A Simple Approach to the Quantum Measurement Problem Using Zeta Function Regularisation and Novel Measurement Theory

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Abstract

This paper shows how an application of zeta function regularisation to a physical model of quantum measurement yields a solution to the problem of wavefunction collapse. A realistic measurement ontology is introduced which is based on particle distinguishability being imposed by the measurement process entering into the classical regime. Based on this, an outcome function is introduced. An outcome counting argument is presented. It is shown how regularisation of this outcome function leads to apparent collapse of the wavefunction.

Keywords: Interpretations of quantum mechanics, Wavefunction collapse, Measurement problem, Zeta regularisation

1 Introduction and Contents of this Paper

This paper begins with some definitions which may serve as a reference for the reader. Section 2 is a background to the measurement problem. Section 3 is a brief overview of the literature of the problem. Section 4 talks about the proposed measurement process ontology. Section 5 develops the derivation of the collapse in mathematical terms. Section 5.1 looks in detail at our maximum outcome function and Hilbert spaces. Section 5.2 is an illustrative discussion of outcomes of a single particle. Section 5.3 is an illustrative discussion of outcomes in m particles. Section 5.4 is an illustrative discussion of many systems of varying complexity. Section 5.5 introduces a realistic outcome function based on a counting function. Section 5.6 derives the expected number of systems of size n across the measurement process. Section 6 shows how regularisation is key to our derivation of collapse. Section 7 shows how our derivation maps onto the measurement operator formalism. Section 8 aims to show how this theory might be experimentally validated. Section 8.1 gives an overview of recent collapse emission experiments. Section 8.2 is an overview of some approaches to show where the theory described in this paper differs from other collapse approaches. Section 9 gives an overview of results. Section 9.1 gives an overview of the measurement process in a general example, according to the theory outlined in this paper. Section 10 is a discussion of some open questions, problems and points of interest.

In the Appendices are discussion of items which are useful but perhaps not necessary to the approach described in this paper. Appendix A describes the formalism of collapsed and uncollapsed wavefunctions. Appendix B contains some figures to help the reader understand the conceptual basis of the proposed approach. Appendix C gives an overview of an alternative approach to interpret the mathematical framework developed in this paper; Fock spaces are introduced. Appendix D discusses the physical reality of some of the basis states that we are counting. Appendix E briefly describes the infinite Hilbert spaces this theory is based upon. Finally, Appendix F discusses the c_n function in further detail.

Definitions

In this section we will define the key terms and objects which we will be using, in the order they will introduced in the text. These can then act as a reference. In the text, we may reiterate these definitions as we give them additional context. In general, we move between quantum representations (wave or matrix) to most simply express our ideas.

Definition 1 Ψ is an arbitrary total wavefunction of a system. ϕ is used to represent an eigenfunction onto which the wavefunction can collapse, whereas ψ is used to represent an uncollapsed wavefunction that could be in a superposition of the possible ϕ's.

Definition 2 λ is an eigenvalue.

Definition 3 $\mathcal{H}$ is a general Hilbert space. $\mathcal{H}_\Psi$ is the Hilbert space of an arbitrary total wavefunction of a system, here typcially referring to the total wavefunction of the many objects invovled in the measruement process. $\mathcal{H}_{d,m}$ is a truncated, or finite, Hilbert space of m particles each with finite dimension, d. $d = \infty$ in the most general Hilbert space.

Definition 4 d is the number of possible states following measurement of an isolated quantum particle, or its dimension, or number of single-particle basis states. For the most general Hilbert space, $d = \infty$.

Definition 5 m is the total number of interacting particles across the measurement process.

Definition 6 p and q are arbitrary numbers which respectively count a number, q, of size p-particle system in an illustrative example.

Definition 7 n is an index which counts the number of particles in each composite system which interact in the measurement process. This index runs from 1 to k.

Definition 8 c_n is the number of many particle systems of size n across the measurement process.

Definition 9 C is the total number of n-particle systems.

Definition 10 k is the size of the largest system involved in the measurement process.

Definition 11 Ω_c is number of micro-states available for the distribution of the counting function, c_n.

Definition 12 $F(\mathcal{H})$ is a general Fock space, while $F'(\mathcal{H}_m)$ is a Fock space for a given number of particles, m.

2 Brief Overview of Background to the Measurement Problem

The reader may use [1] [2], or any other of a number of undergraduate or elementary texts, for a basic treatment of the quantum measurement problem. However, for clarity, we will give a very brief overview of the measurement problem in order to be clear about the problem this paper aims to approach. In terms of [3]'s characterisation of the problem, this paper aims to tackle the 'problem of definite outcomes'.

Quantum theory is meant to be a universal theory to explain all physical phenomenon. However, there appears to be two distinct time evolution phenomenon in quantum mechanics. Firstly, evolution of the wavefunction between measurements, as governed by the time-dependent Schrodinger equation.

$$i\hbar\frac{\partial\Psi}{\partial t} = \hat{H}\Psi \tag{1}$$

And secondly, quantum mechanics under quantum measurement. Under quantum measurement, the wavefunction appears to evolve non-linearly; that is, the total wavefunction will suddenly appear to collapse into a single eigenstate, with corresponding eigenfunction ϕ. To take a simple example, an observable is being measured by the action of an arbitrary Hermitian, linear operator $\hat{O}$. The eigenvalues associated with $\hat{O}$ are λ_a and λ_b, and respective eigenfunctions are ϕ_a and ϕ_b. Take a quantum system, Ψ_0, before measurement that is in the state:

$$\Psi_0 = \frac{1}{\sqrt{2}}(\phi_a + \phi_b) \tag{2}$$

For clarity of exposition, we define the basic postulates and nomenclature of our wavefunction formalism in Appendix A. Notably, we rely on the fact that Ψ is decomposed into a set of orthonormal eigenstates $\{\lambda_i \left|\phi_i\right>\}$ (with observable λ) via the expansion postulate, and after measurement will be in one of these eigenstates. Furthermore, the total measurement system is represented as Ψ, and is clarified in section 9.1, which will involve a plethora of superpositions of states with, for example, the system in equation 2 and the measuring system.

In the example above, in equation 2, after measurement, the quantum system evolves and is projected into either state ϕ_a or state ϕ_b depending on whether the measurement yields the eigenvalue λ_a or λ_b. This is known as the 'collapse postulate'.

These two time evolution phenomena appear to be irreconcilable. The distinction between time evolution dynamics of the two types described above remains to be made out in quantum mechanical terms. In fact this difficulty in reconciliation, the measurement problem, has been described as probably the 'most difficult and controversial conceptual problem' in quantum mechanics [1].

A number of different "interpretations" have been proposed, which aim to help explain this measurement problem, the first of which was the Copenhagen Interpretation [4]. Other popular interpretations include the Many Worlds interpretation [5], hidden variable interpretations [6] and objective collapse interpretations [7] [8].

3 Discussion of the literature

This paper will not offer an exhaustive review of the literature. There are a number of thorough overviews of the various interpretations, for example see [9] [10] for recent overviews of the most popular attempts for solving the measurement problem. [11] gives an up-to-date overview of the problem with a particular focus on collapse model interpretations. Despite the number of existing interpretations, this paper proceeds with the understanding that 'there is no interpretation of (QM) that does not have serious flaws', a view given by [12].
Identifying which broad category of solution this paper describes might be useful, however. Bearing this in mind, this paper outlines an objective collapse theory. Unlike objective collapse theories such as GRW [7] and the CSL model [8], the Schrodinger equation is not explicitly altered. For the quantum formalism used in this paper, please refer to [13] [14] [15] or any of the many other suitable standard texts.

4 Measurement Ontology and Collapse

The measurement ontology, described as a mechanism, is as follows: through measurement, a quantum system interacts with a large number of other quantum systems, of varying sizes and complexity, from the very small and simple, to the large and complex. As these quantum systems interact, the number of possible outcomes from that measurement increases, as the number of superpositions increase. Taking a statistical mechanical model of the number of likely outcomes; as the complexity and size of the interactions increase towards the macro-scale, we approach an infinite number of possible outcomes from a measurement. We also take into account how the particles may be described as distinguishable, since the measurement process spans quantum and classical physics. However, regularisation mediates the divergent infinity of outcomes and, in effect, 'produces' the wave-function collapse phenomenon by reducing the maximum number of possible outcome states to just one. This agrees with apparent real-world observations of physics in the classical and quantum regimes. In Appendix B we provide some diagrams which help clarify the conceptual, mechanistic basis of the proposed measurement model.

4.1 Distinguishable Particles Model

In our derivation we take the principle that classical mechanics requires distinguishable particles. With this principle, and with the understanding that the measurement problem spans both quantum and classical regimes, we examine a measurement process through the standard quantum mechanical formalism but with the principle that particles can be considered distinguishable. Due to this, we use general Hilbert spaces and distinguishable-particle statistical models. In other words, the classical world imposes the principle of distinguishable particles onto the mathematical structure of quantum mechanics, and we show how this leads to apparent collapse.

5 Derivation of the Collapse

In this section we will build up our idea of the outcome function that is critical to our theory. We first examine the relevant Hilbert spaces necessary for our calculations (and briefly discuss Fock spaces in Appendix C. While examination of the Fock space might not be required for the approach described in this paper, we detail these spaces so that some illustrative physical examples can be developed). We will first examine the function quantum mechanically for the most simple case of just one particle. We will then examine the more complex case of multiple, interacting particles. We then, finally, introduce a realistic outcome function based on a counting argument, which counts the number of interacting systems of particles of varying complexity which includes a classical component.

5.1 An Outcome Counting Function and Dimensionality of Hilbert Spaces

5.1.1 $\dim \mathcal{H}_\Psi$

We define a function, $\dim \mathcal{H}_\Psi$ which counts the maximum number of measurement outcomes following a quantum experiment. Quantum mechanically, this function is related to the dimension, or number of basis functions, of the measurement of a many-body system. Importantly for our derivation, since the measurement process is across the quantum and classical regimes, this outcome function includes a classical contribution. This classical contribution is due to the additional number of outcomes due to being able to label individual particles and is apparent in the later discussions around distinguishable particle statistics.

5.1.2 Hilbert Space

More broadly, we interpret the dimension of a Hilbert space, $\dim \mathcal{H}$, of a system as the total possible number of basis states of a system. In principle, quantum mechanics allows experiment to distinguish between these states. Possible outcomes, in other words. For a basic composite system the Hilbert space is defined as:

$$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B \tag{3}$$

and:

$$\dim(\mathcal{H}_{AB}) = \dim(\mathcal{H}_A) \times \dim(\mathcal{H}_B) \tag{4}$$

For a many body system of m particles the Hilbert space is as follows:

$$\mathcal{H}^{\otimes m} \tag{5}$$

with dimension:

$$\dim(\mathcal{H}^{\otimes m}) = d^m \tag{6}$$

We generally consider the most general Hilbert space, which is required to have infinite dimension, $d = \infty$; but also consider finite, or truncated Hilbert spaces, for illustrative purposes. This most general, infinite dimensional Hilbert space is defined for a given number of particles, $\mathcal{H}_{\infty,m}$. See Appendix E for a brief discussion.

5.2 The Maximum Number of Outcomes for a Single Particle

The maximum number of outcomes for a single particle is simple. As a reminder, d is the maximum number of states that may follow a measurement of an isolated single-particle, and we assume this is equal to the number of basis states of the particle. We restrict d to describing fundamental particles, which can include an isolated proton or neutron.

We briefly look at the dimensionality of the truncated Hilbert space (and also in this case the Fock space, see Appendix C for a brief discussion of Fock spaces), since this determines $\dim \mathcal{H}_\Psi$ for quantum systems.

For a fermionic (spin=$\frac{1}{2}$) particle (such as an isolated electron), there will be two degenerate states due to the fermion's intrinsic angular momentum, also referred to as spin ($\mathbf{s}$).

$$\dim \mathcal{H}_\Psi = \dim(F'(\mathcal{H}_{2,1})) = 2\mathbf{s} + 1 = 2^1 = \dim(\mathcal{H}_{2,1}) \tag{7}$$

A physical example of this system under measurement is a Stern-Gerlach like setup, where the intrinsic spin is measured as the magnetic spin projection on a single cartesian axis, by lifting the degeneracy of the two states via the application of an external magnetic field.

Adding an additional fermion to the quantum system will produce either a fermionic or bosonic total wavefunction, which is discussed in Appendix D.

5.3 The Maximum Number of Outcomes for m-particle Systems

For many systems, the maximum number of outcomes is determined by both the maximum value of d for a particle ($\max d$) and by the number of particles that many-particle system comprises. For m-particle systems, it is clear that the maximum number of outcomes is:

$$\dim \mathcal{H}_\Psi = (\max d)^m \tag{8}$$

To expand on our initial example, a finite, $d = 2$ Hilbert space of 2 particles,

$$\dim(\mathcal{H}_{2,2}) = 2^2 = 4 \tag{9}$$

5.4 Example: The Maximum Number of Outcomes for Many Systems, of Varying Complexity

In a realistic model of measurement, a quantum system will interact with objects of varying complexity, themselves which will have already interacted with quantum systems. Thus, take for example a complex system comprising a number, q, of size p-particle systems. In this case, it is clear that:

$$\dim \mathcal{H}_\Psi = (\max d)^{pq} \tag{10}$$

since $m = pq$, with $\max d = \infty$ in the most general case.

The arbitrary variables p and q (different from the ones in section 5.3) are affixed via a process of maximum likelihood in the next section.

For a physical example, take the example of ten uncharged Helium molecules interacting in a vacuum, with each Helium molecule itself made up of six particles (two neutrons, two protons and two electrons). In this example, $q = 10$ and $p = 6$.

5.5 A Realistic Outcome Function

We introduce the idea of a counting function, c_n, which will count the expected number of complex systems of size n, all of which interact through the measurement process. We will then multiply through these expected values in order to find the total number of outcomes. Since each system of size n particles contributes towards the multiplicity of the number of outcomes according to d, we may therefore state:

$$\dim \mathcal{H}_\Psi = \prod_{n=1}^{k} d^{nc_n} \tag{11}$$

with k as the largest n size system involved in the measurement process.

5.6 A n-particle System Counting Function

In order to derive a more accurate representation of c_n, we use some principles from statistical mechanics.

5.6.1 The Approach for Deriving c_n

The goal of this subsection can be simply summarised as being an attempt to find the distribution for c_n (i.e. the number of n-particle systems, for each n, which all interact during the proposed measurement process) which best represents the current state of knowledge about a system, which is the distribution with the maximum entropy. For an example of the mathematical approach we will use, see [16] or a number of other elementary statistical mechanics texts, in which distributions which count occupancy of particles based on a number of physical assumptions and constraints are derived. Figure 2 helps to clarify this proposed physical model of additional outcomes. In terms of Figure 2, we are aiming to further understand the expected number of n-particle systems in each bracket for each n.
We must also account for the additional number of ways that each n-particle system can interact with the total measurement system, Ψ in a classical, distinguishable particle way.

Quite simply, we can see that there are n ways for each n-particle system to interact with the total measurement system. Figure 3 helps to clarify the proposed model of multiplicities of outcomes due to the additional outcomes. This is because since there are n particles, only one of which must interact before that n-particle system interacts with the wider system. We must therefore also count these when counting the number of possible outcomes.

5.6.2 Derivation of c_n

Similar to a statistical mechanics derivation of an expected occupancy distribution, we want to maximise the entropy of this unknown distribution in question within the constraints given. We do this by maximising the number of micro-states available (therefore maximising the entropy) to find the most likely distribution.
Ω_c (given similarities to statistical mechanical Ω) is the total number of micro-states available for the distribution of the counting function, c_n. Physically, Ω_c is the number of possible arrangements of n-particle systems across n layers. We want to find c_n such that Ω_c is maximised.

We are interested in systems of n-particles that can be thought of as distinguishable. We consider n-particle systems from $n = 1$ through to n being very large, here described as n from 1 through to k, where k is a large number. We define c_n as the number of of n-particle systems for each n. We assume a fixed number of n-particle systems and we define this number, C, as:

$$\sum_{n=1}^{k} c_n = C \tag{12}$$

We then define the number of ways of arranging the C n-particle systems as, Ω_c. Since we are interested in placing C distinct objects into k bins, it is clear that the combinatorial function which describes the total number of arrangements (neglecting, for now, to account for the n ways each particle interacts with the total system) is the multinomial coefficient.

Then, also needing to account for the n ways that each n size particle can interact with the wider Ψ, we are introducing a 'degenerate multiplicity' of adding n sub-boxes to each c_n to account for ways to count these [1].

This 'degenerate multiplicity' term is clearly n^{c_n}. With this understanding, we can define Ω as:

$$\Omega_c = \frac{n^{c_n} C!}{c_1! c_2! \ldots c_k!} \tag{13}$$

We want to maximize Ω with respect to c_n. We take logarithms since $\max log(f) = \max f$, and this simplifies calculations.

So we have:

$$\ln(\Omega_c) = \sum_{n}^{k} c_n \ln(n) + \ln(C!) - ln(c_n!) \tag{14}$$

We also use Stirling's approximation, $\ln(x!) = x\ln(x) - x$.

Therefore:

$$\sum_{n}^{k} c_n \ln(n) + \ln(C!) - c_n \ln(c_n) + c_n \tag{15}$$

We use the method of Lagranian multipliers, with the constraint that $\sum c_n = C$, therefore adding the $\alpha(C - \sum c_n)$ term. Since we are going to optimize, we may also ignore the $\ln(C!)$ term since it is a constant. We therefore now have a function f which is to be maximised:

$$\sum_{n}^{k} \Big(c_n \ln(n) - c_n \ln(c_n) + c_n \Big) + \alpha(C - \sum_{n}^{k} c_n) \tag{16}$$

Bringing all the summed terms together:

$$\sum_{n}^{k} \Big(c_n \ln(n) - c_n \ln(c_n) + c_n - \alpha c_n \Big) + \alpha C \tag{17}$$

[1] In physical terms we are accounting for the fact that, in principle, there is a difference between each of the n particles of an n-particle system interacting with the total system, Ψ. Concretely, a physical example is that given the classical mechanics imposed by the measurement process (under the assumptions distinguishability is imposed), it should be possible to determine which of the six fundamental particles (two electrons, two neutrons and two protons) that make up an uncharged helium atom should be the particle to interact with the total system of the measurement process, Ψ.

We now take the partial derivative in order to find the function c_n which maximises Ω_c. Note we also use the fact that each n index in the sum is acted upon only by the corresponding n index in the partial derivative. Different indexed sum terms disappear since they are constants with respect to that n index derivative. We therefore have:

$$\frac{\partial f}{\partial c_n} = \ln(n) - \ln(c_n) - 1 + 1 - \alpha = 0 \tag{18}$$

it is clear using the second derivative test that this is a maximum. So:

$$c_n = \frac{n}{e^{\alpha}} \tag{19}$$

Bringing this c_n into Equation 11, we therefore have:

$$\dim \mathcal{H}_{\Psi} = \prod_{n=1}^{k} d^{n^2 e^{-\alpha}} \tag{20}$$

6 Regularisation of the Outcome function

Using the counting function from equation 20 we can then examine what we would expect to happen under conditions of measurement. Under these conditions, we want to increase k such that the size of the objects are large enough to be perceptible by measuring apparatus and scientists, and perhaps wider, with the environment and universe itself. In the scale dealt with in quantum mechanics this can be represented as $k \to \infty$. We therefore have the following for the maximum number of possible outcomes following measurement:

$$\dim \mathcal{H}_{\Psi} = \prod_{n=1}^{k\to\infty} d^{n^2 e^{-\alpha}} \tag{21}$$

Taking logarithms of both sides:

$$\log \dim \mathcal{H}_{\Psi} = e^{-\alpha} \log d \sum_{n=1}^{\infty} n^2 \tag{22}$$

Using Zeta function regularization to assign a value to the divergent sum, $\sum_{n=1}^{\infty} n^2 = 0$ [17], we find that:

$$\log \dim \mathcal{H}_{\Psi} = 0 \tag{23}$$

and so

$$\dim \mathcal{H}_{\Psi} = 1 \tag{24}$$

Therefore the maximum number of possible outcomes from a quantum measurement following interaction with the environment is one. This is a model of wavefunction collapse as it shows how the non-linear projection into a measured, and single, state might occur. This will be further examined below in terms of measurement operators.

7 Measurement Operators and Selection Criteria

7.1 Single Eigenstate Selection

Finally, it may prove useful to examine this process using the measurement operator approach. We have an arbitrary linear Hermitian measurement operator, $\hat{O}$ acting on our total system, Ψ. Since, by definition, the eigenstates of the operator acting on the system form a complete set of basis states of the system, it is clear that

$$\hat{O}\,|\Psi\rangle = \sum_{i}^{\dim \mathcal{H}_\Psi} a_i(\hat{O}\,|\phi_i\rangle) = \sum_{i}^{\dim \mathcal{H}_\Psi} a_i(\lambda_i\,|\phi_i\rangle) \tag{25}$$

Upon collapse, however, the cardinality of the set of possible eigenstates must reduce to

$$\mathbf{card}(\{\lambda_i\,|\phi_i\rangle\}) = \dim \mathcal{H} = 1 \tag{26}$$

So clearly upon collapse there is only one eigenfunction and eigenvalue.

A brief note around the simultaneity of what we propose: we interpret this measurement operator acting upon the Hilbert space as being simultaneous with $k \to \infty$. That is the number of eigenstates the Hilbert space is projected into increases as the total system interacts with more objects through the measurement process. However, as the dimensions of the spaces are reduced through regularisation, as we have shown above, then $\dim \mathcal{H} = 1$. The collapse occurs as described above and a single eigenstate is selected.

7.2 Born Rule

This single eigenstate is selected from the possible set of eigenstates and this outcome is selected with a probability defined by the Born rule. In bra-ket notation, using our completeness relation defined in appendix A, the probability of measuring an eigenvalue, λ_i, that corresponds to an outcome relating to an isolated system is:

$$|\langle\phi_i|\Psi\rangle|^2 = |a_i|^2 \tag{27}$$

The laws of quantum physics dictate the probabilities associated with the outcomes, and the possible eigenstates, and so the selection process, through the Born rule and the measurement operators, is physically realistic.

8 Approaches to Experiments

[18] gives an overview of some possible experimental tests of some popular collapse models. These experimental methods may be suitably altered to allow for a test of the approach described here.

8.1 Emission Experiments: the Dioso-Penrose Approach and Direct Validation

A recent experimental test has ruled out a parameter-free version of the gravity-collapse, Diosi-Penrose model [19] [20] [21] [22], testing for emissions based on a proposed random diffusion process [23]. This emission process has been derived from the fluctuations the Dioso-Penrose model would predict. The model suggested in this paper does not explicitly involve a random emission process (although we do recognise that neither did the Dioso-Penrose model). It would be interesting to understand how the theorem described in [24] should apply to our model. This theorem proves that given certain assumptions, all collapse theories should induce a diffusion. Understanding the specifics of this theorem, and its application to the regularisation model presented in this paper, might provide a direct route to validation of this approach. Performing the calculations involved in determining the diffusion radiation that might be observed at collapse based on this model, given the complexity of the process described, is beyond the scope of this paper.

8.2 Differentiating Tests Against the GRW Model

An approach to potentially differentially validate this model of measurement against other proposed collapse models would be to highlight key differences between models, that would or would not predict collapse, and where these differences might be experimentally testable. The GRW model is a well known collapse model and is a suitable model for this differential validation.
The GRW model has two parameters: the collapse strength, $\tau_{collapse}$, and the spatial correlation collapse function, r_c.
Fist, let us look at the $\tau_{collapse}$ parameter. $\tau_{collapse}$ gives the collapse rate and is measured in collapses per second. Numerically, GRW suggested $\tau_{collapse,GRW} = 10^{-16}s^{-1}$, [7], while Adler later suggested a value of $\tau_{collapse,Adler} = 10^{-8}s^{-1}$ [25]. The model proposed in this paper does not explicitly have any time parameters associated with the principle theory, and so a differentiating test for our model against the GRW model might be to test for whether collapse is associated with time, or whether, as our model suggests, it is determined solely by the sequence of interacting particle systems, and complexity of those systems. For example, this model would suggest that a small number of particles, kept sufficiently isolated, will not undergo collapse without further interaction. The GRW approach suggests otherwise, however.
Another potential route for differential validation is to look at the spatial correlation function r_c parameter. A proposed value for r_c, according to GRW

was $r_c = 10^{-7}$m [7]. This is the scale at which collapses become apparent. For distances $< r_c$ collapses are not apparent, for distances $> r_c$ collapses are apparent. This paper does not explicitly suggest that collapse should be dependent on length scales. Collapses would be apparent over all length scales, so long as the criteria for complexity of systems interacting and sequence of interactions are met.

9 Overview of Results

The theory in this paper shows how a quantum system, under measurement, is projected into a single state at measurement and "collapses". As a quantum system undergoes interaction with larger and more complex systems, and as these systems approach the classical scales, the total number of possible outcomes from an experiment increases. This increase in number of possible outcomes is due to the increasing dimensionality of the space which describes the whole system, and greater number of possible particles which may interact. The number of outcomes from an experiment can be described as $\dim \mathcal{H}_\Psi = \prod_{n=1}^{k} d^{n^2}$. However, as $k \to \infty$, which is, at the quantum scale, as k approaches classic size, and as this system takes some classical properties, zeta-function regularisation makes collapse apparent. Rather than have an infinite number of outcomes from measurement, this regularisation shows how we observe just one outcome, which is the result of the measurement, selected by the Born rule.

9.1 A Toy Measurement Process

In this section we briefly sketch the the proposed process of measurement using a toy model, which should illustrate the general approach. We examine this toy process at three points in time.

We take a simple example of a Stern-Gerlach type experiment: a single electron is having its spin measured in the z axis. As the electron is accelerated through the magnetic field, the electron interacts with a number of similar sized quantum objects. These quantum objects are n-particle systems and might be environmental photons, electrons and single nuclei. As we saw due to the principle of maximum entropy in subsection 5.6, it is likely there will be c_n, of each of these n-particle systems.

At this point in time ($t = 1$), the electron has not undergone wavefunction collapse, and the wavefunction of the total system would be something similar to:

$$\Psi = \psi_1^{\otimes c_n} \otimes \psi_2^{\otimes c_n} \otimes \psi_3^{\otimes c_n} \otimes \psi_4^{\otimes c_n} ... \otimes \psi_k^{\otimes c_n} \tag{28}$$

where the ψ subscript counts the size of the n-particle system and $c_n = ne^{-\alpha}$.

The total wavefunction then will interact with motes of dust and more complex atmospheric molecules and droplets of larger sizes as it begins to interact with the high complexity of the laboratory environment. The wavefunction, Ψ, is as

just described, but with k much larger at $t = 2$ than at $t = 1$. Similar to $t = 1$, there are c_n of each of these n-particle systems of dust motes, and atmospheric molecules and droplets.
Finally, at $t = 3$ the total wavefunction then interacts with macroscopic objects and $k \to \infty$. Particles become distinguishable and the process described in this paper leads to apparent wavefunction collapse according to the framework above. Ψ is projected into a single eigenfunction associated with the relevant measurement operator acting on that Ψ.

10 Discussion

In this section we highlight some open questions and points of interest.

10.1 Avoiding Problems of Other Interpretations

The approach described, does not create any obvious conflicts with the existing mathematical framework, or require a conscious observer. This new interpretation of quantum mechanical measurement therefore avoids some of the problems associated with other interpretations, which have been widely discussed.

10.2 No Clear Line is Drawn

An important thing to note is that this formulation suggests that it is unclear where the line between quantum and classical worlds may lie, exactly. We have found that it is in the limit, $k \to \infty$, where k is the size of the largest k-particle system involved in measurement, that this formulation produces a physically interesting result. However, it is unclear how to interpret this when trying to understand how large objects might be before they collapse. Perhaps this suggests that so long as there are a finite number of quantum particles in a system then wave-function collapse will not occur? It is also unclear how large k is for $k \to \infty$ in the context outlined in this theory. We are assuming that $k \to \infty$ when larger than the number of particles in an atomic nucleus, but smaller thanthe number of particles in the universe. We have assumed that $k \to \infty$ on the scale of a human person.

10.3 Quantum and Classical Physics Are Mixed

10.3.1 Classical Collapse

Another thing to note is that this formulation includes both quantum (counting the dimensionality of a many-body Hilbert space and resulting outcomes), statistical-mechanical (counting the number of ways for systems of particles to interact with other systems of particles) claims and classical claims, all of which are needed for the regularisation to take place. This approach suggests that the number of quantum outcomes, at measurement, collapses to just one, but also that the number of ways that the classical system of particles can interact collapses to just one. Although this is not unphysical, and it should not be controversial that in a deterministic universe there should only be one possible outcome for the number of ways that systems of classical particles interact, this is a mixing of quantum and classical regimes. This mixing of regimes subtly alters the traditional scope of the quantum measurement problem. One interpretation of this might be that this zeta-function regularisation mediated wave function ‘collapse’ of the number of classical mechanical possibilities is actually a useful thing, since it shows how a deterministic world might appear from a more fundamental quantum, statistical and classical mechanical description. We will not explore the impacts on the classical reduction further but believe this to be an area of interest for further research.

10.3.2 Distinguishable and Indistinguishable Particles

Our argument relies on distinguishable particle statistics. We have worked on the assumption that since the measurement process spans classical and quantum worlds, then this distinguishable property is imposed, and so relevant in calculations. It is possible that this might be seen as a controversial claim, and acknowledge that more work can be done to examine the distinguishable property in physical terms. This is an open area of research, see [26], for example. One argument worthy of note, with regards to exchange of particles, one cannot interchange particles on a different scale (for example, a proton is not interchangable with a large molecule) and so distinguishability across scales is trivial. This supports our argument of the ingressing of the classical property of distinguishability into the collapse mechanism.

10.4 Controversy of Regularisation

This work might also highlight the importance of regularisation, and help us better understand the physical intuition to regularisation in physics, which has historically been controversial. For example, Dirac famously found the (related) renormalization approach to dealing with infinities ‘illogical’ and claimed its empirical success a ‘fluke’ [27]. Putting regularisation at the centre of a quantum theory of measurement might help highlight its importance and confirm its centrality to physics. On the other hand, it might be argued that the approach described in this paper simply hides the mystery of the measurement problem inside the mystery of regularisation, and reveals nothing about either.

10.5 Investigations at Other Scales

Finally, it would be interesting to investigate the theory of the scale changes in physical terms at different scales. For example, taking a simplified but realistic physical model at the quantum, atomic and larger levels, then examining these through the lens of the theory discussed.

10.6 What Particles Are Fundamental in this Approach?

We have not been clear on what constitutes a fundamental particle in the context of example given above. A concrete example, we have talked about neutrons as fundamental particles. We are not clear whether we should consider neutrons or quarks as the fundamental particles relevant for this approach. Whether this theory describes a quark or a neutron as fundamental is still an open question but do not consider this particularly important for the general theory. What is clear is that a fundamental particle in this approach is one which is both indivisible, and can exist on its own; the latter a property not exhibited by quarks.

10.7 Further Work: Experiment

In future, we would like to further develop this work to be able to validate or invalidate its theory, whether through direct experiment or through examination to understand if this theory is incompatible with existing quantum theory and experiments. It would be interesting to calculate the radiation emissions from random diffusion, which is predicted by [24] to directly test the model proposed here. It would also be interested to validate this model against the GRW model by looking at differences in predictions in regards to wavefunction collapse, with time components and length components being particularly of interest.

10.8 Further Work: Theory

We would also like to understand how this theory might work in the broader context of quantum field-theory, which has only been touched upon. In terms of theoretical validation, it would also be useful to understand the role that quantum decoherence might play, given its important role in the foundations of quantum physics. It would also be interesting to examine whether some of the ideas presented in this paper, such as the measurement ontology; outcome counting argument and regularisation approach to mediate wave-function collapse, might be usefully deployed in the frameworks outlined by other interpretations. For example, might the regularisation approach be useful as a potential mechanism in other objective collapse interpretations? It may be interesting to further understand the c_n function. For example, looking at how α varies with k may be of theoretical interest.

A Appendix: Formalism Used to Represent the Collapsed and Uncollapsed Wavefunctions

Below we note some key points on the formalism used to describe collapsed and uncollapsed wavefunctions. See [15], [1] and [13].

A.1 Ket Spaces Contain All Information About a State

We use the following postulate of quantum mechanics: A ket space contains all physical information about a state. Due to this we can say $|\Psi\rangle = \sum_i a_i |\phi_i\rangle$. That is, all the information contained in the uncollapsed wavefunction is the sum of the information physically defined in the possible collapsed wavefunctions.
This necessitates a completeness relation: $\sum_i |a_i|^2 = 1$.

A.2 Superpositions Do Not Appear After Measurement

Superpositions occur before and during the measurement process, but not after. The set of eigenstates that are possible after measurement $\{|\phi_i\rangle\}$ are not in a superposition and do not contain superpositions in the set. As such, the total wavefunction is seen to be normalised via the completeness relation above, involving only collapsed eigenstates.

A.3 Eigenstates

The number of eigenstates is equal to the number of eigenvalues. The eigenstate contains all the information (following collapse) about a physical state: $\lambda_i |\phi_i\rangle$ is a collapsed eigenstate with eigenvalue λ_i and eigenfunction ϕ_i. To sustain the completeness we say that $|\Psi\rangle$ contains a set of possible $|\phi\rangle$ represented as $\{|\phi\rangle\}$. Note that $|a_i|^2$ represents the probability of observing an eigenvalue, yet does not yield any information about the eigenvalue itself, as this would depend on the operator and physical basis used to formulate the eigenfunction. We avoid a physical formulation of ϕ throughout this text as this would distract from the objectives and outcome of the work itself, yet we maintain the standard rigour of quantum mechanics in its representation.

A.4 Bra Spaces

A bra space is used to represent the complex conjugate of a ket space. Since we do not define the structure of operators, only the number of outcomes (represented as the number of eigenstates), we only need to define eigenstates in this text: $\lambda |\phi\rangle$, except to complement our completeness relation with a closure rule: $a_i = \langle\phi_i|\Psi\rangle$. This definition is central to the Born rule used in section 7.2.

B Appendix: Clarification of the Conceptual Basis of the Measurement Model

In this section we further clarify the conceptual basis of the measurement mechanism of the proposed solution.
In Figure 1, we clarify the measurement process by examining the scale of the objects involved. Our conception of quantum measurement is the interaction of quantum objects with larger objects, until those objects are of the scale that

they might be thought of as classical. To the left of the diagram are microscopic objects comprised of a small number of n particles. These are quantum objects. To the right are macroscopic objects comprised of a large number of distinguishable particles. These are classical objects. Measurement is simply the interaction of small, simple objects with increasingly complex objects.
In Figure 2, we clarify the ways in which objects of similar complexities interact, and how these interact across 'layers' with those objects of increasing complexity. The number of objects in each n-layer is a constant according to the calculations above. While for illustrative purposes the n-particle objects are represented as being structured and with each particle joined, in reality these particles may be unstructured with the particles dispersed over space.
In Figure 3, we clarify the added number of outcomes due to the n possible ways that an n size object might be able to interact with another object, or in this case the total system, Ψ.

Fig. 1 Diagram showing the scale of interaction in the proposed measurement model

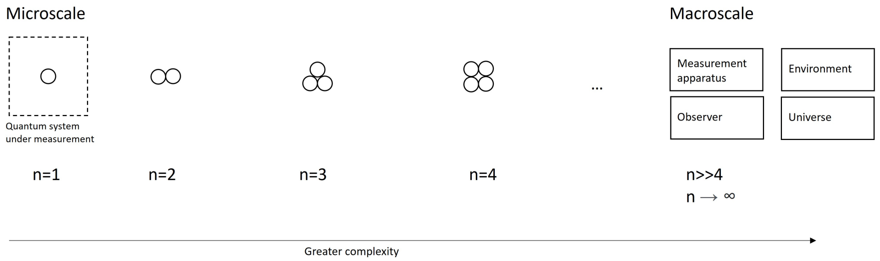


Fig. 2 Diagram showing the proposed number of outcomes due to superpositions across n-layers and inside n-layers

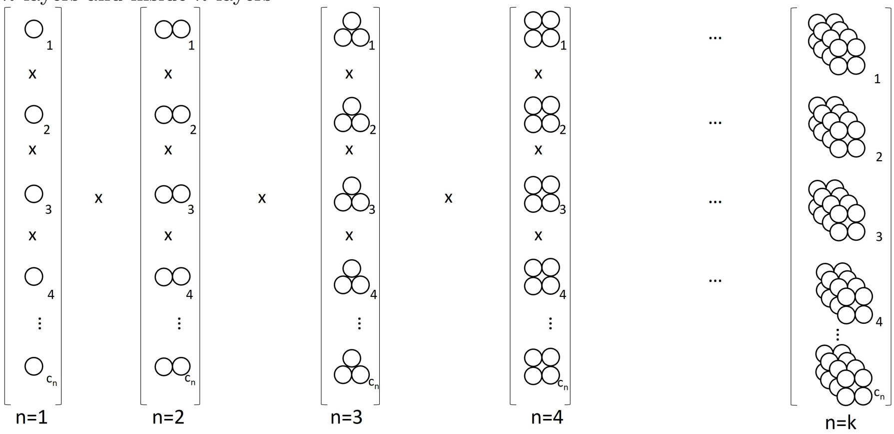
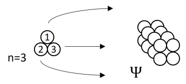


Fig. 3 Diagram showing the additional outcomes due to the additional classical component equal to n



C Appendix: An Alternative Approach with Indistinguishable Particles

While we have dealt with the assumption that the measurement process imposes the property of particles distinguishability onto the particles involved, quantum mechanics is usually formulated with the assumption that particles are indistinguishable. In this Appendix, we briefly discuss how to understand indistinguishable particles in the framework of the theory described.

C.1 Two Models Approach

While in this paper we use a single mathematical framework, we propose two different models to help explain the collapse. These two models involve introducing different ways of thinking about some of the mathematical objects that we will work with. We will briefly expand on these the second of those models and clarify how the assumptions in that model may differ from the approach in the main body of the text. We describe these models because even if the mathematical framework and derivation is similar across both models, either model can help us understand the physical assumptions in play. This requires clarity around the distinguishable or indistinguishable nature of the particles, which is why these two models have been proposed. These sections might be skipped by readers more interested in the mathematical content.

C.1.1 Size of Spaces Model

In this alternative model, we work on the principle that physically measurable spaces are the most important feature of the theory, and so work with these physical spaces as the primary object. We also assume that spaces and subspaces are physically dependent. We deal with this principle below in section C.1.3, assuming that these spaces might impose themselves on one another. That is, with the most general space, the space with its dimension counted by the outcome counting function, $\dim \mathcal{H}_\Psi$, imposes restrictions on the Hilbert sub-space, which then imposes restrictions on the Fock states sub-space. In particular, we suppose that these more general spaces, when they have their dimensionality reduced, reduce the dimensionality of their associated subspaces. It is important to note that according to our derivation, in this model there is still the classical regime ingressing upon the quantum world in terms of imposing distinguishability. However, the distinguishbility is apparent in a qualitatively different way.

C.1.2 Fock Space

Fock spaces, $F(\mathcal{H})$ are Hilbert spaces restricted to fermionic and bosonic allowed states across a number of particles. That is, the states which the relevant symmetry laws allow.

$$F(\mathcal{H}) = \overline{\bigoplus_{n=0}^{\infty} S_\gamma \mathcal{H}^{\otimes n}} \tag{29}$$

with S_γ as the operator which symmetrizes or antisymmetrizes a tensor depending on if the Hilbert space is bosonic or fermionic and n counts the number of particles. We, again, interpret the dimension of this Fock space as containing the states which can, in principle, be differentiated by experiment. For the purposes of this paper we will use a related idea, which accounts for the symmetry of bosons and fermions but is defined only for a given number of particles. That is, there is no sum through the zero, one, two, three etc. particle states, but is defined for m particles. This reduced Fock state is defined as follows:

$$F'(\mathcal{H}_m) = S_+\mathcal{H}^{\otimes p} \oplus S_-\mathcal{H}^{\otimes q} \tag{30}$$

with S_+ the symmetrizer operator acting on a bosonic space, and S_- the antisymmetrizer operator acting on the fermonic space, and $p + q = m$.
The relationship between the more general Hilbert space and the Fock space is clear. The Hilbert decomposes into two subspaces, this Fock space $F'(\mathcal{H}_m)$ and an additional subspace containing states that do not posses any symmetry. This Fock state its-self decomposes into symmetric and anti-symmetric states.

C.1.3 Dimension of Spaces Principle

Given that symmetry restrictions reduce the dimensionality of a Hilbert space to a Fock state, and given that our outcome counting function includes both quantum and a classical component in addition to the quantum Hilbert contributions, we may state the following as a principle, for a given d and m:

$$\dim \mathcal{H}_\Psi \geq \dim(\mathcal{H}_{\infty,m}) \geq \dim(\mathcal{H}_{d,m}) \geq \dim(F'(\mathcal{H}_{d,m})) \tag{31}$$

This is because in general:

$$\dim(A \oplus B) = \dim A + \dim B \tag{32}$$

and for bosonic spaces

$$\dim(S_+\mathcal{H}^{\otimes m}) = \frac{(m+d-1)!}{m!(d-1)!} \tag{33}$$

and for fermionic spaces

$$\dim(S_-\mathcal{H}^{\otimes m}) = \frac{d!}{m!(d-m)!} \tag{34}$$

See footnote [2] for a heuristic proof of inequality 31.

Our outcome counting function for a general quantum system of m particles is larger than or equal to the dimensionality of the (infinite and finite) Hilbert spaces for that same system, which itself is larger than or equal to the Fock state for that same system.

C.2 Interpretation of Collapse Using the 'Sizes of Spaces' Model

We may map the collapse process described in the main body of the text back onto more familiar, less general, quantum spaces, by recalling our principle:

$$\dim \mathcal{H}_\Psi \geq \dim(\mathcal{H}_{\infty,m}) \geq \dim(\mathcal{H}_{d,m}) \geq \dim(F'(\mathcal{H}_{d,m})) \tag{37}$$

see that the reduction in this largest of dimensions, the dimension of our outcome function, $\dim \mathcal{H}_\Psi$ (counting quantum outcomes of the most general Hilbert and classical contributions) could be seen to leading to a 'squashing' of the smaller dimensions of the general Hilbert, truncated Hilbert and Fock state spaces.

$$\max O = 1 \geq \dim(\mathcal{H}) \geq \dim(F(\mathcal{H})) \tag{38}$$

The smaller dimension objects have their dimensions consequently reduced. The approach taken in Section 7 then applies.

D Appendix: Are We Counting Unphysical Basis States?

In this section we briefly discuss an interpretation of the outcome counting function in the context of Hilbert and Fock spaces, which might prove instructive, given discussions in Appendix C. In particular, we show how the redundancy of the Hilbert space with respect to the Fock subspace impacts our

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[2]

The latter part of the inequality in this principle can easily be seen. Firstly, by inspection, for a given d:

$$\frac{(m+d-1)!}{m!(d-1)!} \geq \frac{d!}{m!(d-m)!} \tag{35}$$

So to maximize $\dim(S_+\mathcal{H}^{\otimes p} \oplus S_-\mathcal{H}^{\otimes q})$ we only look at the S_+ contribution. That is, with $p+q = m$, then $p = m$.
Then, by inspection,

$$d^m \geq \frac{(m+d-1)!}{m!(d-1)!} \tag{36}$$

So it is clear that $\dim(\mathcal{H}) \geq \dim(F'(\mathcal{H}_m))$ Since we know that $\infty \geq d$, then $\dim(\mathcal{H}_{\infty,m}) \geq \dim(\mathcal{H}_{d,m})$ follows.

counting argument. The key argument of note is that the probabilities of measuring certain outcomes may be 0, however, these outcomes are still counted as outcomes in our outcome counting function, even if they are unphysical.

D.1 Example: Pauli Exclusion Principle

Take for example, the case of an electron, ψ_A, that might be measured with eignvalue $\lambda_{a,1}$ or $\lambda_{a,2}$ and so $d = 2$. Introducing another electron, ψ_B, which might be measured with eignvalue $\lambda_{b,1}$ or $\lambda_{a,2}$, and following interaction of ψ_A and ψ_B, according to our counting function we should have in principle, $d^m = 2^2 = 4$ outcomes. However, due to the Pauli exclusion principle, it is clear that if these electrons were to share other quantum numbers, and also were confined to a distance comparable to their deBroglie (wave)length, then the antisymetric (fermionic) state would be disallowed, and so the number of possible outcomes should be only those which are symmetric

$$\dim(S_+\mathcal{H}^{\otimes 2}) = \frac{(m+d-1)!}{m!(d-1)!} = \frac{(2+2-1)!}{2!(2-1)!} = 3 \tag{39}$$

However, below we will show how the number of outcomes is solely determined by d and m, even if the probabilities associated with some of those outcomes might be very close to 0. We also count unphysical states which have probability 0 in larger systems than the one considered in this example, i.e. states that are not in the Fock space, but are in the Hilbert space, are still counted.

D.2 Spin Statistics Theorem and Outcome Counting

According to the Spin Statistics Theorem (see [28] for example), which derives the Pauli Exclusion principle, we see that it is the antisymmetric nature of fermions which disallow shared states. The two particle wavefunctions are described by a bosonic (symmetric) two-particle wavefunction :

$$\Psi(A, B) = \psi_A \psi_B \tag{40}$$

According to the Spin Statistics Theorem, for fermions for identical particles, with α as a normalisation factor, we have

$$\Psi(A, B) = \alpha\big(\psi_A \psi_B - \psi_A \psi_B\big) = 0 \tag{41}$$

So according to the Born rule selection process above, in bra-ket notation, we have the probability of finding a wavefunction in a Pauli excluded state as:

$$|\langle \phi_i | \Psi(A, B) \rangle|^2 = 0 \tag{42}$$

We interpret these Pauli excluded states not as outcomes that cannot be counted and included into our counting argument, but we interpret the identical particle fermion excluded states as reduced probability states due to a cancelling action. To reiterate, these low probability wavefunctions represent, in principle, the possibility of outcomes of experiment, and so a contribution towards our counting argument: it just happens that these have probability of occurrence of close to 0 due to the laws of physics. In actual fact the antisymmetric states, for this particular example, will never have exactly 0 probability since we can never have the full physical extent of two wavepackets in exactly the same place, they will repel each other before this can happen.

E Appendix: Infinite Hilbert Spaces

We now briefly discuss the most general infinite dimension Hilbert space for a given number of particles, $\mathcal{H}_{\infty,m}$. In some cases it is clear that for calculations we can define d, such as in the case of magnetic spin projection directly measured on a single cartesian axis, where $d = 2$. This is a truncated, or finite, Hilbert space. However, Hilbert spaces of infinite dimension are necessary in quantum mechanics [14] [13]. We treat the finite dimensional Hilbert space as good approximations to the calculations for the more general, infinite, case. We use these finite examples to help illustrate the theory. However, as above, the infinite dimension of the more general Hilbert space is the quantity used in deriving our counting arguments.

F Appendix: c_n without n multiplicity

It may also be useful to examine the c_n function without the additional degenerate multiplicity due to n ways for the particles to interact with Ψ. We shall call this c'_n

We may find c'_n through the following optimization problem (in this approach examining only the multinomial):

$$\begin{array}{ll} \max & \frac{C!}{c_1!c_2!...c_k!} \\ \text{s.t.} & c'_n > 0 \\ & \sum_{n=1}^{k} c'_n = C \end{array} \tag{43}$$

Since $C!$ is constant, we can invert this optimization. The above maximization equation is clearly equivalent to the minimization of with the same constraints above:

$$\min c'_1!c'_2!...c'_k! \tag{44}$$

To solve this, we can note that for each unequal pair of c'_n, represented here by

$$(c'_n + \delta)$$

and $(c'_n - \delta)$:

$$(c'_n + \delta)!(c'_n - \delta)! \geq c'_n! \tag{45}$$

We notice that in order to minimimze the left hand side of the above, we must have equal pairs of c'_ns. Therefore, the c'_n function which maximises equation 43 must be:

$$c'_1 = c'_2 = c'_n = c' \tag{46}$$

So we have c' is the number of n-particle systems for each n which interact in the proposed measurement model. Also noting that we have k contiguous bins in our derivation, it is clear that:

$$c' = \frac{C}{k} \tag{47}$$

For a simple, but less instructive, proof of this, observe that the maximum entropy distribution with no constraints on the expected value is this uniform distribution ($c'_n = c'$). See, for example, [29]. See footnotes [3] [4] for a brief discussion of c'.

[3]In order to better understand this quantity c', we also know that the total number of particles, m, is the number of n-particle systems, c', multiplied by n, as in equation (10).

$$\sum_{n=1}^{k} nc' = m \tag{48}$$

where k is the number of particles of the largest n-particle system. Using the sum of natural numbers:

$$\frac{k^2+k}{2}c' = m \tag{49}$$

So therefore,

$$c' = \frac{2m}{k^2+k} \tag{50}$$

[4]There is also a natural argument for the uniform c'_n distribution. In nature there are generally a fairly constant number of particles that constitute a composite particle. For example, we see that 12 nucleons constitute ^{12}C, yet also it is highly common to observe around the same number of atoms in a molecule, and furthermore the same number of molecules that constitute lipids and proteins and a similar number of these that constitute a small speck of dust. Matter gathers together in a generally consistent way. Although the c'_n may vary vastly between scale changes, it may also be seen to reduce back to the uniform pattern periodically and therefore average at a fairly constant value.

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