \approx 1.16(\frac{G_N M_P}{c^2}) = 1.16\sqrt{\frac{\hbar G_N}{c^3}} = 1.16 l_P \tag{78}$$

At $R_m = R_{gs-GR-1st}$, $G(k) = 0$, marking the Planck scale where divergences vanish.

If $R_m < R_{gs-GR-1st}$, then $G(k) < 0$, which means that the system is in a negative mass state. Therefore, the Planck scale acts as a boundary energy where an object is converted to a negative energy state by the gravitational self-energy of the object. In a theoretical analysis, a negative mass state may be allowed, although the system can temporarily enter a negative mass state, the mass distribution expands again because there is a repulsive gravitational effect between the negative masses. Thus, the Planck scale (l_P) serves as a boundary preventing negative energy states driven by gravitational self-energy.

4.6.2. Uncertainty principle and total energy with gravitational self-energy

To elucidate the interplay between quantum fluctuations and gravitational effects, we apply the energy-time uncertainty principle ($\Delta E \Delta t \geq \frac{\hbar}{2}$) to the total energy of a system, incorporating gravitational self-energy.

The energy-time uncertainty principle provides:

$$\Delta E \Delta t \geq \frac{\hbar}{2} \tag{79}$$

At the Planck time, $\Delta t = t_P$, energy fluctuation is:

$$\Delta E \geq \frac{\hbar}{2t_P} = \frac{1}{2} M_P c^2 \tag{80}$$

During Planck time, let's suppose that quantum fluctuations of $\frac{5}{6}M_P$ mass have occurred. Since all mass or energy is combinations of infinitesimal masses or energies, positive mass or positive energy has a negative gravitational self-energy. The total energy of the system, including the gravitational self-energy, is

$$E_T = \sum_i m_i c^2 + \sum_{i<j} -\frac{G m_i m_j}{r_{ij}} = Mc^2 - \frac{3}{5}\frac{GM^2}{R} \tag{81}$$

Here, the factor $\frac{3}{5}$ arises from the gravitational self-energy of a uniform mass distribution. Substituting $\frac{5}{6}M_P$ and $R = \frac{ct_P}{2}$ (where $c\Delta t$ represents the diameter of the energy distribution, constrained by the speed of light (or the speed of gravitational transfer). Thus, $\Delta x = 2R = c\Delta t$.

When calculated using the Newtonian mechanical binding energy equation, (t_P,$\frac{5}{6}M_P$)

$$E_T = Mc^2 - \frac{3}{5}\frac{GM^2}{R} \simeq \frac{5}{6}M_P c^2 - \frac{3}{5}\frac{G(\frac{5}{6}M_P)^2}{\frac{ct_P}{2}} = \frac{5}{6}M_P c^2 - \frac{5}{6}M_P c^2 = 0 \tag{82}$$

This demonstrates that at the Planck scale, the negative gravitational self-energy balances (or can be offset) the positive mass-energy, defining a cut-off energy $\Lambda \sim M_P c^2$. For energies $E > \Lambda$, the system enters a negative energy state ($E_T < 0$), which is generally prohibited due to the repulsive gravitational effects of negative mass states. Repulsive gravity prevents further collapse, dynamically enforcing the Planck scale as a minimal length.

[Quantum fluctuations at different mass scales]

We evaluate Δt, R, and E_T for three representative masses: the Planck mass ($M_P = \sqrt{\frac{\hbar c}{G}} \approx 2.17 \times 10^{-8} kg$), the proton mass ($M_{proton} \approx 1.67 \times 10^{-27} kg$), and the electron mass ($M_{electron} \approx 9.10 \times 10^{-31} kg$).

1) Planck mass & Planck time
If $R_m = \frac{c\Delta t}{2} = \frac{ct_P}{2} = \frac{1}{2}l_P$

$$E_T \approx M_P c^2 - \frac{3G_N {M_P}^2}{5(\frac{1}{2}l_P)}\left(1 + \frac{15}{14}\frac{G_N M_P}{(\frac{1}{2}l_P)c^2}\right) = M_P c^2 - 3.77 M_P c^2 = -2.77 M_P c^2 \tag{83}$$

If $R_m = l_P$

$$E_T \approx M_P c^2 - \frac{3G_N {M_P}^2}{5(l_P)}\left(1 + \frac{15}{14}\frac{G_N M_P}{(l_P)c^2}\right) \approx M_P c^2 - 1.24 M_P c^2 = -0.24 M_P c^2 \quad (84)$$

This negative E_T indicates that $R_m (= \frac{1}{2} l_P) < R_{gs-GR-1st} (= 1.16 l_P)$, where $R_{gp-GR-1st} \sim l_p$ is the critical radius at which $E_T = 0$. Increasing $\Delta t \sim t_p$, $R_m \to R_{gs-GR}$, and $E_T \to 0$, suggesting that the Planck scale is where gravitational self-energy can balance the mass-energy, supporting a physical cut-off at $\Lambda \sim M_P c^2$.

2) Proton mass
For $M = M_{proton}$, $\Delta E \approx 938 MeV$, and $\Delta t \approx 3.5 \times 10^{-25} s \sim 6.5 \times 10^{18} t_P$, $R_m \approx 5.254 \times 10^{-17} m \sim 10^{18} l_p$
The total energy is

$$E_T \approx M_{proton} c^2 - \frac{3}{5}\frac{G M_{proton}^2}{R_m} \approx M_{proton} c^2 \quad (85)$$

Here, $R_m \gg R_{gs-GR-ist} \approx 7.450 \times 10^{-55} m$, and the gravitational self-energy ($\sim 10^{-48} J$) is negligible compared to $M_{proton} c^2 \approx 1.504 \times 10^{-10} J$.

3) Electron mass
For $M = M_{electron}$, $\Delta E \approx 0.511 MeV$, and $\Delta t \approx 6.439 \times 10^{-22} s \sim 10^{22} t_p$, $R_m \approx 9.652 \times 10^{-14} m \sim 10^{21} l_p$

$$E_T \approx M_{electron} c^2 - \frac{3}{5}\frac{G M_{electron}^2}{R_m} \approx M_{electron} c^2 \quad (86)$$

Here, $R_m \gg R_{gs-GR-1st} \approx 4.058 \times 10^{-58} m$, and the gravitational self-energy ($\sim 10^{-58} J$)) is negligible compared to $M_{electron} c^2 \approx 8.187 \times 10^{-14} J$. For protons and electrons, since the gravitational self-energy is negligibly small compared to the mass energy, the gravitational self-energy calculations obtained from Newtonian mechanics are sufficient.

The Planck scale exhibits a unique characteristic: only for $M \sim M_P$, $t \sim t_p$, and $R \sim l_p$ does the gravitational self-energy (U_{gp-GR}) approach the mass-energy, enabling $E_T \approx 0$. This balance (or offset) suggests that the QFT cut-off $\Lambda \sim M_P c^2$ acts as a physical boundary where quantum and gravitational effects converge. In contrast, for proton or electron masses, $R_m \gg R_{gs-GR} - 1st$, rendering gravitational effects negligible and aligning with QED/QCD cut-offs ($\Lambda \sim GeV$).

4.6.3. Generalization and exceptions

The mechanism of balancing (or offset) positive mass-energy with negative gravitational self-energy applies primarily to systems dominated by gravitational effects, such as gravitational effective field theories or quantum gravity scenarios [5]. In non-gravitational theories like QED or ϕ^4, where binding energies are positive (e.g., electrostatic self-energy, $U_{es} > 0$), cut-offs are unrelated to the Planck scale and are determined by other physical scales. Thus, a Planck scale cut-off emerges only when quantum gravitational effects are significant.

While negative energy states are generally avoided in localized systems, different situations exist on cosmological scales. The observable universe is estimated to have a negative total energy, potentially due to mechanisms like cosmic inflation or dark energy [14] [19]. In the universe, as time progresses, surrounding matter and energy also become involved in gravitational interactions. In such scenarios, positive mass-energy scales proportionally to M, whereas negative gravitational potential energy scales as $-M^2/R$. Consequently, a mechanism exists whereby the absolute value of gravitational potential energy increases more rapidly than mass-energy [14]. As a result, negative mass states may persist unresolved for extended periods.

4.6.4. In gravitational problems, the physical meaning of cut-off energy

The cut-off energy $\Lambda \sim M_P c^2$ is not merely a mathematical artifact but a physical boundary driven by the balance (or offset) between positive mass-energy and negative gravitational self-energy. This mechanism offers a novel perspective on the Planck scale as the natural cut-off in gravitational systems, addressing the long-standing question of the physical origin of QFT cut-offs and providing a unified understanding of quantum-gravitational interactions.

5.Quantum gravity combining Effective Field Theory and the running coupling constant G(k)

The Effective Field Theory (EFT) approach, pioneered by John F. Donoghue [5], provides a robust and consistent framework for calculating low-energy quantum corrections to general relativity. The foundational principle of EFT is that the Einstein-Hilbert action is merely the lowest-order term in a more general action, organized as an expansion in powers of the curvature. The most general action consistent with general coordinate invariance is given by :

$$S = \int d^4x\sqrt{-g}\left\{\frac{2}{\kappa^2}R + c_1R^2 + c_2R_{\mu\nu}R^{\mu\nu} + O(R^3)\right\} \tag{87}$$

(This is adapted from Eq.(46) in Donoghue [5])

Here, the R term is the familiar Einstein-Hilbert action, while the higher-derivative terms, parameterized by unknown coefficients c_1 and c_2, encapsulate the effects of high-energy (UV) physics. Crucially, these higher-order terms are not merely theoretical possibilities; they are required to renormalize the theory. In their landmark 1974 paper [20], 't Hooft and Veltman demonstrated that one-loop quantum calculations in gravity, involving graviton and ghost loops, produce UV divergences that are not proportional to the original R term. Their result for the divergent part of the one-loop effective action is:

$$L^{(div)}_{1loop} = \frac{1}{8\pi^2\varepsilon}\left\{\frac{1}{120}\bar{R}^2 + \frac{7}{20}\bar{R}_{\mu\nu}\bar{R}^{\mu\nu}\right\} \tag{88}$$

(This is adapted from Eq.(49) in Donoghue [5])

This divergence must be absorbed by renormalizing the coefficients c_1 and c_2. Thus, these coefficients act as necessary counter-terms, parameterizing our ignorance of the physics that would ultimately render these calculations finite in a UV-complete theory. The standard EFT, by design, does not predict the values of c_1 and c_2; it accepts them as empirical inputs and proceeds to make reliable low-energy predictions.

Our work builds upon this powerful framework but proposes a physical resolution to the very problem that EFT parameterizes. We argue that by incorporating gravitational self-energy via an effective mass ($M \to M_{eff}$), the gravitational coupling $G(k)$ itself vanishes at a critical scale. This self-renormalization mechanism eliminates divergences at their source, thereby rendering the infinite tower of counter-terms, including c_1 and c_2, unnecessary.

The concept of effective mass (M_{eff}), which inherently includes binding energy, is a core principle embedded within both Newtonian mechanics and general relativity. From a differential calculus perspective, any entity possessing spatial extent is an aggregation of infinitesimal elements. A point mass is merely a theoretical idealization; virtually all massive entities are, in fact, bound states of constituent micro-masses.

Consequently, any entity with mass or energy inherently possesses gravitational self-energy (binding energy) due to its own existence. This gravitational self-energy is exclusively a function of its mass (or energy) and its distribution radius R_m. Furthermore, this gravitational self-energy becomes critically important at the Planck scale. Thus, it is imperative for the advancement of quantum gravity that alternative models also integrate, at the very least, the concept of gravitational binding energy or self-energy into their theoretical framework.

By integrating this principle, we can construct a unified model that not only aligns with the predictions of EFT at low energies but also resolves its high-energy limitations, leading to a UV-complete theory of gravity.

5.1. The Standard EFT prediction for the gravitational potential

Donoghue's seminal work on gravitational EFT culminates in the calculation of the leading quantum corrections to the Newtonian potential between two heavy masses. The result, expressed in momentum space, elegantly separates the different physical contributions: (Donoghue's (54) [5])

$$V(q) \sim \frac{G_Nm_1m_2}{q^2}\left(1 + aG_Nq^2\sqrt{\frac{m^2}{-q^2}} + bG_N\hbar q^2\ln(-q^2) + cG_Nq^2 + \cdots\right) \tag{89}$$

1)Classical Newtonian potential: The leading term, $\frac{G_N m_1 m_2}{q^2}$, is the Fourier transform of the standard $\frac{1}{r}$ Newtonian potential.

2)Classical general relativistic correction: The non-analytic term $\sim \sqrt{\frac{m^2}{-q^2}}$ corresponds to the leading classical correction from general relativity. In coordinate space, this term gives rise to the $\frac{1}{r^2}$ correction.

3)Leading quantum correction: The non-analytic term $\sim \ln(-q^2)$ is the most significant result. It is the genuine, unambiguous quantum prediction of the theory, independent of the unknown high-energy physics. It contains $\hbar$ explicitly and corresponds to a $\frac{1}{r^3}$ correction to the potential in coordinate space.

4)Local/analytic term: The term $\sim q^2$ is a local, analytic term. Contributions to this term can arise from both the low-energy loop calculation and the unknown coefficients of high-derivative terms in the original Lagrangian. As these two sources cannot be disentangled, this term is not a prediction of the effective theory.

This result brilliantly demonstrates that even a non-renormalizable theory like gravity can yield concrete, finite quantum predictions at low energies.

5.2. A Unified Model: Integrating the running coupling constant $G(k)$

Our model is built upon the physical principle that the gravitational source is not the free mass (M_{fr}) but the effective mass (M_{eff}), which includes the gravitational self-energy (or binding energy). This principle naturally leads to a running gravitational coupling constant, $G(k)$, which depends on the energy scale of the interaction.

The logical and powerful next step is to incorporate this running coupling into the EFT calculation. This is achieved by replacing every instance of the Newtonian constant G_N in the interaction vertices with our effective coupling $G(k)$. This reflects the idea that all gravitational interactions, including those that generate quantum fluctuations, are governed by the same scale-dependent coupling.

Applying this principle to Donoghue's result, we arrive at a new, unified expression for the gravitational potential.

$$V_{unified}(q) \sim \frac{G(k)m_1 m_2}{q^2}\left(1 + aG(k)q^2\sqrt{\frac{m^2}{-q^2}} + bG(k)\hbar q^2 \ln(-q^2) + cG(k)q^2 + \cdots\right) \tag{90}$$

This unified potential represents a profound synthesis of the two approaches, yielding a richer physical picture.

5.3. Physical implications of the Unified Model

The unified model not only preserves the successes of both frameworks but also offers new, powerful insights.

5.3.1. Consistency at low energies

In the low-energy limit (k is small, or r is large), our running coupling $G(k)$ approaches the Newtonian constant, $G(k) \to G_N$. In this regime, our unified potential equation seamlessly reduces to Donoghue's original equation. This demonstrates that our model is fully consistent with the standard, experimentally tested predictions of quantum gravity at low energies.

5.3.2. Suppression of all interactions at high energies

This is the most remarkable feature of the unified model. As the energy scale k approaches the critical scale k^*(where $R_m \to R_{gs-GR-1st}$), our running coupling $G(k)$ goes to zero. The consequences are profound:

1) The classical interaction ($\sim \frac{G(k)}{q^2}$) vanishes.

2) The classical GR correction ($\sim [G(k)]^2$) vanishes.

3) The quantum correction ($\sim [G(k)]^2\hbar$) also vanishes.

This leads to a beautiful and self-consistent physical picture. As the gravitational interaction itself is turned off at the critical scale, all of its associated quantum fluctuations are simultaneously and naturally suppressed.

5.3.3. Resolution of fundamental problems

- Divergence Problem: The unified model provides a physical mechanism for self-renormalization. The problematic local terms ($\sim cG(k)q^2$) that require renormalization in the standard EFT approach are driven to zero by the vanishing coupling $G(k)$. The perturbative expansion parameter itself, $\kappa(k) = \sqrt{32\pi G(k)}$, goes to zero, thus eliminating the divergences at their source without the need for an infinite series of counter-terms.

- Singularity Problem: As established in previous chapters, for scales smaller than the critical radius ($R_m < R_{gs-GR-1st}$), the coupling $G(k)$ becomes negative, inducing a repulsive force that prevents the formation of a singularity.

In summary, by integrating the physical principle of gravitational self-energy with the robust framework of Effective Field Theory, we have constructed a more complete and powerful description of quantum gravity. This unified model not only reproduces the confirmed low-energy predictions of the standard approach but also provides a compelling physical mechanism for resolving the long-standing problems of divergences and singularities, all while offering new predictions about the behavior of quantum effects at the high energy scales.

5.4. Application to the quantum corrections of the gravitational potential

The true predictive power of the Effective Field Theory (EFT) framework is most brilliantly demonstrated in the calculation of quantum corrections to the gravitational potential. By systematically separating the predictable long-range (non-local) quantum effects from the unknown short-range (local) physics, Donoghue and his collaborators derived the leading-order quantum correction to the Newtonian potential between two heavy masses, m_1 and m_2 [5].

5.4.1. The standard EFT prediction: A landmark result

The calculation in standard gravitational EFT yields a potential that includes not only the classical Newtonian term but also the leading corrections from both general relativity (classical) and quantum mechanics. The final result, in coordinate space, is (Donoghue's (64) adapted [5])

$$V(r) = -\frac{G_N m_1 m_2}{r}\left(1 - C_{GR}\frac{G_N(m_1+m_2)}{rc^2} - C_Q\frac{G_N\hbar}{r^2c^3}\right) \tag{91}$$

Here, the coefficients are unambiguously predicted by the theory to be $C_{GR} = 1$ (from the non-analytic $\sim \sqrt{\frac{m^2}{-q^2}}$ term in momentum space) and $C_Q = \frac{127}{30\pi^2}$ (from the $\sim \ln(-q^2)$ term).

This equation is a landmark achievement, as it proves that concrete, finite quantum predictions can be extracted from a non-renormalizable theory. The physical meaning of each term is clear:

1) The first term is the standard Newtonian potential.

2) The second term is the leading classical correction from general relativity's non-linear nature.

3) The third term, proportional to $\hbar$, is the leading quantum gravity correction.

5.4.2. A unified gravitational potential: Renormalizing the source

The central tenet of our work is that the physical source of gravity is not the free state mass (m_{fr}) but the effective mass (m_{eff}), which includes self-energy. This naturally leads to a running coupling $G(k)$. To apply this principle to the two-body problem described by Donoghue, we must renormalize the source masses (m_1, m_2) themselves, rather than applying a distance-dependent $G(r)$ to the interaction.

1)Effective Mass as the Gravitational Source: Each interacting particle i (where $i = 1, 2$) has its bare mass $m_{i,fr}$ modified by its own gravitational self-energy, resulting in an effective mass:

$$m_{i,eff} = m_{i,fr}(1 - \frac{U_{gp,i}}{m_{i,fr}c^2}) \approx m_{i,fr}(1 - \frac{R_{gs-GR-1st,i}}{R_{m,i}}) \tag{92}$$

where $R_{m,i}$ is the radius of the mass distribution for particle i, and $R_{gs-GR-1st,i} \approx 1.16(\frac{G_N m_{i,fr}}{c^2})$.

2)The Unified Potential: The interaction potential between these two "dressed" particles is then given by the standard EFT result, with the bare masses replaced by their effective counterparts.

$$V_{unified}(r) \approx -\frac{G_N m_{1,eff} m_{2,eff}}{r}\left(1 - \frac{G_N(m_{1,eff}+m_{2,eff})}{rc^2} - \frac{127}{30\pi^2}\frac{G_N\hbar}{r^2c^3}\right) \tag{93}$$

This formula is more physically robust and leads to profound new insights.

5.4.3. Physical implications of the source renormalization

This model perfectly separates the physics of the source from the physics of the interaction, yielding a clear and consistent picture across all scales.

1)Case 1: Interaction between macroscopic objects (e.g., stars, planets)

For any macroscopic object, its physical radius R_m is vastly larger than its critical radius $R_{gs-GR-1st}$. For the Earth, $R_{Earth} \approx 6.37 \times 10^6 m$ while $R_{gs-GR-1st,Earth} \approx 5.15 \times 10^{-3} m$. Therefore, $R_{m,i} \gg R_{gs-GR-1st,i}$, which leads to $m_{i,eff} \approx m_{i,fr}$

In this limit, our unified potential exactly reduces to Donoghue's original equation (64). This confirms that for all astrophysical and everyday scenarios, our model is in perfect agreement with the established predictions of general relativity and its quantum corrections. The running coupling effect is negligible because the objects are not compact enough.

2)Case 2: Interaction between Planck-scale particles

Now consider two particles whose mass m_{fr} is the Planck mass (M_P) and whose size R_m is on the order of the Planck length (l_P). As shown previously, for these particles, $R_m \approx R_{gs-GR-1st}$. This leads to a dramatic consequence: $m_{eff} \approx 0$ for both particles.

Substituting this into our unified potential, we find

$$V_{unified}(r) \approx -\frac{G_N(0)\cdot(0)}{r}\left(1 - \frac{G_N(0+0)}{rc^2} - \frac{127}{30\pi^2}\frac{G_N\hbar}{r^2c^3}\right) = 0 \quad (94)$$

The entire interaction, including the classical potential and all quantum corrections, vanishes completely!

6. Conclusion

In this study, we have proposed a new framework for gravity based on the fundamental physical principle of gravitational self-energy. By incorporating this energy into an effective mass, M_{eff}, we derived a running gravitational coupling constant $G(k)$, that dynamically regulates the strength of the gravitational interaction.

Our analysis, progressing through models of increasing complexity −from Newtonian to Post-Newtonian and full General Relativity including rotation− has robustly demonstrated the existence of a critical radius $R_{gs-GR-1sr} \approx 1.16(\frac{G_N M_{fr}}{c^2})$, where $G(k)$ vanishes. This mechanism provides an intrinsic solution to the long-standing problem of gravitational divergences, achieving self-renormalization without the need for external quantum corrections. Furthermore, for scales smaller than this critical radius, the emergence of a repulsive force ($G(k) < 0$) naturally resolves the singularity problem within black holes.

In addition, at the Planck scale, the balance between positive mass-energy and negative gravitational self-energy establishes a physical cut-off ($\Lambda \sim M_P c^2$), providing a novel perspective on the origin of quantum field theory cut-offs.

Furthermore, we have successfully unified this framework with the standard Effective Field Theory (EFT) of gravity. By applying our principle to the gravitational source itself (renormalizing the mass, $M \to M_{eff}$), our model not only reproduces the canonical low-energy quantum corrections predicted by EFT but also offers a profound new insight: the complete suppression of all gravitational interactions, both classical and quantum, as the system approaches the critical scale. This demonstrates that our model is not only consistent with established low-energy quantum predictions but also completes the picture by providing a physical mechanism that governs the theory's behavior at the high energies.

In conclusion, the single principle of gravitational self-energy provides a unified solution to some of the most profound problems in physics. It resolves the issues of singularities and divergences, establishes the physical origin of the Planck-scale cutoff, and offers a novel perspective on cosmological phenomena such as cosmic acceleration. This work presents a compelling case for a self-consistent, complete theory of gravity, valid across all energy scales.

The concept of effective mass (M_{eff}), which inherently includes binding energy, is a core principle embedded within both Newtonian mechanics and general relativity. From a differential calculus perspective, any entity possessing spatial extent is an aggregation of infinitesimal elements. A point mass is merely a theoretical idealization. Consequently, any entity possessing mass or energy inherently has gravitational self-energy (or binding energy) due to the presence of that mass or energy. Furthermore, this gravitational self-energy becomes critically important at the Planck scale. Thus, it is imperative for the advancement of quantum gravity that alternative models also integrate, at the very least, the concept of gravitational binding energy or self-energy into their theoretical framework.

[**Important problems related to gravity in the fields of physics and astronomy**]
1) Black hole singularity problem
2) Dark energy problem
3) Problem with the cause and mechanism of inflation
4) Gravitational divergence problem and gravity renormalization problem

The mainstream recognizes all four problems as different problems, and therefore presents as-hoc hypotheses for each of them. But these four problems may actually be different aspects of one problem. That is, problems that can be explained by the existence of repulsion or antigravity in the gravitational problem.

The singularity problem, inflation problem, divergence problem, and dark energy seem to be on different scales, right? So it seems like multiple sources are needed?

The only thing we need is a mechanism that creates repulsion or anti-gravity in the problem of gravity. And, this anti-gravity effect can be achieved by gravitational binding energy or gravitational self-energy with a negative value.

For a simple analysis, let's assume a spherical uniform distribution, and look at the gravitational potential energy or gravitational self-energy.

$$U_{gp} = -\frac{3}{5}\frac{GM^2}{R} \tag{95}$$

The total energy, including the gravitational potential energy or gravitational self-energy, is

$$E_T = \sum_i m_i c^2 + \sum_{i<j} -\frac{Gm_i m_j}{r_{ij}} = Mc^2 - \frac{3}{5}\frac{GM^2}{R} \tag{96}$$

When the energy distribution radius R is very small and the mass M is large, the negative gravitational potential energy term can be larger than the positive mass energy. This applies to the singularity problem [6], inflation problem [19] [21], and divergence problem.([6] & this paper).

The negative gravitational potential energy term can be larger than the positive mass energy when M is very large. It applies to the dark energy problem, which accelerates the expansion of the universe [15] [16].

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