The House of St. Nicholas

VOLKER W. THÜREY
Bremen, Germany *

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Abstract

We introduce two constants. They are suitable for finite graphs. They can serve as 'characteristic numbers' or 'dimensions'. Further, we define for each finite graph nine sequences. We will not calculate them.

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The defined numbers can be used as a 'characteristic number' for a finite graph.

The following well-known graph is called ' The House of St. Nicholas', and it can be drawn without lifting the pencil from the paper.

This idea can be extended.

In the following graphs, '$\times$' means a vertex.

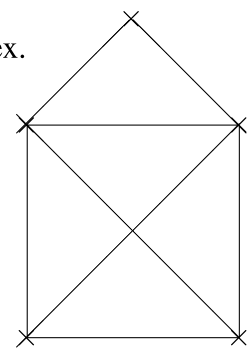

Figure 1:

The graph 'The House of St. Nicholas'.

It consists of 5 vertices and 8 edges.

The edges are straight lines.

We call a *traverse line* a graph which can be drawn without lifting the pencil from the paper.

*49 (0)421 591777, volker@thuerey.de

Let G be a finite graph, i.e. G has a finite number of vertices and edges, and let n be a natural number, i.e. $\mathsf{n} \in \mathbb{N} = \{1, 2, 3, \ldots\}$.

The graph ‘ The House of St. Nicholas’ can be drawn without lifting the pencil from the paper. It is a traverse line. You can speak one syllable of the German sentence ‘Das ist das Haus vom Nikolaus’ when you draw each edge.

Definition 1. We define an n-*Nicholas* as n Houses of St. Nicholas, which are put in a row. They touch each other at one edge. See Figure 2.
We define an n-*Rectangle* as n rectangles which are put in a row, and they touch at one edge. The shape of an n-Rectangle is a narrow rectangle. See Figure 3.

Remark 1. An 1-Nicholas is just the House of St. Nicholas. An 1-Rectangle is just a square.

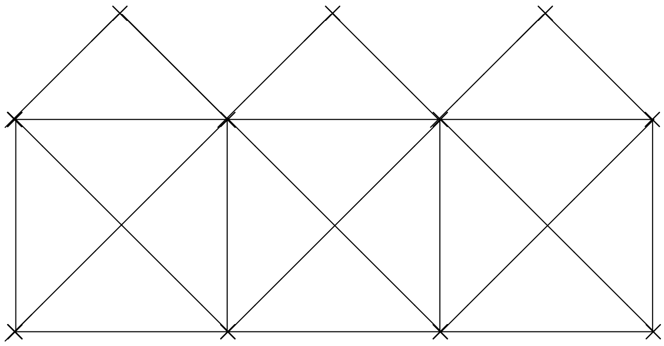

Figure 2:

The graph 3-Nicholas.

It consists of 11 vertices and 22 edges.

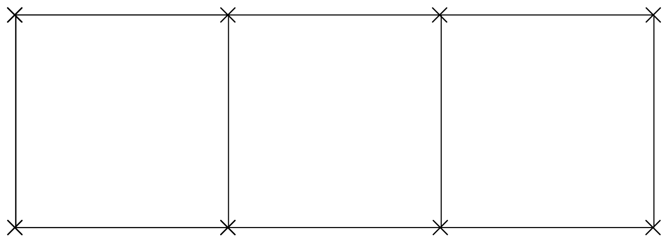

Figure 3:

The graph 3-Rectangle.

It consists of 8 vertices and 10 edges.

Remark 2. n-Nicholas is a traverse line if and only if $\mathsf{n} = 1$. n-Rectangle is a traverse line if and only if n is less than 3, i.e. either it is a 2-Rectangle or a square. This is shown by the old theorem of Leonard Euler and the problem of the bridges of Königsberg since only in a 2-Rectangle the number of vertices which have an odd number of edges is two. This number is larger than two in an n-Nicholas if $\mathsf{n} > 1$.

Definition 2. We define for a finite graph G the *Nicholas dimension* (in short *nicdim*) as the minimum natural number k such that G is composed of k traverse lines.

We define for a finite graph G which consists of k edges, where $\mathsf{k} > 0$, the *order dimension* (in short *orddim*) as the number of possibilities that G can be drawn.

Remark 3. The order dimension for a finite graph which consists of k edges, where $\mathsf{k} > 0$, is the natural number $\mathsf{k}!$.

An edge e and a vertex $\vec{v}$ are *incident* if and only if $\vec{v}$ is a vertex of e.
Two vertices are *adjacent* if and only if they belong to the same edge.
We say that two edges are *neighboring* if and only if they are not identical and they have a common vertex.

Now we generate for every finite graph G for each n nine constants. They are natural numbers or zero. Therefore, we get for each graph G nine sequences.

A coloring of G means that we color both the vertices and the edges of G.

We say that two colorings C_1 and C_2 are different if and only if there is a vertex or an edge of G such that the color it has by C_1 differs from the color that it gets from C_2.

To color an edge means for a concrete graph (i.e. a graph which is a subset of $\mathbb{R}^p$ for any p), that we color all inner points with the same color (i.e. all points of the edge except the two incident vertices). For an abstract graph it is just a definition to color it.

We assume that we have n colors.

Definition 3.
Let *vertices* (G, n) be defined as the number of possibilities to color the vertices of G.

Let *adjacent* (G, n) be defined as the number of possibilities to color the vertices of G, where adjacent vertices have different colors.

Let *edges* (G, n) be defined as the number of possibilities to color the edges of G.

Let *neighboring* (G, n) be defined as the number of possibilities to color the edges of G, where neighboring edges get different colors.

Let *graph* (G, n) be defined as the number of possibilities to color both the vertices and the edges of the graph G.

Let *graph, no* (G, n) be defined as the number of possibilities to color the graph G, where adjacent vertices, and neighboring edges, and also pairs of an incident vertex and edge get different colors.

Let *graph, adjacent* (G, n) be defined as the number of possibilities to color the graph G, where adjacent vertices get different colors.

Let *graph, neighboring* (G, n) be defined as the number of possibilities to color the graph G, where neighboring edges get different colors.

Let *graph, incident* (G, n) be defined as the number of possibilities to color the graph G, where incident pairs of a vertex and an edge get different colors.

References

[1] Joan M. Aldous and Robin J. Wilson: *Graphs and Applications*, Springer (2000)