Problem of Principal Axis and DBZC: $\tan(\pi/2) = 0$

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Abstract: In this note, we would like to see the fundamental result $\tan(\pi/2) = 0$ from the famous problem of principal axis in connection with the division by zero calculus $\frac{f(x)}{(x-a)^n}|_{x=a} := \frac{f^{(n)}(a)}{n!}$.

Key Words: Division by zero, division by zero calculus, $1/0 = 0/0 = 0$, $\tan(\pi/2) = 0$, principal axis, eigen value problem, diagonalization of matrices.

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1 Result

We assume the elementary properties of the division by zero and division by zero calculus:

$$\frac{1}{0} = \frac{0}{0} = 0$$

and

$$\frac{f(x)}{(x-a)^n}|_{x=a} := \frac{f^{(n)}(a)}{n!}.$$

See the basic references. However, here, we would like to point out the simple and fundamental result

$$\tan\frac{\pi}{2} = 0,$$

in order to see its naturalness from the famous problem of principal axis in connection with the division by zero calculus.

For

$$ax^2 + 2hxy + by^2,$$

by

$$A = \begin{pmatrix} a & h \\ h & b \end{pmatrix},$$

and

$$P = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$

we set

$${}^tPAP = \begin{pmatrix} \alpha & \gamma \\ \gamma & \beta \end{pmatrix}$$

where

$$\alpha = a\cos^2\theta + 2h\cos\theta\sin\theta + b\sin^2\theta$$
$$\gamma = h\cos^2\theta - (a-b)\cos\theta\sin\theta - h\sin^2\theta$$

and

$$\beta = a\sin^2\theta - 2h\cos\theta\sin\theta + b\cos^2\theta.$$

If $a \neq b, h \neq 0$, with θ:

$$\tan 2\theta = \frac{2h}{a-b}$$

we have

$$\gamma = 0, \alpha = a, \beta = b.$$

If $h = 0$, then, of course, $\theta = 0$ and $\tan 0 = 0$.

If $h \neq 0$ and $a = b$, then $\gamma = 0$ with $\theta = \pi/4$; that is.

$$\tan\frac{\pi}{2} = \frac{2h}{0} = 0.$$

Acknowledgement

The author is gathering examples on the division by zero and division by zero calculus, and this example was listed with No. 1319 on 22th, September 2024. Of course, the author knew the fact in many years ago, however, the result is too important, for many peoples.

References

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