A NEW APPROACH TO THE GOLDBACH CONJECTURE

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ABSTRACT

Abstract In 1742 Christian Goldbach suggested that any even number four or greater is the sum of two primes. The Goldbach conjecture remains unproven to the present day, although it has been verified for all even numbers up to 4×10^{18}. Previously this problem has been attacked using deep analytical methods and with complicated integer sieves. This paper takes an entirely new approach to the Goldbach conjecture linking pairs of prime numbers (prime pairs) to matching pairs of composite integers (composite pairs) where both pairs sum to the same even natural number.

1 Introduction

In the past one hundred years more and more complicated analytical and sieve methods have been used to find partial answers to the Goldbach conjecture. In 1924 assuming the Riemann Hypothesis, Hardy and Littlewood showed that the Goldbach conjecture was true for all even numbers up to n for all but $n^{1/2+c}$ for small c [1]. In 1948 with sieve theory Alfred Renyi showed that every sufficiently large even number is the sum of a prime and a composite number having not more than k factors [2]. J. Pintz and I. Z. Ruzsa set k at 8 in 2020 [3]. In 1973 Chen Jingrum used sieve theory to prove that every sufficiently large even number is the sum of two primes or a prime and a composite number with two factors [4].

We take an altogether different approach to proving the Goldbach conjecture by linking pairs of primes (prime pairs) with matching pairs of composites (composite pairs), where both sets of pairs sum to the same even natural number. For an even number n the number of composites c that can be paired with primes p forming mixed pairs can be no more than the total number of primes. We remove composite pairs from the set of composite numbers until the number of composites remaining equals the total number of primes. After that each composite pair removed will be matched by a prime pair.

For an even n let there be $2z$ odd numbers between 3 and $n-3$. Note: If n equals $4b+2$, a composite or prime $2b+1$ is counted twice. Since $c+p$ equals $2z$, the number of composites and primes must be even or odd. $c-p$ equals $2a$ for some integer a. If $a<0$, there are at least two more primes than composites. There would be at least one prime pair $(p, n-p)$ that sums up to n. The crossover point is $n=122$ with exactly 30 primes and composites between 3 and $n-3$.

$$(c+p)+(c-p)=2z+2a=2c \text{ giving } z+a \text{ composites.} \tag{1}$$

$$(c+p)-(c-p)=2z-2a=2p \text{ giving } z-a \text{ primes.} \tag{2}$$

2 Composite prime and mixed pairs

Powers of two contain the smallest proportion of composite and prime pairs. Let n equal $(s)(2^t)$ and g+h equal 2^t with an odd g, h, and s. (sg, sh) and (c_1, c_2) with s not a factor are generally a larger proportion of composite pairs of $(s)(2^t)$ than the proportion of (j, l) composite pairs when n equals 2^k. There are always many prime pairs.1

For $a>0$ we remove a composite pairs from the $z+a$ composites leaving $z-a$ primes and composites. The primes $p<n/2$ dividing n are in mixed (prime, composite) pairs $(p, n-p)$. Other primes p_2 that leave the same remainder as n when both are divided by p_1 form mixed pairs with p_1 dividing $n-p_2$ and

$$3 \leq p_1 \leq \sqrt{n-p_2} \tag{3}$$

$$3 \leq p_2 \leq n-p_1^2. \tag{4}$$

Because the primes are irregularly distributed throughout the natural numbers, for some prime p there is always a composite c that leaves the same remainder as n, when both are divided by p. If $p > c$, then c is the common remainder. c and $n - c$ are a composite pair with p multiplied by another odd number producing $n - c$. Since this makes the number of primes two greater than the number of unmatched composites, there is a matching prime pair. Next are some prime pair calculations.

For $n = 200$ there are 54 odd composites and 44 primes between 3 and 197. Subtracting 26 from 54 by removing 13 composite pairs leaves 28 pairs with a prime and a composite summing to 200. There are 49 total pairs of odd numbers that sum up to 200. Subtracting 41 (28+13) from 49 leaves 8 prime pairs: (3, 197), (7, 193), (19, 181), (37, 163), (43, 157), (61, 139), (73, 127), (97, 103).

For $n = 202$ there are 54 odd composites and 45 primes between 3 and 199. Subtracting 26 from 54 by removing 13 composite pairs leaves 28 pairs with a prime and a composite summing to 202. There are 50 total pairs of odd numbers that sum up to 202. Subtracting 41 (28+13) from 50 leaves 9 prime pairs: (3, 199), (5, 197), (11, 191), (23, 179), (29, 173), (53, 149), (71, 131), (89, 113), (101, 101).

The following equations link primes p, composites c, composite pairs $a + m$, and prime pairs m for odd numbers between 3 and $n - 3$, and $n \geq 6$.

$$\text{composite pairs } (a + m) - m \text{ prime pairs} \tag{5}$$

$$c = z + a \text{ minus } p = z - a \text{ is } 2a \tag{6}$$

$$(c - p)(c + p) = c^2 - p^2 \tag{7}$$

$$(2)((a + m) - m)(c + p) = c^2 - p^2 \tag{8}$$

For $n = 4b + 2$ a prime (101) or composite $2b + 1$ is counted twice.

For n=200 (2)$((5 + 8) - 8)$(54 + 44) = $54^2 - 44^2$

For n=202 (2)$((4 + 9) - 9)$(54 + 46) = $54^2 - 46^2$

3 Primes and prime pairs

Table 1: Primes and prime pairs.

2^{23} **to** 2^{27}	**pripair**	**tprime**	**odd 3 to n-3**	**pp/tp**	**tp/odd**	**(pp/tp)/(tp/odd)**
134217728	283746	7603552	67108862	3.7%	11.3%	32.9%
67108864	153850	3957808	33554430	3.9%	11.8%	33.0%
33554432	83467	2063688	16777214	4.0%	12.3%	32.9%
16777216	45746	1077870	8388606	4.2%	12.8%	33.0%
8388608	24928	564162	4194302	4.4%	13.5%	32.9%

(prime pairs / total primes) / (total primes / total odd numbers)
remains proportionally the same at 32.9%, when n is increased from 2^{23} to 2^{27}.

Powers of two have the smallest proportion of prime pairs.

3.1 The (pp/tp)/(tp/odd) proportionality factor calculation

The 2^{23} to 2^{27} proportion increase for

prime pairs (pp) = 283746/24928 = 11.383

primes (tp) = 7603552/564162 = 13.478

odd numbers = 67108862/4194302 = 16.0

$(11.383/13.478)/(13.478/16.0) = 1.003$

n is increased from 2^{23} to 2^{24} (1.835 / 1.911) / (1.911 / 2.0) = 1.005

n is increased from 2^{24} to 2^{25} (1.825 / 1.915) / (1.915 / 2.0) = 0.995

n is increased from 2^{25} to 2^{26} (1.843 / 1.918) / (1.918 / 2.0) = 1.002

n is increased from 2^{26} to 2^{27} (1.844 / 1.922) / (1.922 / 2.0) = 0.998

Each increment of the powers of two doubles the size of both n and the total count of odd numbers, yet each incremental proportionality factor remains close to 1. This suggests that **(pp/tp)/(tp/odd)** will remain approximately 33% for all large powers of two and the same or greater for all large n.

4 Conclusion

Most mathematicians believe the Goldbach conjecture is true. The numerical evidence for it is overwhelming. The relationship between prime pairs and composite pairs gives us a solid reason why the Goldbach conjecture is true.

2020 MSC: 11P32 Goldbach type questions summing primes

Keywords: prime pairs, composite pairs, proportionality factor, Goldbach conjecture

Statements and declarations
The author declares that there are no affiliations with or involvement in any organization or entity with any financial interest in the subject matter or materials discussed in this manuscript.

There is no data available with this manuscript

I am an independent researcher and a data analyst with no university affiliations, having spent 45 years with various corporations primarily in the automotive industry. I have received no outside funding.

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