Multi-dimensions Universes

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Abstract

The impact of a photon hitting a surface is determined. Newton law of dynamics is demonstrated from thermodynamics considerations in which Planck oscillator is considered as a 4-space dimensions oscillator. Wave-corpuscle duality is remodeled. Vacuum is a sea of positive charges and negative charges.

Key words: Photon impact, fundamental law of dynamics, Planck oscillator, absolute time, extra-dimensions Universe, wave-corpuscle duality, quantum cosmology, quantum foam, quantum gravitation.

1-Introduction :

The "radiancy" of a black body is given by Cardoso & de Castro law as a generalized Stefan-Boltzmann law in $D-Dimensional$ Universe [1]:

$E_T = R_T = \sigma_D T^{D+1}$ (1)

With $\sigma_D = \left(\frac{2}{c}\right)^{D-1}\left(\sqrt{\pi}\right)^{D-2}\frac{k^{D+1}}{h^D}D(D-1)\Gamma\left(\frac{D}{2}\right)\zeta(D+1)$: generalized Stefan-Boltzmann constant .

$D > 1$: spatial dimension of the Universe.

$k = 1.38\ 10^{-23} Joule.K^{-1}$: Boltzmann constant ;

$h = 6.626\ 10^{-34}\ Joule\ .s$: Planck constant ;

$c = 3\ 10^8 m.s^{-1}$; speed of light in vacuum.

T: Temperature of equilibrium of the black body;

$\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$: Gamma function;

$\zeta(x) = \frac{1}{\Gamma(x)}\int_0^\infty \frac{t^{x-1}}{e^t-1}dt$: Zeta function

-For $D = 3$ we have $\sigma_3 = \sigma = \frac{2\pi^5 k^4}{15c^2h^3}$: Stefan-Boltzmann constant given in thermo dynamical analysis and proved by experience.

-For $D = 4$

$E = \frac{4\pi k^5}{c^3h^4}T^5\zeta(5)\Gamma(5) \approx \frac{120k^5}{c^3h^4}T^5$ (2)

We can obtain (2) as:

$$E = \int_0^\infty \frac{4\pi\nu^2}{c^3}\frac{h\nu^2}{\exp\left(\frac{h\nu}{kT}\right)-1}d\nu \qquad (3)$$

Or as:

$$E = \int_0^\infty \frac{8\pi\nu^2}{c^3}\frac{\frac{1}{2}h\nu^2}{\exp\left(\frac{h\nu}{kT}\right)-1}d\nu \qquad (4)$$

But

$\frac{8\pi\nu^2}{c^3}.\frac{1}{\exp\left(\frac{h\nu}{kT}\right)-1}d\nu$ is the number of oscillators per unit volume in the frequency intervalν & $\nu + d\nu$ in the three dimensional space.

Equation (2) is the density of power of the black body in $[Watt.m^{-3}]$.

What does it mean?:

Equation (3) mean that the mean power of an oscillator is:

$$W = \frac{h\nu^2}{\exp\left(\frac{h\nu}{kT}\right)-1} \qquad (5)$$

And so in a black body oven that at any time only 50% of Planck resonators radiate energy, the others (also 50%) are absorbing energy. It is logic in an equilibrium state.

Equation (4) mean that all oscillators radiate energy & the mean power of an oscillator is $\frac{\frac{1}{2}h\nu^2}{\exp\left(\frac{h\nu}{kT}\right)-1}$. We reject this description because the black body will explode by this manner and there is no equilibrium.

If we take in consideration Planck assumption that only $\eta = 1 - \exp\left(-\frac{h\nu}{kT}\right)$ oscillators radiate energy in the frequency interval ν & $\nu + d\nu$ we should have that[2]:

$$W_{Planck} = \int_0^\infty \frac{8\pi\nu^2}{c^3}\frac{h\nu^2}{\exp\left(\frac{h\nu}{kT}\right)-1}\left(1-\exp\left(-\frac{h\nu}{kT}\right)\right)d\nu$$

And this leads us after replacing $\frac{h\nu}{kT}$ $by\ x$ to:

$$W_{Planck} = \frac{8\pi k^5}{c^3h^4}T^5\zeta(5)\Gamma(5) - \frac{8\pi k^5}{c^3h^4}T^5\int_0^\infty \frac{x^4.e^{-x}}{e^x-1}dx$$

$\int_0^\infty \frac{x^4.e^{-x}}{e^x-1}dx = \int_0^\infty \frac{x^4.e^{-2x}}{1-e^{-x}}dx = \Gamma(5)\zeta(5,2) \approx 0.864$

With $\zeta(s,q) = \frac{1}{\Gamma(s)}.\int_0^\infty \frac{t^{s-1}e^{-tq}}{1-e^{-t}}dt$ Hurwitz zeta function with $s = 5$ & $t = 2$

We have also for the function poly-gamma:

$\psi_m(x) = (-1)^{m+1}m!\,\zeta(m+1,x)$

With $\psi_m(x) = (-1)^{m+1}\int_0^\infty \frac{t^m e^{-xt}}{1-e^{-t}}dt$ Poly-gamma function

This function verify the recurrence relation:

$$\psi_m(x+1) = \psi_m(x) + (-1)^m m!\,x^{-(m+1)}$$

So we have :

$\psi_4(2) = (-1)^5 4!\,\zeta(5,2) = -4!\,\zeta(5,2)$

$\psi_4(2) = \psi_4(1) + (-1)^4 4!\,1^{-5} = (-1)^5\int_0^\infty \frac{t^4 e^{-t}}{1-e^{-t}}dt + 4! = 4! - \Gamma(5)\zeta(5)$

Which mean that: $\zeta(5,2) = -1 + \frac{\Gamma(5)\zeta(5)}{4!}$

So:

$\int_0^\infty \frac{x^4.e^{-2x}}{1-e^{-x}}dx = -\Gamma(5) + \frac{\Gamma(5)^2\zeta(5)}{4!} = \Gamma(5)[\zeta(5)-1] = 4!\,[1.036-1] \approx 0.864$

Than:

$$W_{Planck} = \frac{8\pi k^5}{c^3h^4}T^5\zeta(5)\Gamma(5) - \frac{8\pi k^5}{c^3h^4}T^5\Gamma(5)[\zeta(5)-1] = \frac{8\pi k^5}{c^3h^4}T^5\Gamma(5) \neq \frac{4\pi k^5}{c^3h^4}T^5\zeta(5)\Gamma(5)$$

If now we consider Planck theory of heat radiation we have for the mean energy of the oscillator [3]:

$$\frac{dU}{dt} = Constant$$

But we have also that $U = \frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right)-1}$ than we get:

$$\frac{dU}{dt} = \frac{d(h\nu)}{dt}.\frac{1}{\exp\left(\frac{h\nu}{kT}\right)-1} + h\nu.\frac{-\frac{\frac{d(h\nu)}{dt}}{kT}.e^{\frac{h\nu}{kT}}}{[\exp\left(\frac{h\nu}{kT}\right)-1]^2}$$

And to be conform with Planck approximation ($h\nu \ll kT$ or for large U) we get:

$$\frac{dU}{dt} \approx \frac{d(h\nu)}{dt}.\frac{kT}{h\nu} + h\nu.\left(\frac{kT}{h\nu}\right)^2.\left(-\frac{\frac{d(h\nu)}{dt}}{kT}\right).\left(1+\frac{h\nu}{kT}\right) = -\frac{d(h\nu)}{dt} = -\alpha_0 \quad (6)$$

α_0 : is declared a new universal constant (but Planck don't do this declaration).

2-The fundamental law of dynamics:

Planck oscillators are classic oscillators. We can conclude from equation (5) that the power radiated by a single oscillator is:

$$\delta = h.\nu^2 \qquad (7)$$

The energy absorbed by an oscillator is as a multiple integer of the quantity:

$$\varepsilon = h.\nu \qquad (8)$$

This energy is as:

$$\varepsilon = \int \delta.dt \qquad (9)$$

So:

$$d\varepsilon = \delta.dt$$

Than:

$$dt = \frac{d\nu}{\nu^2} \qquad (10)$$

In classic mechanics by definition the power is the force scalar the speed of the corpuscle (we suppose that motion is in a straight line):

$$W = f.v = h\nu^2$$

So the force acting on the corpuscle is:

$$f = \frac{1}{v}.h\nu^2$$

Duality of wave-corpuscle implies that:

$$\frac{1}{v} = \frac{d\tilde{k}}{d\omega}$$

with $v = v_g$: the group speed of the packet of waves assimilated as a corpuscle;

$\tilde{k}$: wave-vector of the packet of waves

$\omega = 2\pi\nu$: the frequency of the packet of waves.

So:

$$f = h\nu^2.\frac{d\tilde{k}}{d\omega} = h\nu^2.\frac{d\tilde{k}}{2\pi d\nu} = \hbar\nu^2.\frac{d\tilde{k}}{d\nu} = \frac{d\left(\hbar\tilde{k}\right)}{dt}$$

With : $\hbar = \frac{h}{2\pi}$: reduced Planck constant.

$\hbar\tilde{k}$: have the dimension of a moment. So:

$f = \frac{dp}{dt}$ with $p = mv$: is the moment of the corpuscle.

This relation is generalized as:

$\boldsymbol{f} = m\boldsymbol{\gamma}$ (11)

With: $\boldsymbol{\gamma} = \frac{d\boldsymbol{v}}{dt}$the acceleration of the corpuscle

m : the mass of the corpuscle.

Equation (11) is the fundamental law of dynamics or the Newton first law. It is invariant by Galilean Transformations of space-time.

For a relativist corpuscle we have the invariant:

$E^2 - p^2c^2 = m^2c^2$ (12)

With $E = \frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}}$ & $\boldsymbol{p} = \frac{m.\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}}$

Wave-corpuscle duality: $E = h\nu$ so $h^2\nu^2 - p^2c^2 = m^2c^2$

Differentiate this equation and taking account about (10) than we get:

$\frac{d\boldsymbol{p}}{dt}.\boldsymbol{v} = h\nu^2$ (13)

Of course $\frac{d\boldsymbol{p}}{dt}$ is not invariant by Lorentz Transformations of space-time.

For a photon we get: $\frac{dp}{dt} = \frac{\fbox{?}\nu^2}{c}$ (14)

From a classical view of the force equation (14) can be considered as the impact of the photon hitting directly an orthogonal surface.

4.Universal time:

From equation (6) we can deduce that for an oscillator:

$h\nu = \alpha_0\tau$ (15)

with: τ: a characteristic time of the oscillator &$d\tau = d\tilde{\zeta}$when the energy of the oscillator is varying;

$\tilde{\zeta}$: Universal time (the time τ is the "position" of the oscillator in the axle of the absolute time). It is like a fifth dimension of the oscillator.

Since the radiancy (2) of the black body have the dimension of $Watt.m^{-3}$ than the Universe in which the black body exist have four space dimensions (Do the analogy with Kurlbaum measurements: since the radiancy have the dimension of $Watt.m^{-2}$ than the Universe in which the black body exist have three space dimension).

The time t is relative .

Since we declare that the speed of light c is an universal constant than Lorentz transformations of space & time are applicable.

The Planck formulae $\varepsilon = h\nu$ holds for classic dynamics and for relativist dynamics. In relativist dynamics the relation $\hbar\widetilde{\boldsymbol{k}} = \boldsymbol{p}$ holds also with $\boldsymbol{p} = \frac{m.\boldsymbol{v}}{\sqrt{1-\frac{v^2}{c^2}}}$ but the relation of dynamics (11) doesn't hold because it is not invariant by Lorentz transformations. Which is invariant in relativist dynamics is the transformations of energy & moment. Energy of a corpuscle in relativist mechanics is $\varepsilon = \frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}}$.

The time τ have also another meaning: it signify that there is damping for the oscillator. Of course there is damping for a charged oscillator but for a non charged oscillator it means that vacuum is not vacuum there is always a backup energy (electromagnetic) which damp the motion of the oscillator and this energy is the fundamental state of the electromagnetic field. Equation (13) is general for any type of corpuscle.

5.Wave-corpuscle duality:

Planck oscillator should have enough time to absorb energy from radiation . The Planck condition for this is that the frequency of the oscillator times time should be a great integer or in other manner:

$$\nu.t \gg 1 \qquad (16)$$

A corpuscle should be considered as a pulse and not as a packet of waves because in the model of the last one waves are reinforced an destroy each other in many regions of space-time . The model of the pulse is limited in space-time and so the pulse should not spread so much in time. This condition implies that:

$$\nu.t < 1 \qquad (17)$$

The condition satisfying the two models is:

$\nu . t \approx 1$ (18)

Which means that:

$dt = -\frac{d\nu}{\nu^2}$ (19)

We take always dt positive and so we can omit the sign minus in (19) or its absolute value.

6-Generalasation of the notion of density of power:

The number of modes with frequencies between $\nu \& \nu + d\nu$in a $D - volume\ V$is :

$$N(\nu) = (D-1)V.\frac{2}{\Gamma\left(\frac{D}{2}\right)}\left(\frac{\sqrt{\pi}}{c}\right)^D \nu^D d\nu \quad (20)$$

The density of "energy" of a black body in $D - 1\ space\ dimensional$ Universe is :

$$\rho_T = \int_0^\infty \rho_T(\nu) d\nu \text{ with } \rho_T(\nu)d\nu = \frac{N(\nu)}{V}.\frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right)-1}d\nu = (D-2).\frac{2}{\Gamma\left(\frac{D-1}{2}\right)}.\left(\frac{\sqrt{\pi}}{c}\right)^{D-1}\frac{h\nu^D}{\exp\left(\frac{h\nu}{kT}\right)-1}d\nu \quad (21)$$

The "radiancy" of a black body in $D - dimensional$ Universe is :

$$R_T = \frac{\Gamma\left(\frac{D}{2}\right)}{\Gamma\left(\frac{D+1}{2}\right)}.\frac{c}{2\sqrt{\pi}}.\rho_T = \frac{\Gamma\left(\frac{D}{2}\right)}{\Gamma\left(\frac{D+1}{2}\right)}.\frac{c}{2\sqrt{\pi}}.\int_0^\infty 2\left(\frac{\sqrt{\pi}}{c}\right)^D.\frac{D-1}{\Gamma\left(\frac{D}{2}\right)}.\frac{h\nu^D}{\exp\left(\frac{h\nu}{kT}\right)-1}d\nu = \frac{1}{\Gamma\left(\frac{D+1}{2}\right)}.\int_0^\infty 2\left(\frac{\sqrt{\pi}}{c}\right)^{D-1}.\frac{D-1}{\Gamma\left(\frac{D}{2}\right)}.\frac{\nu^{D-2}h\nu^2}{\exp\left(\frac{h\nu}{kT}\right)-1}d\nu \quad (22)$$

This means the density of power per unit $D - 1$ volume.

The mean power of an oscillator is $\frac{h\nu^2}{\exp\left(\frac{h\nu}{kT}\right)-1}$.

The percentage of power radiant oscillators is:

$$\eta_{D-1} = \frac{\frac{1}{\Gamma\left(\frac{D+1}{2}\right)}.(D-1)}{(D-2)\frac{2}{\Gamma\left(\frac{D-1}{2}\right)}}$$

Using the identities $\Gamma(z+1) = z\Gamma(z)$ and $\Gamma(2z) = \frac{2^{2z-\frac{1}{2}}}{\sqrt{2\pi}}\Gamma(2z)\Gamma\left(z+\frac{1}{2}\right)$ than we get:

$\eta_{D-1} = \frac{1}{D-2}$ (23)

With $D \geq 3$

For $D = 2$ there is another treatment to do referring to Cardoso-De Castro theory.

7-Vacuum energy:

7-1-The mean energy of an oscillator:

Plank oscillator is a classic oscillator. The curve of the oscillator in the phase space position-moment is elliptic. The phase space of Planck oscillator in a black body is divided in regions $C_0, C_1, \dots C_n, \dots . etc.$ where the mean energy of the oscillator in the n^{th} region is :

$E_n = nh\nu N_n$ (24)

With $N_n = N. w_n$

w_n : probability of the oscillator to have the energy E_n

N : total number of oscillators

Of course we should have:

$1 = w_0 + w_1 + \cdots + w_n + \cdots$ (25)

$N = \sum_{n=0}^{n=\infty} N_n$ (26)

The region N_0 correspond to the region when all the oscillators are in their fundamental states.

The mean energy of the an oscillator is :

$$E = h\nu \sum_{n=0}^{n=\infty} nN_n$$

So:

$$\frac{E}{h\nu} - N = \sum_{n=0}^{n=\infty} nN_n - N = \sum_{n=0}^{n=\infty} (n-1)N_n = -N_0 + P$$

With $P = 0N_1 + 1N_2 + \cdots$ a great integer.

To get the same formulae as Planck did in his theory of black body in 1900 we should have $N_0 = N$ i.e at $T = 0K$ all Planck oscillators lies in the region C_0-

The mean energy of Planck oscillator is :

$U = \frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right) - 1}$ (27)

At $T = 0K$ all Planck oscillators lies in the region C_0 and they are all in their fundamental states.

Planck oscillators are classic oscillators.

According to quantum mechanics an oscillator in its fundamental state have the energy:

$$E = \frac{1}{2} h\nu \qquad (28)$$

Equation (28) is a full equation as Planck formulae for an oscillator at any temperature different from zero $(E = h\nu)$.

We can take the same Planck model for a cavity at a temperature different to zero and equal to zero with a little difference in definitions.

The mean energy of quantum oscillator at a temperature equal to zero is:

$$U = \frac{\frac{1}{2}h\nu}{\exp\left(\frac{\nu}{\nu_0}\right)-1} \qquad (29)$$

With :ν_0 is that when $\nu \ll \nu_0$ the mean energy of the oscillator is $U \approx \frac{1}{2} h\nu_0$. This frequency is declared as an universal constant.

7-2 -Vacuum energy from cosmology:

The energy density of vacuum as given by General Relativity is as follows[4] :

$$U_0 = \frac{\Lambda.c^4}{8\pi G} \approx 10^{-9} Joule.m^{-3} \quad (30)$$

With :

$\Lambda = 1{,}088\ 10^{-52} m^{-2}$: cosmological constant ;

$c = 3\ \ 10^{8} m.s^{-1}$: light celerity in vacuum ;

$G = 6{,}67\ \ 10^{-11}\ SI\ units$: gravitationnel constant ;

The total energy density of the vacuum is then according to the black body theoryat $T = 0K$:

$$U_0 = \int_0^{\infty} \frac{8\pi\nu^2}{c^3}.U d\nu = \int_0^{\infty} \frac{4\pi h}{c^3}.\frac{\nu^3}{\exp\left(\frac{\nu}{\nu_0}\right)-1} d\nu = \frac{4\pi^5 h}{15.c^3}.\nu_0^4 \qquad (31)$$

Equating (30) & (31) we get:

$$\nu_0 = \left[\frac{15\ \Lambda.c^7}{32.\pi^6.G.h}\right]^{\frac{1}{4}} \approx 0{,}7\ 10^{12} Hz \qquad (32)$$

7-3-Vacuum energy from atoms:

According to Bohr model of the atom, the electron in an hydrogen atom is moving in planetary motion (circular) as the speed of the electron is equal to αc where $\alpha = \frac{1}{137}$ the fine structure constant. The vacuum in atoms is the same vacuum in the cosmos. Vacuum which is fill with energy has a certain mechanical impedance & with negative pressure there is no friction in the motion of any corpuscle is it is given by General Relativity. However we can get the value of the constant ν_0 when we suppose that the hydrogen atom is in a medium which its temperature is near zero.

The energy exchanged with vacuum for the electron is:

$$\varepsilon = \int_0^{\alpha c} a\boldsymbol{v}.\boldsymbol{v}dt = \int_0^{\alpha c} a.v^2.dt \qquad (33)$$

With :$a(\nu) = \frac{h\nu^2}{c^2}$ the mechanical impedance of vacuum for the frequency ν.

ν : is the frequency of the fundamental state of the electromagnetic field filling the space. It is also the frequency of the oscillator "electron" as a packet of waves.

Planck oscillator is a classic oscillator and the speed of the electron can be considered as non relativist. From equations (15) &(19) injected in (33) one can deduce that:

$$\varepsilon = \int_0^{\alpha c} \frac{h\nu^2}{c^2}.v^2.\frac{d\nu}{\nu^2} = \int_0^{\alpha c} \frac{h}{c^2}.v^2.d\left(\frac{1}{2}m\frac{v^2}{h}\right) = \left[\frac{1}{4}m.\frac{v^4}{c^2}\right]_0^{\alpha c} = \frac{1}{4}\alpha^4 mc^2 \qquad (34)$$

With: $m = 9{,}1\ 10^{-31}Kg$ the mass of the electron.

Electromagnetic field is two dimensions oscillator. Its energy at low frequency & low temperature (zero Kelvin) is equal to $\frac{1}{2}h\nu_0$. For a quantum of energy we should multiply this by two.

To get the density of power in black body we had considered as the black body have four space dimensions i.e. Planck oscillator have four space dimensions. It means that a corpuscle assimilated to a point have zero dimension is without any sense . To associate four space dimensions to Planck oscillator we should that the real electron is having the three ordinary space dimensions and another compactified dimension. Planck oscillator is two space dimensions. The energy (34) is calculated only for one space dimension. To get a full quantum of energy $h\nu_0$ we should multiply (34) by 8 and so:

$$\nu_0 = 2\frac{\alpha^4 mc^2}{h} \approx 0.7\ 10^{12}\ Hz \qquad (35)$$

It is the same value given by cosmology.

$\alpha_0 = h\nu_0^2$ is the quantum of power of Planck oscillator at very low temperature.

The expression of fine structure constant in the cgs system is $\alpha = \frac{e^2}{\hbar c}$

Where: $e = 4.8\ 10^{-10}\ ues\ cgs$ the electric charge

$\hbar = \frac{h}{2\pi} = 1.034\ 10^{-27}\ erg.s$: Reduced Planck constant

Equation (33) means that the electric charge is a characteristic of vacuum.

Vacuum can be considered as a sea of positive charges $+e$ and negative charges $-e$ having a moment of inertia as:$J_0 = \frac{h}{4\pi^2\nu_0}$ which is an universal constant.

The energy of the fundamental state of electromagnetic field at zero Kelvin is:

$$E_0 = \frac{1}{2} J_0 \omega_0^2 = \frac{1}{2} . \frac{h}{4\pi^2 \nu_0} . (2\pi\nu_0)^2 = \frac{1}{2} h\nu_0$$

In a general case we have the exchanged energy for the electron with vacuum as follows:

$\varepsilon = \int_0^{\alpha c} a. v^2. dt = \int_0^{\alpha c} a. v^2. d\tau$

With: $a = \frac{h\nu_0^2}{c^2}$: universal constant

$\tau = \frac{\frac{m}{a}}{\sqrt{1-\frac{v^2}{c^2}}}$: inertial time of the electron with $d\tau = dt$ when the energy of the electron is varying .

with : m : mass of the corpuscle (here the electron)

We have always:

$h\nu = ac^2\tau = \frac{m.c^2}{\sqrt{1-\frac{v^2}{c^2}}}$ than: $d\tau = \frac{h}{ac^2} d\nu$

So

$$\varepsilon = \int_0^{\alpha c} a v^2 d\tau = \int_{\frac{m}{a}}^{\frac{\frac{m}{a}}{\sqrt{1-\alpha^2}}} ac^2 (1 - \frac{(\frac{m}{a})^2}{\tau^2}) d\tau = ac^2 [\frac{\frac{m}{a}}{\sqrt{1-\alpha^2}} - \frac{m}{a} + (\frac{m}{a})^2 \left(\frac{\sqrt{1-\alpha^2}}{\frac{m}{a}} - \frac{1}{\frac{m}{a}} \right)]$$

In a first order approximation ($\alpha \sim 10^{-2}$) we get:

$$\varepsilon = mc^2 \left[\frac{1}{\sqrt{1-\alpha^2}} - 1 + \sqrt{1-\alpha^2} - 1 \right] \approx mc^2 \left(1 + \frac{1}{2}\alpha^2 - 1 + 1 - \frac{1}{2}\alpha^2 - 1 \right) = 0$$

In a second order approximation we get:

$$\varepsilon \approx mc^2 \left(1 + \frac{1}{2}\alpha^2 + \frac{3}{8}\alpha^4 - 1 + 1 - \frac{1}{2}\alpha^2 - \frac{1}{4}\alpha^4 - 1 \right) = \frac{1}{4}\alpha^4 mc^2$$

It is the same value as the electron move in a low speed.

But for low speed and high speed we have that:

$dt = \frac{d\nu}{\nu^2} = d\tau = \frac{h}{ac^2} d\nu$

It comes that: $\nu = \sqrt{\frac{a}{h}}. c = \nu_0$

If $\nu \gg \nu_0$: than the treatment of the corpuscle motion should be in the ordinary life (classic approximation or relativist case).

If $\nu \ll \nu_0$: than the treatment of the corpuscle motion should be in the quantum mechanics

It is evident that to get a more precise value of ν_0 is as follows:

$$\frac{1}{2}h\nu_0 = 4mc^2\left[\frac{1}{\sqrt{1-\alpha^2}} - 1 + \sqrt{1-\alpha^2} - 1\right]$$

With m: the mass of the electron.

In general for a corpuscle in motion the exchanged energy with vacuum is :

$$\varepsilon = \int_0^v a\boldsymbol{v}.\boldsymbol{v}dt = \int_0^v a.v^2.d\tau = \int_{\tau_0}^{\frac{\frac{m}{a}}{\sqrt{1-\frac{v^2}{c^2}}}} a.c^2\left(1-\frac{(\frac{m}{a})^2}{\tau^2}\right).d\tau = a.c^2\left[\frac{\frac{m}{a}}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{m}{a} + (\frac{m}{a})^2\left(\frac{\sqrt{1-\frac{v^2}{c^2}}}{\frac{m}{a}} - \frac{1}{\frac{m}{a}}\right)\right] = mc^2\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 + \sqrt{1-\frac{v^2}{c^2}} - 1\right] = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\left[1 - 2\sqrt{1-\frac{v^2}{c^2}} + \sqrt{1-\frac{v^2}{c^2}}^2\right) = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}(1 - \sqrt{1-\frac{v^2}{c^2}})^2$$

It is evident that in a first order approximation ($v \ll c$) we get:

$$\varepsilon \approx \frac{1}{4}m.\frac{v^4}{c^2}$$

The forth dimension of this corpuscle can be considered as a little sphere. This sphere is considered as a Universe with 4th spatial dimensions. The exchanged energy with vacuum in one direction of this Universe is (of course the corpuscle is considered as in its fundamental state because it reach the speed zero in a very close little sphere: the 4th spatial dimension):

$$\frac{1}{2}\Delta\varepsilon = \frac{\varepsilon}{4} = \frac{1}{16}m.\frac{v^4}{c^2} \quad \text{as} \quad \Delta\varepsilon = \frac{\varepsilon}{2} = \frac{1}{8}m.\frac{v^4}{c^2} = \frac{4}{32}m.\frac{v^4}{c^2} = \frac{4\varepsilon}{8}$$

The total energy of the corpuscle should be (relativist case):

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \pm \frac{\Delta\varepsilon}{8} = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \pm \frac{1}{8}\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\left(1 - \sqrt{1-\frac{v^2}{c^2}}\right)^2$$

We can consider the new Universe as having an extra-dimension assimilated as a little sphere around a point which modelized the corpuscle and the uncertainty of the exchanged energy of the corpuscle in one previous dimension is equal to:

$$\frac{1}{2}\Delta\varepsilon = \frac{\varepsilon}{16} \quad \text{as} \quad \Delta\varepsilon = \frac{\varepsilon}{8} = \frac{4\varepsilon}{32}$$

The total energy of the corpuscle should be (relativist case):

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \pm \frac{1}{8}\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\left(1 - \sqrt{1-\frac{v^2}{c^2}}\right)^2 \pm \frac{1}{32}\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\left(1 - \sqrt{1-\frac{v^2}{c^2}}\right)^2$$

And so on.

The term $\frac{1}{8}\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)^2 \approx \frac{1}{32}m\frac{v^4}{c^2}$ is for the corpuscle the beginning of *the quantum foam*.

We can add many terms of this expression as the dimensionality of the Universe grow. For $D-dimensional$ Universe we have:

$$E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \pm \frac{D-3}{8}\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}\left(1-\sqrt{1-\frac{v^2}{c^2}}\right)^2$$

Let's pose : $\xi = \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}$ the inertia of the corpuscle .

So $E = \xi c^2 \pm \frac{D-3}{8}\ \xi c^2(1-\frac{m}{\xi})^2$

Let's take the case $E = \xi c^2 + \frac{D-3}{8}\ \xi c^2(1-\frac{m}{\xi})^2$

The minimum of the energy by referring to the inertia ξ of the corpuscle is when:

$\frac{\partial E}{\partial \xi} = 0$ which mean that $\xi^2 = m^2.\frac{\frac{D-3}{8}}{1+\frac{D-3}{8}}$

The corpuscle is considered as a packet of plane waves which are reinforced in limited space and destroy each other away. The corpuscle have not an intrinsic moment (or spin).

De Broglie wave-function of the corpuscle is as follows:

$\psi(\boldsymbol{x},t) = A\ \exp(i\boldsymbol{k}.\boldsymbol{x} - i\omega t)$

With : A : amplitude of the wave-function

$\hbar\boldsymbol{k} = \boldsymbol{p} = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$: the relationship between the moment of the corpuscle and the wave-vector

$\hbar\omega = \xi c^2$: the relationship between the frequency of the packet of waves and the inertia of the corpuscle

Klein-Gordon equation of the wave-function is as follows (equation of propagation of the packet):

$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi(\boldsymbol{x},t) - \nabla^2\psi(\boldsymbol{x},t) = -\frac{m^2c^2}{\hbar^2}\psi(\boldsymbol{x},t)$

Or in other words:

$\nabla^2\psi(\boldsymbol{x},t) = -(\frac{\omega^2}{c^2} - \frac{m^2c^2}{\hbar^2})\psi(\boldsymbol{x},t)$

Let's suppose that the wave-function of the corpuscle is independent from time and have a spherical symmetry . This function will be a potential energy that describe the interaction between two stationary corpuscles or the potential which can generate this corpuscle itself. Let's denote this potential by $V(r)$ where r is the distance in spherical coordinates between the two corpuscles or the distance from the corpuscle itself in the center.

So: $\nabla^2 V(r) = -(\frac{\omega^2}{c^2} - \frac{m^2c^2}{\hbar^2})V(r)$ with : $\omega^2 = \xi^2 . \frac{c^4}{\hbar^2} = \frac{m^2c^4}{\hbar^2} \frac{\frac{D-3}{8}}{1+\frac{D-3}{8}}$

Than : $\nabla^2 V(r) = \frac{m^2c^2}{\hbar^2}\left(\frac{1}{1+\frac{D-3}{8}}\right)V(r) = \frac{1}{r^2} . \frac{d}{dr}(r^2 \frac{dV(r)}{dr})$

After integration:

$V(r) = \frac{K}{r}\exp(-\frac{mc}{\hbar\sqrt{1+\frac{D-3}{8}}}.r)$ with : K: $constant, D \geq 3$

-For: $m = 0$: the potential will be the electrical potential

-For :$D \to \infty$: the potential will be the gravitational potential generated by the mass m in the center.

Of course we can take the other case where $E = \xi c^2 - \frac{D-3}{8}\ \xi c^2 (1 - \frac{m}{\xi})^2$ than we get:

$V(r) = \frac{K}{r}\exp(-\frac{mc}{\hbar\sqrt{1-\frac{D-3}{8}}}.r)$ with : K: $constant, 3 \leq\ D \leq 10$

Of course we can take for the energy of the corpuscle the second order of the quantum foam and so after development we obtain the Newtonian potential and a term which depends on the dimensionality: this is *quantum gravitation*.

We can continue indefinitely with other orders of the quantum foam.

8-The meaning of entropy:

In general we mean by entropy in thermodynamics the degree of disorder. More the disorder of a gas is great more its entropy is high. To derive the black body radiation law Planck had the ingenuous idea to link the thermodynamic entropy as defined by Boltzmann to the statistical entropy (or mathematical entropy) defined as the sum of single entropy of all Planck oscillators. To get the entropy of a single oscillator Planck consider how many manner to distribute P parts of energy ε on N box (or resonator) . The mean energy of an oscillator is the contribution of all single resonator in the boxes to the total energy of the oscillator. Considering the second law of thermodynamics with constant volume as $T\ dS = dU$ and Wien displacement law in thermal equilibrium Planck get Its law of black body radiation.

We have:

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right) - 1}\right) = \frac{d}{d\nu}\left(\frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right) - 1}\right) . \frac{d\nu}{dt} = h\nu^2 . \frac{\exp\left(\frac{h\nu}{kT}\right) - 1 - \frac{h\nu}{kT} . \exp\left(\frac{h\nu}{kT}\right)}{(\exp\left(\frac{h\nu}{kT}\right) - 1)^2}$$

And to be conform with Planck approximation ($h\nu \ll kT$ or for large U) we get:

$\frac{dU}{dt} \approx -h\nu^2$ (36)

Equation (34) is not equal to a constant as Planck said . It is equal to constant only when the temperature approaches zero Kelvin.

For the entropy of the oscillator:

$$dS = \frac{dU}{T} \cong -\frac{h\nu^2}{T}.\frac{d\nu}{\nu^2} = -\frac{hd\nu}{T} = -\frac{h\nu^2}{c^2}.\frac{c^2 d\nu}{\nu^2 T}$$

Or in other manner:

$TdS = -ac.cdt$ (37)

With $a = \frac{h\nu^2}{c^2}$: the mechanical impedance of the medium (it becomes the mechanical impedance of the vacuum for temperature approaches to zero Kelvin).

$dt = \frac{d\nu}{\nu^2}.$

From (35) we deduce that the variation of the entropy is the work of a friction force equal to "ac" along the length "cdt". It is like that the oscillator is moving with a speed "c" in a medium with a friction coefficient equal to "a".

9-The dimension of the 4th space dimension:

Around the Bohr radius the electron will execute vibration in the 4th dimension which can be closed or open. Its value is:

$r_1 = \int_0^{\alpha c} v dt = \int_0^{\alpha c} v \frac{d\nu}{\nu^2}$ with $h\nu \approx mc^2 + \frac{1}{2}mv^2$

It is very easy to get: $r_1 = \frac{1}{3}.\frac{\alpha^3 h}{mc} = \frac{1}{3}.\frac{6.62\ 10^{-34}}{137^3 x 9.1\ 10^{-31} x 3\ 10^8} = 3.14349\ 10^{-19}\ meter$ (38)

For the electron in the first layer of an atom which have an atomic number Z the extra-dimension have a radius as to replace the constant α by $Z\alpha$ in in the last equation:

$r_Z = \frac{1}{3}.\frac{Z^3\alpha^3 h}{mc} = Z^3 x 3.14\ 10^{-19}\ meter$ (39)

The forth dimension can be considered as a little sphere of radius r_Z and the electron is a point oscillating in this sphere with maximum speed equal to $Z\alpha c$ .The pressure in this sphere is negative.

We can consider the little sphere as a Universe in itself and so the exchanged energy of the electron for this extra-dimension can be divided by 4 and the uncertainty of the exchanged energy of the electron in one previous dimension is equal to :

$\frac{1}{2}\Delta\varepsilon = \frac{1}{16}\alpha^4 mc^2$ as : $\Delta\varepsilon = \frac{1}{8}\alpha^4 mc^2 = \frac{4}{32}\alpha^4 mc^2$

So for the bound energy of the electron there is an uncertainty as:

$E = -R_\infty \pm \frac{\alpha^4}{32} mc^2$ with $R_\infty = \frac{1}{2}\alpha^2 mc^2$: hydrogen ionization energy.

So:

$$E = -R_\infty \pm \frac{\alpha^2}{16} R_\infty$$

We can consider the new Universe as having an extra-dimension assimilated as a little sphere around a point which modelized the electron and the uncertainty of the exchanged energy of the electron in one previous dimension is equal to:

$\frac{1}{2}\Delta\varepsilon = \frac{1}{64}\alpha^4 mc^2$ as $: \Delta\varepsilon = \frac{1}{32}\alpha^4 mc^2 = \frac{4}{128}\alpha^4 mc^2$

So for the energy of the electron there is an uncertainty as:

$$E = -R_\infty \pm \frac{\alpha^2}{16} R_\infty \pm \frac{\alpha^2}{64} R_\infty$$

We can consider the new Universe as having an extra-dimension assimilated as a little sphere around a point which modelized the electron and the uncertainty of the exchanged energy of the electron in one previous dimension is equal to:

$\frac{1}{2}\Delta\varepsilon = \frac{1}{128}\alpha^4 mc^2$ as $: \Delta\varepsilon = \frac{1}{64}\alpha^4 mc^2 = \frac{4}{256}\alpha^4 mc^2$

So for the energy of the electron there is an uncertainty as:

$E = -R_\infty \pm \frac{\alpha^2}{16} R_\infty \pm \frac{\alpha^2}{64} R_\infty \pm \frac{\alpha^2}{128} R_\infty$ and so on.

With those expression of the bound energy we can get extra-dimensions more and more less than $r_1 = 3.14349\ 10^{-19}\ meter$ from the same procedure of calculations in equation (38).

Let’s take the case:

$$E = -R_\infty + \frac{\alpha^2}{16} R_\infty = -R_\infty \left(1 - \frac{\alpha^2}{16}\right) = -E_{kinetic}$$

So the speed of the electron is: $v = \alpha c\sqrt{1 - \frac{\alpha^2}{16}}$

The first radius of the fine structure extra-dimension is :

$$r_{fs1} = \int_0^{\alpha c\sqrt{1-\frac{\alpha^2}{16}}} v dt = \int_0^{\alpha c\sqrt{1-\frac{\alpha^2}{16}}} v \frac{dv}{v^2} = \frac{1}{3}.\frac{\alpha^3 h}{mc}\sqrt{1 - \frac{\alpha^2}{16}}^3 - \frac{1}{5}.\frac{\alpha^5 h}{mc}\sqrt{1 - \frac{\alpha^2}{16}}^5$$

The second radius of the fine structure extra-dimension is when we take :

$$E = -R_\infty - \frac{\alpha^2}{16}R_\infty = -R_\infty\left(1+\frac{\alpha^2}{16}\right) = -E_{kinetic}$$

So the speed of the electron is: $v = \alpha c\sqrt{1+\frac{\alpha^2}{16}}$

Than:

$$r_{fs2} = \int_0^{\alpha c\sqrt{1+\frac{\alpha^2}{16}}} vdt = \int_0^{\alpha c\sqrt{1+\frac{\alpha^2}{16}}} v\frac{dv}{v^2} = \frac{1}{3}.\frac{\alpha^3 h}{mc}\sqrt{1+\frac{\alpha^2}{16}}^3 - \frac{1}{5}.\frac{\alpha^5 h}{mc}\sqrt{1+\frac{\alpha^2}{16}}^5$$

10-The quantum of power from thermodynamics:

Thiesen equation of the density of energy of black body radiation is [5]:

$$E_\lambda d\lambda = T^5\psi(\lambda T)d\lambda \qquad (40)$$

Where λ is the wavelength , $E_\lambda d\lambda$ the spatial density of energy of black body radiation in the interval λ and $\lambda + d\lambda$,T the temperature and $\psi(x)$ an universal function of the single argument x.

Introducing the density of energy in the frequency interval ν and $\nu + d\nu$ as $u_\nu d\nu$ and doing the substitution $E_\lambda d\lambda$ by $u_\nu d\nu$ and $\lambda, \lambda + d\lambda$ by , $\nu, \nu + d\nu$, replace $d\lambda$ by $\frac{c}{\nu^2}d\nu$ Thiesen gives:

$$u_\nu d\nu = T^5\psi\left(\frac{cT}{\nu}\right)\frac{c}{\nu^2}d\nu \qquad (41)$$

Let's introduce the density of power radiation in a black body as:

$$p_\nu = C_1\, u_\nu.\nu \qquad (42)$$

Where $C_1: a\ constant\ without\ dimension$

It comes that:

$$p_\nu = C_1\, T^5\psi\left(\frac{cT}{\nu}\right)\frac{c}{\nu} \qquad (43)$$

According to Kirchoff-Clausius law the rate of emission of energy of a black body surface in a thermal medium at temperature T is inversely proportional to c^2 so :

$$\int u_\nu d\nu \sim \frac{1}{c^2} \qquad (44)$$

By analogy to equation (44) the rate of the rate of emission of energy of a black body surface in a thermal medium at temperature T should be inversely proportional to c^2 so :

$\int p_\nu d\nu \sim \frac{1}{c^2}$ (45)

Combining the thermodynamic law of Stefan-Blotzmann $E = \sigma . T^4$ and Wien empirical displacement law $\lambda_{max} . T = Constant$ implying the maximum of energy emission we get that:

$E_\lambda = \frac{dE}{d(\lambda T)} = \sigma . T^4 \psi(\lambda T)$ (46)

Where $\psi(\lambda T)$ is an universal function of an unique argument $x = \lambda T$.

Than the energy density is inversely proportional to c^3 and we get:

$u_\nu = \frac{T^5}{c^3 \nu^2} g_1\left(\frac{T}{\nu}\right) = \frac{\nu^3}{c^3} g_2\left(\frac{T}{\nu}\right)$ (47)

Where g_1 & g_2 are universal functions independent from c i.e. $u_\nu \lambda^3$ is the same at $T = Constant$.

Also the power density is inversely proportional to c^3 and we get:

$p_\nu = \frac{T^5}{c^3 \nu} f_1\left(\frac{T}{\nu}\right) = \frac{\nu^4}{c^3} f_2\left(\frac{T}{\nu}\right)$ (48)

Where f_1 & f_2 are universal functions independent from c i.e. $p_\nu \lambda^4$ is the same at $T = Constant$.

Let's continue with Planck for the intensity of radiation as:

$I = \frac{\nu^2}{c^2} U$ (49)

Where U is the mean energy of the oscillator with resonant frequency ν in the radiation field .

The density of energy per unit volume and per unit frequency is:

$u_\nu = \frac{8\pi I}{c} = \frac{8\pi \nu^2}{c^3} U$ (50)

There for:

$p_\nu = C_1 \frac{8\pi \nu^3}{c^3} U$ (51)

From (51) & (48) we get:

$U = \frac{\nu}{8\pi C_1} f_2\left(\frac{T}{\nu}\right) = T f_3\left(\frac{T}{\nu}\right)$ (52)

Where c does not appear and f_3 is another universal function.

Introducing the entropy of the oscillator as Planck did:

$$\frac{1}{T} = \frac{dS}{dU} \qquad (53)$$

Than:

$$\frac{dS}{dU} = \frac{1}{U} f_3\left(\frac{T}{\nu}\right) \qquad (54)$$

From (50) & (47) it follows that:

$$U = \nu\, g_3\left(\frac{T}{\nu}\right) \qquad (55)$$

Where c does not appear at all.

In place of this we write:

$$T = \nu g_4\left(\frac{U}{\nu}\right) \qquad (56)$$

From (51) & (48) it follows that:

$$U = \nu f_4\left(\frac{T}{\nu}\right) \qquad (57)$$

Where c does not appear at all.

In place of this we write:

$$T = \nu f_5\left(\frac{U}{\nu}\right) \qquad (58)$$

From (53) & (56) we get:

$$\frac{dS}{dU} = \frac{1}{\nu} g_5\left(\frac{U}{\nu}\right) \qquad (59)$$

From (53) & (58) we get:

$$\frac{dS}{dU} = \frac{1}{\nu} f_6\left(\frac{U}{\nu}\right) \qquad (60)$$

Integrate (59) or (60) , it follows that:

$$S = F\left(\frac{U}{\nu}\right) \qquad (61)$$

Continuing with Planck that the statistical entropy is the same thermodynamic entropy, Planck obtain that:

$$U = \frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right) - 1} \qquad (62)$$

Than:

$$u_\nu = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right)-1} \qquad (63)$$

$$p_\nu = C_1\frac{8\pi\nu^2}{c^3}\frac{h\nu^2}{\exp\left(\frac{h\nu}{kT}\right)-1} \qquad (64)$$

Equation (64) is like that every oscillator have a mean power as:

$$W = \frac{h\nu^2}{\exp\left(\frac{h\nu}{kT}\right)-1} \qquad (65)$$

The coefficient C_1 can be considered as a coefficient to modulate the number of oscillators to emit or absorb energy per unit time.

We can write (64) as:

$$p_\nu d\nu = C_1\frac{8\pi\nu^3}{c^3}\frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right)-1}d\nu = C_1 c\frac{8\pi\nu^3}{c^4}\frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right)-1}d\nu \qquad (66)$$

Equation (66) is like that the oscillator have a mean energy of $\frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right)-1}$ in a four dimension space i.e.p_ν will be the radiated energy per unit time from a 4-space dimension black body.

The first part of equation (66) the coefficient $C_2 = C_1 c$ is a modulation coefficient and after integration it will be as a part of Boltzmann-Stefan coefficient for black body radiating from 4-space volume. Than we can considerate the electron as a particle with 4-space dimensions.

We can continue this analysis and attribute to the electron more than three space dimensions in Planck theory or more than 4-space dimensions in this model.

Equations (62) & (65) means that:

-The quantum of energy is : $\varepsilon = h\nu$ (67)

-The quantum of power is: $\delta = h\nu^2$ (68)

Of course (62) & (65) can be obtained by statistical manner as follows:

$$U = \frac{\sum_0^\infty n\varepsilon.\exp(\frac{-n\varepsilon}{kT})}{\sum_0^\infty \exp(\frac{-n\varepsilon}{kT})} = \frac{\varepsilon}{\exp\left(\frac{\varepsilon}{kT}\right)-1}$$

$$W = \frac{\sum_0^\infty n\delta.\exp(\frac{-n\varepsilon}{kT})}{\sum_0^\infty \exp(\frac{-n\varepsilon}{kT})} = \frac{\delta}{\exp\left(\frac{\varepsilon}{kT}\right)-1}$$

It always works for any quantum as $Q = h\nu^{1+l}$ with l integer i.e.:

$$K = \frac{\sum_0^\infty nQ.\exp(\frac{-n\varepsilon}{kT})}{\sum_0^\infty \exp(\frac{-n\varepsilon}{kT})} = \frac{Q}{\exp\left(\frac{\varepsilon}{kT}\right)-1}$$

Because:

Let's pose that: $x = e^{-\frac{\varepsilon}{kT}} \quad so \quad x < 1.$

$\sum_{n=0}^{\infty} x^n = \frac{first\ element\ in - first\ element\ out}{1 - raison} = \frac{1-0}{1-x} = \frac{1}{1-x}$ because it is a sum of a geometric serial with a raison equal to x.

$\frac{d}{dx}\sum_0^{\infty} x^n = \sum_1^{\infty} n x^{n-1} = \frac{1}{(1-x)^2}$

So for example:

$U = h\nu \frac{\sum_0^{\infty} n x^n}{\sum_0^{\infty} x^n} = \frac{h\nu}{e^{\frac{-h\nu}{kT}} - 1}$ CQFD.

Let's define the radiancy of a black body in a Universe of $3 + n$ dimensions with n the number of space extra-dimensions:

$$R_n = C_n u_\nu \nu^n \qquad (69)$$

$C_n: a\ coefficient\ without\ dimension$

It comes that:

$$R_n = C_n\, T^5 \psi\left(\frac{cT}{\nu}\right) c\nu^{n-2} \qquad (70)$$

By analogy to equation (44) "the rate of the rate…(n+1) times" of emission of energy of a black body surface in a thermal medium at temperature T should be inversely proportional to c^2 so :

$$\int R_n d\nu \sim \frac{1}{c^2} \qquad (71)$$

Also "the rate of the rate…(n+1) times" of emission of energy is inversely proportional to c^3 and we get:

$$R_n = \frac{T^5}{c^3 \nu^{2-n}} k_1\left(\frac{T}{\nu}\right) = \frac{\nu^{3+n}}{c^3} k_2\left(\frac{T}{\nu}\right) \qquad (72)$$

Where k_1 & k_2 are universal functions independent from c i.e. $R_n \lambda^{3+n}$ is the same at $T = Constant$.

There for:

$$R_n = C_n \frac{8\pi \nu^{2+n}}{c^3} U \qquad (73)$$

From (73) & (72) we get:

$$U = \frac{\nu}{8\pi C_n} k_2\left(\frac{T}{\nu}\right) = T k_3\left(\frac{T}{\nu}\right) \qquad (74)$$

Where c does not appear and k_3 is another universal function.

Introducing the entropy of the oscillator as Planck did:

$$\frac{1}{T} = \frac{dS}{dU} \qquad (75)$$

Than:

$$\frac{dS}{dU} = \frac{1}{U} k_3\left(\frac{T}{\nu}\right) \qquad (76)$$

From (74) it follows that:

$$U = \nu k_4\left(\frac{T}{\nu}\right) \qquad (77)$$

Where c does not appear at all.

In place of this we write:

$$T = \nu k_5\left(\frac{U}{\nu}\right) \qquad (78)$$

From (78) & (75) we get:

$$\frac{dS}{dU} = \frac{1}{\nu} k_6\left(\frac{U}{\nu}\right) \qquad (79)$$

Integrate (79) , it follows that:

$$S = H\left(\frac{U}{\nu}\right) \qquad (80)$$

Continuing with Planck that the statistical entropy is the same thermodynamic entropy, Planck obtain that:

$$U = \frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right)-1} \qquad (81)$$

Than:

$$R_n = C_n \frac{8\pi\nu^2}{c^3}\frac{h\nu^{1+n}}{\exp\left(\frac{h\nu}{kT}\right)-1} = C_n \frac{8\pi\nu^{1+n}}{c^3}\frac{h\nu^2}{\exp\left(\frac{h\nu}{kT}\right)-1} \qquad (82)$$

Equation (82) is like that every oscillator have a mean power as:

$$W = \frac{h\nu^2}{\exp\left(\frac{h\nu}{kT}\right)-1} \qquad (83)$$

We can write (82) as:

$$R_n d\nu = C_n c^n . \frac{8\pi\nu^{2+n}}{c^{3+n}}\frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right)-1} d\nu = C_n c^{n-1} . \frac{8\pi\nu^{1+n}}{c^{2+n}}\frac{h\nu^2}{\exp\left(\frac{h\nu}{kT}\right)-1} d\nu \qquad (84)$$

Equation (84) is like that the oscillator have a mean energy of $\frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right)-1}$ in a $3+n$ space dimensions black body. The first term the coefficient $C'_n = C_n c^n$ is a modulation coefficient and after integration it will be as a part of Boltzmann-Stefan coefficient for black body radiating from $3+n$ -space volume. It is also that the oscillator have a mean power of $\frac{h\nu^2}{\exp\left(\frac{h\nu}{kT}\right)-1}$ in a $2+n$ -space dimensions black body. The term $C''_n = C_n c^{n-1}$ is a modulation coefficient and after integration it will be as a part of Boltzmann-Stefan coefficient for a black body radiating from $3+n$-space volume to $2+n$-surface.

12-Quantum Cosmology:

The equations of gravitational field according to General Relativity are:

$$R_{ik} - \frac{1}{2} R g_{ik} = \frac{8\pi G}{c^4} T_{ik} + \Lambda g_{ik} \qquad (85)$$

With R_{ik} : curvature tensor;

R : scalar (curvature of space-time)

T_{ik} : momentum-energy tensor of matter

g_{ik} : metric tensor with signature (+,-,-,-)

$i, k = 0,1,2,3$: tensor indices

Equating again (30) & (31) we deduce that:

$$\Lambda = \frac{32\pi^6 hG}{15.c^7} . \nu_0^4 \qquad (86)$$

For a space with D space-dimensions ($D \geq 4$) the half quantum of vacuum energy is:

$$\frac{1}{2} h\nu_0 = D . \frac{1}{4} \alpha^4 mc^2$$

So $\nu_0 = \frac{D}{2h} \alpha^4 mc^2$ (87)

Replace (87) in (86) we get :

$$\Lambda = \frac{16\pi^6 \alpha^{16} m^4 cG}{15.h^4} D^4 \qquad (88)$$

With: $m = 9{,}1\ 10^{-31} Kg$ the mass of the electron.

It is clear from equation (88) that constant Λ of General Relativity depends on the structure of space-time precisely it depends on the number of extra-dimensions to add to the three ordinary space dimensions.

The most volume for space of D dimensions at a distance R is in the exterior layer $\epsilon = \frac{R}{D}$ [6]. For those layers there will be high vacuum energy when D is high so the gravitational field will differ from layer to layer and the Cosmos will be as a superposition of layers or we can say that when the Universe is expanding its space-dimensionality change.

Equation (88) works well for $D = 4$. Instead to choose the Hydrogen atom for the speed of the electron as Bohr did we can also choose any atom having atomic number Z . The speed of the first electron of the first layer is deduced from Bohr atom model : only replace the constant α by αZ in the expression of the speed of the electron so energy vacuum for 4-space dimension electron is:

$$\frac{1}{2} h\nu_0 = 4.\frac{1}{4}\alpha^4 Z^4 mc^2$$

Which mean that:

$$\nu_0 = \frac{Z^4}{2h}\alpha^4 mc^2 \qquad (89)$$

Which is not so different from equation (88).

We can more complicate this equation for D space dimensions Universe as:

$$\nu_0 = \frac{D}{2h}\alpha^4 Z^4 mc^2 \qquad (90)$$

It is evident that the Universe will be in accelerated expansion. No need for a demonstration. All what we deal in physics is models. I think I had propose a good model for Universe.

13- An experiment to detect the forth space dimension:

Let's take the experiment of the photo-electric effect. We have a photo cell illuminated by light with a frequency ν . The photo-electrons are accelerated or decelerated by a tension V applied on its ends and the correspondent current is measured.

Einstein equation for the photo-electrons:

$$eV = h\nu - W_s \qquad (91)$$

$W_s = h\nu_s$: the work done by light when $V = 0\ Volt$

ν_s : starting frequency for the photo-electric phenomena of the cell.

The uncertainty of the measuring of the potential V is:

$$e\Delta V = h\Delta\nu - \Delta W_s \qquad (92)$$

If we suppose that the frequency of the incident light is highly precise than in absolute manner we have:

$$\Delta V \sim \frac{\Delta W_s}{e} \qquad (93)$$

With: $\Delta W_s = 2\alpha^4 mc^2$ & m: $the\ mass\ of\ the\ electron$

So from equation (93) we deduce that the curve $I = f(V)$ never coincide with the $V - axis$ at exactly when the current $I = 0$.

Grosso modo it is easy to estimate it as : $\Delta V \approx 10\ milli - Volt$.

References:

[1] Cardoso & de Castro "The blackbody radiation in D-dimensional Universes" arxiv 0510002v1 ,

https://arxiv.org/abs/quant-ph/0510002

[2] Max Planck "Theory of Heat Radiation" page 269, https://www.gutenberg.org/files/40030/40030-pdf.pdf

[3] Max Planck « On the Law of Distribution of Energy in the Normal Spectrum"

http://strangepaths.com/files/planck1901.pdf

[4] M. P. Hobson, G. P. Efstathiou and A. N. Lasenby 2006 "General Relativity" page187 , Cambridge University.https://wigner.hu/~barta/GRcourse2020/Hobson%20M.P.,%20Efstathiou%20G.P.,%20Lasenby%20A.N.%20-%20GR%20-%20An%20Introduction%20for%20Physicists(Cambridge,2006).pdf

[5] M.Thiesen , Verhandl.d. Deutch. Phys. Gesellsch. 2,66 (1900) https://www.lawebdefisica.com/arts/improvementwiens.pdf

[6]High-dim-space.pdf, ttps://www.cs.cmu.edu/~venkatg/teaching/CStheory-infoage/chap1-high-dim-space.pdf