Fundamental properties of Dirac supersingletons in particle theory

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Abstract

As shown in the famous Dyson's paper "Missed Opportunities", even from purely mathematical considerations (without any physics) it is clear that Poincare quantum symmetry is a special degenerate case of de Sitter quantum symmetries. Then the question arises why in particle physics Poincare symmetry works with very high accuracy. The usual answer to this question is that a theory in de Sitter space becomes a theory in Minkowski space in the formal limit when the radius of de Sitter space tends to infinity. However, de Sitter and Minkowski spaces are purely classical concepts, and in quantum theory the answer to this question must be given only in terms of quantum concepts. At the quantum level, Poincare symmetry is a good approximate symmetry if the eigenvalues of the representation operators $M_{4\mu}$ of the anti-de Sitter algebra are much greater than the eigenvalues of the operators $M_{\mu\nu}$ $(\mu,\nu = 0,1,2,3)$. We show that an explicit solution with such properties exists within the framework of the approach where standard elementary particles are bound states of two Dirac supersingletons.

Keywords: irreducible representations; de Sitter supersymmetry; Dirac supersingletons; accuracy of Poincare symmetry

1 Problem statement

In the literature, relativistic (Poincare) symmetry in quantum field theory (QFT) is usually explained as follows. Since Poincare group is the group of motions of Minkowski space, the system under consideration should be described by unitary representations of this group. This implies that the representation generators commute according to the commutation relations of the Poincare group Lie algebra:

$$[P^\mu, P^\nu] = 0, \quad [P^\mu, M^{\nu\rho}] = -i(\eta^{\mu\rho}P^\nu - \eta^{\mu\nu}P^\rho),$$
$$[M^{\mu\nu}, M^{\rho\sigma}] = -i(\eta^{\mu\rho}M^{\nu\sigma} + \eta^{\nu\sigma}M^{\mu\rho} - \eta^{\mu\sigma}M^{\nu\rho} - \eta^{\nu\rho}M^{\mu\sigma}) \quad (1)$$

where $\mu, \nu = 0, 1, 2, 3$, $\eta^{\mu\nu} = 0$ if $\mu \neq \nu$, $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = 1$, P^μ are the four-momentum operators and $M^{\mu\nu}$ are the Lorentz angular momentum operators. This approach is in the spirit of the Erlangen Program proposed by Felix Klein in 1872 when quantum theory did not yet exist. However, although the Poincare group is the group of motions of Minkowski space, the description (1) does not involve this group and this space.

As noted in [1, 2], in quantum theory, background space is only a mathematical concept because here each physical quantity should be described by an operator but there are no operators for the coordinates of background space. *There is no law that every physical theory must involve a background space.* For example, it is not used in nonrelativistic quantum mechanics and in relativistic quantum theory for describing irreducible representations (IRs) for elementary particles. In particle theory, transformations from the Poincare group are not used because, according to the Heisenberg S-matrix program, it is possible to describe only transitions of states from the infinite past when $t \to -\infty$ to the distant future when $t \to +\infty$. In this theory, systems are described by observable physical quantities — momenta and angular momenta. So, *symmetry at the quantum level is defined not by a background space and its group of motions but by commutation relations of the symmetry algebra* (see [1, 2] for more details). In particular, Eqs. (1) can be treated *as the definition of Poincare invariance at the quantum level.*

In his famous paper "Missed Opportunities" [3] Dyson notes that:

- a) Relativistic quantum theories are more general than nonrelativistic quantum theories even from purely mathematical considerations because Poincare group is more symmetric than Galilei one: the latter can be obtained from the former by contraction $c \to \infty$.

- b) de Sitter (dS) and anti-de Sitter (AdS) quantum theories are more general than relativistic quantum theories even from purely mathematical considerations because dS and AdS groups are more symmetric than Poincare one: the latter can be obtained from the former by contraction $R \to \infty$ where R is a parameter with the dimension *length*, and the meaning of this parameter will be explained below.

- c) At the same time, since dS and AdS groups are semisimple, they have a maximum possible symmetry and cannot be obtained from more symmetric groups by contraction.

As noted above, symmetry at the quantum level should be defined in terms of the the corresponding Lie algebras, and in [2], the statements a)-c) have been reformulated in such terms. It has also been shown that the fact that quantum theory is more general than classical theory follows even from purely mathematical considerations because formally the classical symmetry algebra can be obtained from the symmetry algebra in quantum theory by contraction $\hbar \to 0$. For these reasons, the most general description in terms of ten-dimensional Lie algebras should be carried out in terms of quantum dS or AdS symmetry.

The definition of those symmetries is as follows. If M^{ab} $(a, b = 0, 1, 2, 3, 4,\ M^{ab} = -M^{ba})$ are the angular momentum operators for the system under consideration, they should satisfy the commutation relations:

$$[M^{ab}, M^{cd}] = -i(\eta^{ac}M^{bd} + \eta^{bd}M^{ac} - \eta^{ad}M^{bc} - \eta^{bc}M^{ad}) \qquad (2)$$

Here the tensor η^{ab} is such that $\eta^{ab} = \eta_{ab}$, $\eta^{ab} = 0$ if $a \neq b$, $\eta^{00} = -\eta^{11} = -\eta^{22} = -\eta^{33} = 1$, $\eta^{44} = \mp 1$ for the dS and AdS symmetries, respectively, and this tensor is used to raise and lower the indices of operators M^{ab}.

Although the dS and AdS groups are the groups of motions of dS and AdS spaces, respectively, the description in terms of (2) does not involve those groups and spaces, and *it is a definition of dS and AdS symmetries at the quantum level* (see the discussion in [1, 2]).

The procedure of contraction from dS or AdS symmetry to Poincare one is defined as follows. If we *define* the momentum operators P^μ as $P^\mu = M^{4\mu}/R$ $(\mu = 0, 1, 2, 3)$ then in the formal limit when $R \to \infty$, $M^{4\mu} \to \infty$ but the quantities P^μ are finite, Eqs. (2) become Eqs. (1). Here R *is a parameter which has nothing to do with the dS and AdS spaces*. As seen from Eqs. (2), quantum dS and AdS theories do not involve the dimensional parameters $(c, \hbar, R)$ because (kg, m, s) are meaningful only at the macroscopic level.

At the classical (non-quantum) level, the transition from dS or AdS symmetry to Poincare one is explained as follows. When the radius R of dS or AdS space becomes infinitely large, the angular momentum

M of a particle moving in this space also becomes infinitely large. In the formal limit $R \to \infty$ when dS or AdS space transform into flat Minkowski space, the motion of a particle in such space must be described by momentum $p = M/R$ which is finite in this limit.

One can raise the question why Poincare symmetry works with high accuracy in particle physics. At the classical level, the explanation of this fact is that we live in dS or AdS space whose radius R is very large. However, as noted above, the concept of background space is purely classical. Therefore, the question arises whether the answer to the above question can be given within the framework of purely quantum theory, without involving classical concepts. As follows from the above definition of contraction from dS or AdS algebra to Poincare one, Poincare invariance works with high accuracy, provided that in nature such states are realized in which the eigenvalues of the operators $M^{4\mu}$ are much greater than the eigenvalues of the operators $M^{\mu\nu}$ (μ, ν=0,1,2,3). In this paper, we propose a scenario that describes such a situation.

The paper is organized as follows. In Sec. 2 we explain why supersymmetric AdS symmetry is more general (fundamental) than standard AdS symmetry. In Sec. 3 we explicitly describe the construction of the IR for Dirac supersingletons and explicit relations between representation operators in this IR and representation operators of the AdS algebra. Then in Sec. 4 it is explicitly shown that there exists a scenario when Poincare symmetry works with a high accuracy.

2 Supersymmetry

A problem discussed in a wide literature is that supersymmetric generalization exists in the AdS case but does not exist in the dS one. As shown in [4], in standard quantum theory (over the field of complex numbers), dS symmetry is more general than AdS one, and it may be a reason why supersymmetry has not been discovered yet. However, as shown in [2], standard quantum theory is a special degenerate case of a quantum theory over a finite ring of field (FQT) of characteristic p in a formal limit $p \to \infty$, and in FQT, dS and AdS symmetries are equivalent.

Note that representations of the standard Poincare superalgebra are described by 14 operators. Ten of them are the representation operators of the Poincare algebra—four momentum operators and six operators of the Lorentz algebra, and in addition, there are four fermionic operators. The anticommutators of the fermionic operators are linear combinations of the Lorentz algebra operators, the commutators of the fermionic operators with the Lorentz algebra operators are linear combinations of the fermionic operators and the fermionic operators commute with the momentum operators. However, the latter are not bilinear combinations of fermionic operators.

From the formal point of view, representations of the AdS so(2,3) superalgebra osp(1,4) are also described by 14 operators — ten representation operators of the so(2,3) algebra and four fermionic operators. There are three types of relations: the operators of the so(2,3) algebra commute with each other as usual (see Eqs. (2)), anticommutators of the fermionic operators are linear combinations of the so(2,3) operators and commutators of the latter with the fermionic operators are their linear combinations. However, in fact, representations of the osp(1,4) superalgebra can be described exclusively in terms of the fermionic operators. The matter is as that in the general case, the anticommutators of four operators form ten independent linear combinations. Therefore, ten bosonic operators can be expressed in terms of fermionic ones. This is not the case for the Poincare superalgebra since the Poincare algebra operators are obtained from the so(2,3) ones by contraction. One can say that the representations of the osp(1,4) superalgebra is an implementation of the idea that supersymmetry is the extraction of the square root from the usual symmetry (by analogy with the treatment of the Dirac equation as a square root from the Klein-Gordon equation).

We use $(d_1', d_2', d_1'', d_2'')$ to denote the fermionic operators of the osp(1,4) superalgebra. They should satisfy the following relations. If (A, B, C) are any fermionic operators, [...,...] is used to denote a commutator and $\{..., ...\}$ to denote an anticommutator then

$$[A, \{B, C\}] = F(A, B)C + F(A, C)B \tag{3}$$

where the form $F(A, B)$ is skew symmetric, $F(d_j', d_j")=1$ $(j = 1, 2)$ and the other independent values of $F(A, B)$ are equal to zero.

As shown by various authors (see e.g., [2, 5]), the operators M^{ab} in Eqs. (2) can be expressed through bilinear combinations of the fermionic operators as follows:

$$h_1 = \{d_1', d_1''\},\ h_2 = \{d_2', d_2''\},\ M_{04} = h_1 + h_2,\ M_{12} = L_z = h_1 - h_2$$

$$
\begin{aligned}
&L_+ = \{d'_2, d''_1\},\ L_- = \{d'_1, d''_2\},\ M_{23} = L_x = L_+ + L_- \\
&M_{31} = L_y = -i(L_+ - L_-),\ M_{14} = (d''_2)^2 + (d'_2)^2 - (d''_1)^2 - (d'_1)^2 \\
&M_{24} = i[(d''_1)^2 + (d''_2)^2 - (d'_1)^2 - (d'_2)^2] \\
&M_{34} = \{d'_1, d'_2\} + \{d''_1, d''_2\},\ M_{30} = -i[\{d''_1, d''_2\} - \{d'_1, d'_2\}] \\
&M_{10} = i[(d''_1)^2 - (d'_1)^2 - (d''_2)^2 + (d'_2)^2] \\
&M_{20} = (d''_1)^2 + (d''_2)^2 + (d'_1)^2 + (d'_2)^2
\end{aligned} \tag{4}
$$

where $\mathbf{L} = (L_x, L_y, L_z)$ is the standard operator of three-dimensional rotations.

The fact that the representation of the osp(1,4) superalgebra is fully defined by Eqs. (3,4) and the properties of the form $F(.,.)$, shows that osp(1,4) is a special case of the superalgebra.

We require the existence of the generating vector e_0 satisfying the conditions :

$$
d'_j e_0 = d'_2 d''_1 e_0 = 0, \quad d'_j d''_j e_0 = q_j e_0 \quad (j = 1, 2) \tag{5}
$$

These conditions are written exclusively in terms of the d operators. The full representation space can be obtained by successively acting by the fermionic operators on e_0 and taking all possible linear combinations of such vectors. The theory of self-adjoint IRs of the osp(1,4) algebra has been developed by several authors (see e.g., [5]), and in [2] this theory has been generalized to the case of FQT.

3 Dirac supersingleton

When describing elementary particles within the framework of AdS symmetry, the following problems arise.

If m is the mass of the particle in Poincare invariant theory then its mass μ in AdS theory is dimensionless and the relation between μ and m is $\mu = mR$ where R is the contraction parameter for the transition from AdS to Poincare symmetry. As explained in [4], R has nothing to do with the radius of classical AdS space, R is fundamental to the same extent as c and $\hbar$, and the problem why the value of R is as is does not arise. The data on cosmological acceleration show that, at the present stage of the universe, R is of the order of $10^{26}m$ [4]. Therefore, even for elementary particles, the AdS masses are very large. For example, the AdS masses of the electron, the Earth and the

Sun are of the order of 10^{39}, 10^{93} and 10^{99}, respectively. The fact that even the AdS mass of the electron is so large might be an indication that the electron is not a true elementary particle. In addition, the present upper level for the photon mass is $10^{-17}ev$ or less. This value seems to be an extremely tiny quantity. However, the corresponding AdS mass is of the order of 10^{16} and so, even the mass which is treated as extremely small in Poincare invariant theory might be very large in AdS invariant theory.

As shown in [4], in standard quantum theory, dS symmetry is more general than AdS one but in the framework of this symmetry it is not possible to describe neutral elementary particles, i.e., particles which are equivalent to the their antiparticles. In FQT, dS and AdS symmetries are equivalent and, as shown in [2], in this theory also there are no neutral elementary particles. In particular, even the photon is not elementary.

This problem has been discussed by several authors. In Standard Model (based on Poincare invariance) only massless particles are treated as elementary. However, as shown in the seminal paper by Flato and Fronsdal [6] (see also [7]), in standard AdS theory, each massless IR can be constructed from the tensor product of two singleton IRs discovered by Dirac in his paper [8] titled "A Remarkable Representation of the 3 + 2 de Sitter group", and the authors of [6] believe that this is indeed a truly remarkable property.

The IR describing the supersingleton is constructed as follows. In Eq. (5), we choose q_1 and q_2 the same and equal q_0 where $q_0 = 1/2$ in standard theory over complex numbers and $q_0 = (p+1)/2$ in FQT, where p is the characteristic of the field or ring and, in the latter case, p is odd.

The authors of [6] and other publications treat singletons as true elementary particles because their weight diagrams has only a single trajectory (that's why the corresponding IRs are called singletons). However, one should answer the following questions:

- a) Why singletons have not been observed yet.
- b) Why such massless particles as photons and others are stable and their decays into singletons have not been observed.

There exists a wide literature (see e.g. [9, 10] and references therein) where this problem is investigated from the point of view of standard AdS QFT. For example, in AdS QFT, singleton fields live on the boundary at infinity of the AdS bulk (boundary which has one dimension less than the bulk). However, as noted in Sec. 1, the explanation in the framework of quantum theory should not involve classical spaces.

In standard theory, an IR characterized by (q_1, q_2) can be constructed from tensor products of two IRs characterized by $(q_1^{(1)}, q_2^{(1)})$ and $(q_1^{(2)}, q_2^{(2)})$ only if $q_1 \geq (q_1^{(1)} + q_1^{(2)})$ and $q_2 \geq (q_2^{(1)} + q_2^{(2)})$. Since no interaction is assumed, a problem arises whether a particle constructed from a tensor product of other two particles will be stable. In standard theory, a particle with the mass m can be a stable composite state of two particles with the masses m_1 and m_2 only if $m < (m_1 + m_2)$ and the quantity $(m_1 + m_2 - m)c^2$ is called the binding energy. The greater the binding energy is the more stable is the composite state with respect to external interactions.

As argued in [2], in FQT, the properties of Dirac singletons are even more remarkable than in standard theory. Here the eigenvalues of the operators h_1 and h_2 for singletons in FQT are $(p+1)/2, (p+3)/2, (p+5)/2...$, i.e. huge numbers if p is huge. Hence Poincare limit for Dirac singletons in FQT has no physical meaning and they cannot be observable.

For answering question b) we note the following. In standard theory, the binding energy is a measure of stability: the greater the binding energy is, the greater is the probability that the bound state will not decay into its components under the influence of external forces.

If a massless particle is a composite state of two Dirac singletons, and the eigenvalues of the operators h_1 and h_2 for the Dirac singletons in FQT are $(p+1)/2, (p+3)/2, (p+5)/2...$ then, since in FQT the eigenvalues of these operators should be taken modulo p, the corresponding eigenvalues for the massless particle are $1, 2, 3....$ Hence an analog of the binding energy for the operators h_1 and h_2 is p, i.e., a huge number. This phenomenon can take place only in FQT: although, from the formal point of view, the Dirac singletons comprising the massless state do not interact with each other, the analog of the binding energy for the operators h_1 and h_2 is huge. In other words, the fact that all the quantities in FQT are taken modulo p implies a very strong effective interactions between the singletons. It explains why the massless state does not decay into Dirac singletons and why free Dirac singletons effectively interact pairwise for creating their bound state.

As noted in the literature on singletons (see e.g., the review [9] and references therein), the possibility that only singletons are true elementary particles but they are not observable has some analogy with quarks. However, the analogy is not full. According to Quantum Chromodynamics, forces between quarks at large distances prevent quarks from being observable in free states. In FQT, Dirac singletons cannot be in free states even if there is no interaction between them; the effective interaction between Dirac singletons arises as a consequence of the fact that FQT is based on arithmetic modulo p. In addition, quarks and gluons are used for describing only strongly interacting particles while in standard AdS theory and in FQT, quarks, gluons, leptons, photons, W and Z bosons can be constructed from Dirac singletons.

In standard AdS theory, there exist four Dirac singletons which in the literature are called Di singleton, Rac singleton and their antiparticles. In the case of supersymmetry, Di and Rac singletons are combined into one superparticle - the Dirac supersingleton, so that there are two supersingletons - the Dirac supersingleton and its antiparticle. However, in FQT those supersingletons are combined into one object and so there is only one supersingleton. Here, one of the remarkable properties of supersingletons is the following. The physical meaning of division comes from classical physics, which assumes that every object can be divided into any arbitrarily large number of arbitrarily small parts. However, standard division loses its standard physical meaning when we reach the level of elementary particles since, for example, the electron cannot be divided into two, three, and so on parts. As shown in [2], in FQT, the theory of singletons can be built over a ring in which there is no division, but only addition, subtraction and multiplication.

As shown in [2], the operators d_1'' and d_2'' commute in the space of the supersingleton IR. The basis of this IR can be chosen as [2] $e(j,k) = (d_1'')^j (d_2'')^k e_0$ where $j,k = 0,1,...\infty$ in standard theory and $j,k = 0,1,...p-1$ in FQT. Then it can be shown [2] that

$$d_1' e(j,k) = \frac{1}{2} j e(j-1,k),\ \ d_2' e(j,k) = \frac{1}{2} k e(j,k-1) \qquad (6)$$

in standard theory, and $\frac{1}{2}$ should be replaced by $(p+1)/2$ in FQT.

The important property of supersingletons is that the above results can be immediately generalized to the case of higher dimensions.

Consider a superalgebra defined by the set of operators (d'_j, d''_j) where $j = 1, 2, ...J$ and formally any triplet of the operators (A, B, C) satisfies the commutation-anticommutation relation Eq. (3) where the form $F(A, B)$ is skew symmetric, $F(d'_j, d''_j) = 1$ $(j = 1, 2, ...J)$ and the other independent values of $F(A, B)$ are equal to zero. The higher-dimensional analog of the supersingleton IR can now be defined such that the representation space contains a vector e_0 satisfying the conditions

$$d'_j e_0 = 0, \quad d'_j d''_j e_0 = \frac{1}{2} e_0 \quad (j = 1, 2, ...J)$$

in standard theory and $\frac{1}{2}$ should be replaced by $(p+1)/2$ in FQT. The basis of the representation space can be chosen in the form

$$e(n_1, n_2, ...n_J) = (d''_1)^{n_1} (d''_2)^{n_2} \cdots (d''_J)^{n_J} e_0$$

where the operators $(d''_1, ...d''_J)$ mutually commute on the representation space. The fact that singleton physics can be directly generalized to the case of higher dimensions has been indicated by several authors (see e.g., [9] and references therein). It is interesting to explore the possibility that spatial and internal quantum numbers are combined within the framework of the theory of supersingletons in which $J > 2$.

4 Why Poincare symmetry in particle theory works with a high accuracy

Now we can consider the problem posed in Sec. 1: why in particle theory, the eigenvalues of the operators $M_{4\mu}$ are much greater than the eigenvalues of the operators $M_{\mu\nu}$ (μ, ν=0,1,2,3). As noted in Sec. 1, this problem must be solved exclusively within the framework of quantum theory, without involving such classical concepts as dS or AdS space.

Even when we work in FQT and consider states of supersingletons in which the quantum numbers j, k are much less than p, then, with high accuracy, we can apply standard mathematics. We assume that, although the numbers j, k can be very large, they are still much less than p. Therefore, in what follows we consider Dirac supersingletons only within the framework of standard mathematics.

We now treat $(d'_1, d'_2, d''_1, d''_2)$ as the operators in the Hilbert space related by Hermitian conjugation as $(d'_1)^* = d''_1$ and $(d'_2)^* = d''_2$. Then, as follows from Eq. (6), the squared norm of the element $e(j,k)$ is equal to

$$||e(j,k)||^2 = \frac{j!k!}{2^{j+k}} \tag{7}$$

Therefore, the normalized basis vectors can be defined as

$$\tilde{e}(j,k) = (\frac{2^{j+k}}{j!k!})^{1/2} e(j,k) \tag{8}$$

We consider the supersingleton wave functions $\sum_{jk} c(j,k)\tilde{e}(j,k)$ in the framework of semiclassical approximation when the coefficients $c(j,k)$ are not equal to zero only at $j \in (j_1, j_2),\ k \in (k_1, k_2)$ where $j_2 - j_1 \ll j_1,\ k_2 - k_1 \ll k_1$ and the values of $|c(j,k)|$ at such j,k are approximately the same. We define the angular dependence of the coefficients as $c(j,k) = |c(j,k)|exp[i(j+k)\chi + i(j-k)\varphi]$. Then taking into account Eqs. (4,6,8), and the definition of the basis elements and the coefficients $c(j,k)$, direct calculation shows that, in semiclassical approximation, the operators M_{ab} can be replaced by their numerical values:

$$\begin{aligned}
&L_x = 2(jk)^{1/2}cos(2\varphi),\ L_y = -2(jk)^{1/2}sin(2\varphi),\ L_z = j-k \\
&M_{10} = jsin(2\varphi+2\chi) + ksin(2\varphi-2\chi) \\
&M_{20} = jcos(2\varphi+2\chi) + kcos(2\varphi-2\chi) \\
&M_{30} = 2(jk)^{1/2}sin(2\chi),\ M_{34} = 2(jk)^{1/2}cos(2\chi) \\
&M_{14} = kcos(2\varphi-2\chi) - jcos(2\varphi+2\chi) \\
&M_{24} = jsin(2\varphi+2\chi) - ksin(2\varphi-2\chi),\ M_{04} = j+k+1
\end{aligned} \tag{9}$$

As can be seen from these expressions, for a single supersingleton there is no scenario when the eigenvalues of the operators $M_{4\mu}$ are much greater than the eigenvalues of the operators $M_{\mu\nu}$ (μ,ν=0,1,2,3). This is an additional argument why singletons cannot exist in free states.

The eigenvalues of the operators in Eq. (9) satisfy the property that when one applies the transformations

$$j \leftrightarrow k,\ \chi \to -\chi,\ \varphi \to \varphi + \pi/2 \tag{10}$$

then all the eigenvalues of the operators $M_{4\mu}$ do not change while all the eigenvalues of the operators $M_{\mu\nu}$ change their sign.

Let us now consider a system of two free supersingletons 1 and 2. The AdS superalgebra representation for such a system is the tensor product of the representations for supersingletons 1 and 2, and the representation operators are the sums of the corresponding operators: $M_{ab} = M_{ab}^{(1)} + M_{ab}^{(2)}$. The question arises under what conditions a system consisting of such supersingletons can be described in the framework of Poincare symmetry with a high accuracy. As noted above, a necessary condition for this is that for the system as a whole, the eigenvalues of the operators $M_{4\mu}$ must be much greater than the eigenvalues of the operators $M_{\mu\nu}$.

If the eigenvalues of $M_{ab}^{(1)}$ are described by Eqs. (9) with the parameters $(j,k,\chi,\varphi) = (j_1,k_1,\chi_1,\varphi_1)$ and the eigenvalues of the operators $M_{ab}^{(2)}$ are described by Eqs. (9) with the parameters $(j,k,\chi,\varphi) = (j_2,k_2,\chi_2,\varphi_2)$ then, as follows from the remarks after Eq. (10), if

$$j_2 \approx k_1,\ k_2 \approx j_1,\ \chi_2 \approx -\chi_1,\ \varphi_2 \approx \varphi_1 + \pi/2 \tag{11}$$

then the eigenvalues of the operators $M_{4\mu}^{(1)}$ and $M_{4\mu}^{(2)}$ will be approximately equal for each μ while for each μ,ν the eigenvalues of the operators $M_{\mu\nu}^{(1)}$ and $M_{\mu\nu}^{(2)}$ will approximately differ by sign. Therefore, for the operators describing the tensor product, the eigenvalues of the operators $M_{4\mu}$ will be much greater than the eigenvalues of the operators $M_{\mu\nu}$, and this guarantees that Poincare symmetry will be a good approximate symmetry.

5 Conclusion

As shown in the famous Dyson's paper [3], even from purely mathematical considerations (without any physics) it is clear that Poincare quantum symmetry is a special degenerate case of de Sitter quantum symmetries. As shown in [2, 4], in standard quantum theory (over the field of complex numbers), dS symmetry is more general than AdS one but in the framework of this symmetry it is not possible to describe neutral elementary particles, i.e., particles which are equivalent to the their antiparticles. In particular, even the photon cannot be a neutral elementary particle. This problem has been discussed by several authors.

In Standard Model (based on Poincare invariance) only massless particles are treated as elementary. However, as shown by Flato and Fronsdal [6] (see also [7]), in standard AdS theory, each massless IR can be constructed from the tensor product of two singleton IRs discovered by Dirac in his seminal paper [8]. As explained in Sec. 2, AdS theory based on supersymmetry is more general (fundamental) than standard AdS theory.

Therefore, the question arises why in particle physics, Poincare symmetry works with a very high accuracy. The usual answer to this question is that a theory in de Sitter space becomes a theory in Minkowski space in the formal limit when the radius of de Sitter space tends to infinity. However, de Sitter and Minkowski spaces are purely classical concepts, and in quantum theory the answer to this question must be given only in terms of quantum concepts.

As noted in Sec. 1, at the quantum level, Poincare symmetry is a good approximate symmetry if the eigenvalues of the operators $M_{4\mu}$ are much greater than the eigenvalues of the operators $M_{\mu\nu}$ ($\mu, \nu = 0, 1, 2, 3$). As shown in Sec. 4, for a single supersingleton there is no scenario when these conditions are met but an explicit solution with such properties exists within the framework of the approach where standard elementary particles are bound states of two Dirac supersingletons such that their states satisfy the conditions (11). The problem that needs to be investigated is the accuracy with which these conditions must be met for existing elementary particles.

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