Collatz Conjecture Explored
Counter-Example Examination Leads to a '98%' Proof

Updated : July 231, 2023
James DeCoste
jbdecoste@eastlink.ca

Section - Abstract

This is a mathematical analysis of everything Collatz. I've come up with a revolutionary way of representing the counting numbers as an infinite set of equations. From these I am able to make some provable connections between these equations that not only show all counting numbers are used once in the Collatz Tree structure; where any additional loops originate; among other realizations. I also show that there can only be one unbroken chain of continuous “3n+1 / 2” growing to infinite number sizes approaching infinity but never actually getting there. This would be the only counter-example that is possible

Using the induction method where we assume x= 1 is true; also assume that x from 1 to k are also true; then k+1 is also true. That is a complicated way of saying that if we know or assume all numbers from 1 to k are true, then the very next number k+1 is also true in as much as we apply the two rules correctly so the number reduces to one that is in the already in the proven set!

The first three equations of my infinite set of equations are easy to apply this induction to and cover 87.5% of the counting number set. I change things up a bit for the upper level equations but I am able to prove through the same induction method that any number that is not a multiple of 3 (falling in these levels/equations) is also provable. All said and done I am able to prove that 98% are provable leaving only 2%! And that 2% are multiple of 3 numbers only. If only I could find a method to handle this condition the proof would be complete.

I've covered off on the loop issue part of the proof by showing how additional loops come about in the Collatz Tree structure. There is only one loop in Collatz and that is the trivial { 1 – 2 – 4 } loop. No others are possible no matter how close to infinity one gets and all numbers will reduce to this trivial loop.

The detailed discussion of how I arrived at these different conclusions is outlined below. I apologize if some sections are difficult to follow. I am not a mathematician by nature or profession. I do love maths though. I hope you enjoy my self awakening process on this subject as I did further exploring.

This is an updated version of my original document with a new section near the end that gets into the details of the proof. The remainder of the original report remains intact and unchanged.

Section 1 - Introduction

The Collatz conjecture is a sequence of numbers generated by applying two rules; if the number is Odd multiply it by 3 and add 1 (3n+1); if the number is even then divide by 2 (n/2). So the Collatz sequence is { 3n+1 ; n/2 }.

The conjecture states that if you start at any number from 1 to infinity (natural counting numbers) you will eventually end up in a { 1 – 2 – 4 } loop.

Sounds simple enough. It is, but proving that this is infact true over the entire set of natural numbers is quite difficult.

My attempt is to approach the proof from a slightly different angle and look at the natural numbers in a more confined fashion. This will allow for the observation that something fundamental is occuring. That will become clear in the following sections.

I am not a mathematician per say... but a computer scientist … and we all know computers are just large computational devices that rely on maths. I do not have access to a maths addon for publishing in the correct format so I will make due with what I can get off the keyboard (symbol wise).

Section 2 – Infinite Sequence of Equations to create all Natural Numbers (Primes)

The basis of my observations and subsequent conclusion is the understanding that all the natural numbers (1 to infinity) can be represented by the following infinite set of equations.

- 0 + 2x { 0 + (2^1)x } { (((2^1) / 2) -1) + (2^1)x }
- 1 + 4x { 1 + (2^2)x } { (((2^2) / 2) -1) + (2^2)x }
- 3 + 8x { 3 + (2^3)x } { (((2^3) / 2) -1) + (2^3)x }
- 7 + 16x { 7 + (2^4)x } { (((2^4) / 2) -1) + (2^4)x }
- …
- (((2^y) / 2) -1) + (2^y)x
- …
- (((2^infinity) / 2) -1) + (2^infinity)x

As seen above this is an infinite sequence of equations and it will cover all the natural numbers (1 to infinity). I expanded out the first few equations to show how they are formed noting that 'powers of 2' play a very important role. Now, there is an unexpected reality to these equations in that 0 + 2x contains all the even numbers (a subset that contains exactly half (½) of the natural number set). For example { 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, … }. The next equation 1 + 4x spawns the following subset: { 1, 5, 9, 13, 17, 21, … } This subset contains exactly one quarter (¼) of the entire natural number set. So the first 2 equations account for (¾) of the natural number set. You will find that the next equation subset will contain only (1/8) of the natural numbers: { 3, 11, 19, 27, … }. And the following equation has (1/16) of the natural numbers { 7 + 16x } { 7, 23, 39, 55, … }. Do you see a pattern here? The subset for any equation contains (1/2^y): { (½) for 2^1; (¼) for 2^2; (1/8) for 2^3; … }. As we approach the infinity power of 2 we find that that subset contains only (1/infinity) elements...a very tiny number. So just for kicks, let's calculate how what proportion of the natural number set are included with the first 10 equations (½) + (¼) + (1/8) + (1/16) + (1/32) + (1/64) + (1/128) + (1/256) + (1/512) + (1/1024) = (1023/1024). Interesting, indeed. The vast majority of all the natural numbers can be created using only the first 10 equations. We will come back to this point later.

Just so we are all on the same page I've listed the first several equations with the numbers they create:

{ 0 + 2x } → 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, 44, 46, ...
{ 1 + 4x } → 1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 45, 49, 53, 57, 61, 65, 69, 73, 77, 81, 85, 89, ...
{ 3 + 8x } → 3, 11, 19, 27, 35, 43, 51, 59, 67, 75, 83, 91, 99, 107, 115, 123, 131, 139, 147, 155, ...
{ 7 + 16x } → 7, 23, 39, 55, 71, 87, 103, 119, 135, 151, 167, 183, 199, 215, …
{ 15 + 32x } → 15, 47, 79, 111, 143, 175, 207, 239, 271, 303, 335, 367, ...
{ 31 + 64x } → 31, 95, 159, 223, 287, 351, 415, 479, ...
{ 63 + 128x } → 63, 191, 319, 447, 575, 703, 831, ...
{ 127 + 256x } → 127, 383, 639, 895, 1151, 1407, 1663, ...
{ 255 + 512x } → 255, 767, 1279, 1791, 2303, ...
{ 511 + 1024x } → 511, 1535, 2559, 3583, ...

This is likely as good a spot as any to show how primes work into my equations. The negative natural numbers shown is subsequant sections work in the same fashion. I'm going to list off the first 21 equations:

{ 0 + 2x }	→	0	+ 2x
{ 1 + 4x }	→	1	+ 4x
{ 3 + 8x }	→	**3**	+ 8x
{ 7 + 16x }	→	7	+ 16x
{ 15 + 32x }	→	**5 * 3**	+ 32x
{ 31 + 64x }	→	31	+ 64x
{ 63 + 128x }	→	**7 * 3 * 3**	+ 128x
{ 127 + 256x }	→	127	+ 256x
{ 255 + 512x }	→	**17 * 5 * 3**	+ 512x
{ 511 + 1024x }	→	73 * 7	+ 1024x
{ 1023 + 2048x }	→	**31 * 11 * 3**	+ 2048x
{ 2047 + 4096x }	→	89 * 23	+ 4096x
{ 4095 + 8192x }	→	**13 * 7 * 5 * 3 * 3**	+ 8192x
{ 8191 + 16384x }	→	8191	+ 16384x
{ 16383 + 32768x }	→	**127 * 43 * 3**	+ 32768x
{ 32767 + 65536x }	→	151 * 31 * 7	+ 65536x
{ 65535 + 131072x }	→	**257 * 17 * 5 * 3**	+ 131072x
{ 131071 + 262144x }	→	131071	+ 262144x
{ 262143 + 524288x }	→	**73 * 19 * 7 * 3 * 3 * 3**	+ 524288x
{ 524287 + 1048576x }	→	524287	+ 1048576x
{ 1048575 + 2097152x }	→	**41 * 31 * 11 * 5 * 5 * 3**	+ 2097152x

The important thing to notice here is that the first part of every equation is simply some {2^x –

1 } and that each of them in turn is formed by nothing but PRIME factors. The ultra important realization is that starting at 3 every second equation after that is comprised of factors that contain at least one 3. All the other equations do not include a factor of 3. This makes every second equation a 'multiple of 3' equation? We will see that any odd number that is a multiple of 3 can not form further branches; it is a dead end row. I love how primes have made an appearance. Later we will see the appearance of 3^x = 2^y +1. Again a connection with powers of 3 and powers of 2. Note there are only two cases where this is true; 3^1 = 2^1 + 1; 3^2 + 2^3 +1. The above primes discussion play with 2^x – 1. Quite a coincidence, isn't it? Every second equation is the same as saying add 3 multiplied by ' 4 ' or ' 2^2 '. 3+(3*4)=15; 15+(3*16)=63; 63+(3*64)=255;... Note that as we jump to next equation we are multiplying by 4 more...3*4; 3*4*4; 3*4*4*4;... This is how we skip over every other equation and why we see branches separated by ' 4 ' or ' 2^2 '.

Now, another item that may be important to explore here before going futher is the relationship between 3 and 2. This relationship fits in with how the Collatz tree propagates. If you multiply a number (say 1) by three and add one (3n+1) you are in effect doing 3+1=4. 4 is simply 2+2=4. 4 is in important transition point in the tree. Let's do another iteration of 3n+1 but not by multiplying but simply adding the effect. 3n+1+3n+1 = 3+3+2 = 8. Can we mirror this with 2? Yes, 2+2+2+2 or 4+4 =8. 3, 6 and 2, 4 are all an important numbers when building tables for Collatz:

Odd number	3n+1	n/2
1	4	2
3	10	5
5	16	8
7	22	11
9	28	14
11	34	17
13	40	20
15	46	23
17	52	26
19	58	29
21	64	32

See that the Odd number column is separated by 2 in each step up (+2). 3n+1 is (+6) in each step up. And just for kicks, n/2 is (+3) in each step up. Interesting INDEED! So there is a definate link between 3n+1 and n/2; that is 3 and 6.

What happens on the third itteration is very important to note. This is an important transition step. 3n+1+3n+1+3n+1 = 3+3+3 +3 = 12. So the excess 1's give an even 3 after 3 iterations. That is important because it becomes evenly divisible by 3. And it's connection to 2 is 2+2+2+2+2+2 = 12 or 6+6. or 4+4+4.

You are likely saying we can't use this and you are likely right but it was a stepping stone to show what I really intended. Again, suppose n=1 for ease of understanding. 3n+1 if n=1 is 4. Now apply 3n+1 to that and do it a second time ending up with 3(3(3n+1)+1)+1 or 27n+13. This is just three iterations of 3n+1. Lets rearrange 27n+13 to 27n+9+4 and factor out 9 giving 9(3n+1)+4 and since 4 is actually 3n+1, replace the 4 giving 9(3n+1)+(3n+1). This is the case so long as we keep n=1. You can now note that we actually have 10(3n+1). This means that after 3 consecutive iterations of 3n+1 we should be able to divide out an extra 2 (n/2). BUT, actually what is happening is (3n+1)/2. So to complicate things a tad bit what happens if we add in the n/2 each iteration. Should be nothing, really. First yields (3n+1)/2. Next yields (3((3n+1)/2)+1)/2. And the third gives (3((3((3n+1)/2)+1)/2)+1)/2. Multiplied through we get (27n+19)/8. If we try to do like above to factor out 9 we get (9(3n+1)+10)/8. Separate out a 4 from the 10 to give (9(3n+1)+(3n+1)+6)/8 or (10(3n+1)+6)/8. And we can still mathematically strip out a 2 as follows: 2(5(3n+1)+3)/8. In essence we continue to get an extra n/2 every three iterations. This observation must provide statistical advantage to increase the overall number of (n/2). Something similar must be happening when n is other than 1. I am unable to make that leap at this point.

I will come back to this connection later in this discussion.

Why have I discussed any of this in the first place. It was to show that all natural counting numbers are included in the tree structure. None are missed. As well, it is to show how powers of 2 and 3 play an important role in the construction of this tree. Since all odd numbers are in the tree implies that all even numbers are as well (since any even number can be formed by multiplying an odd number by two or another even number by 2). Again, this is a multiple of 2 (2^1).

Section 3 – Cascading effect

You are likely asking why this is important. That's where this gets very interesting. The structure of the tree is dictated by the odd number at any of the nodes; a 'node' being designated by it's location in the tree – in this case anywhere where you can go right by multiplying by two and up by multiplying by three and adding one. There are only two paths. Other nodes with two paths contain only two multiply by 2. So I call them connector nodes. I also call all other nodes with 3 paths connector nodes; they have a 'minus one and divide by three' and a 'divide by two' and a 'multiply by two'.

"node"

{even number = node * 3 + 1}
|
{ node } – { node * 2 }

"connector node"

{ connector node / 2 } – { connector node } – { connector node * 2 }

"connector node (all other nodes)"

```
{ connector node / 2 } – { connector node } – ( connector node * 2}
                                |
                         {     node      }
```

```
1 – 2 – 4 – 8 – 16 – …
                 |
                05 – 10 – 20 – 40 – ...
                      |          |
                      |         13 – 26 – 52 – ...
                      |                    |
                      |                   17 – 34 – 68 – ...
                      |                         |
                     03 – 06 – 12 – …          11 – 22 – 44 – ...
                                                      |
                                                     07 – 14 – 28 – ...
                                                               |
                                                              09 – 18 – ...
```

1, 5, 3, 13, 17, 11 – nodes (1 is included as a node because it loops back to 4)
2, 4, 8, 16, 10, 20, 40, 6, 12, 26, 52, 34, 68, 22, 44, 14, 28, 18 – connector nodes

If a node contains an odd number from say { 7 + 16x } … the very next odd number will be (3n+1)/2 and will be a number contained in { 3 + 8x } … with the very next odd number a further (3n+1)/2 and it will fall in { 1 +4x } … till it finally falls into { 0 + 2x }. This is the case no matter what subset you were to start at. If you started at {511 + 1024x } it would cascade uninterupted through each prior equation one-by-one till it gets to { 0 + 2x }.

Note that any odd number that is a multiple of '3' is a starting node. No other node can migrate through it on it's way back to the { 1 – 2 – 4 } loop. 3 & 9 shown above in red are such nodes. Do you see why this is the case? Any multiple of 3 can not be arrived at by applying 3n+1 to another odd.

This may be an appropriate place to point out another mathematical oddity that occurs in this trees. An Odd node will also cover itself times 4 plus 1. 4*(Odd Node) + 1... and that new node will have the same applied to it and so forth all the way though the tree for all ODD nodes.

```
1 – 2 – 4 – 8 – 16 – 32 – 64 – 128
        |       |          |
        |       |         21 – 42 - 84
        |       |
        |      05 – 10 – 20 – 40 – ...
        |            |          |
        1            |         13 – 26 – 52 – ...
                     |                    |
                     |                   17 – 34 – 68 – ...
                     |                         |
                    03 – 06 – 12 – …          11 – 22 – 44 – ...
```

```
                                                                              |
                                                                             07 – 14 – 28 – ...
                                                                                          |
                                                                                         09 – 18 – …
```

Drawn slighty different with the '1' hanging where it should be you can see this... 1, 5, 21, … or 1*4+1 = 5; 5*4+1=21; etc. 3, 13, 53, … or 3*4+1 =13; 13*4+1 = 53; etc. All nodes display this feature. This occurs because of the way the tree is constructed and branches form...namely that after the first branch on any row is formed, 2^2 or multiply by 4 to get the next branch on the row. And example is 10 and 40 on that row. 10*2*2 = 40. The branch at 10 gives a node of 3. The branch at 40 gives a node of 13. And the next branch at 160 (40*2*2 = 160) will give a node of 53 which is 53*3+1 = 160! And 53 is 13*4+1. All rows that can have branches do this indefinately.

Section 4 – Validating the Cascade Mathematically

Now I will take a moment to show how this works. Let's start with { 7 + 16x }. Any number created from this equation will be odd so one must apply the 3n+1 followed by n/2.

(3 ({ 7 + 16x }) + 1) / 2
(21 + 48x + 1) / 2
(22 + 48x) / 2
11 + 24x
3 + 8 + 24x
3 + 8 (1 + 3x) or { 3 + 8x since 1+3x is actually an 'x' after applying 3n+1 }

So as you can see from the above the very next odd number will fall in the prior equation { 3 + 8x }. Since it falls in this subset it is automatically an odd and can't be further divided by 2. Replace 1+3x with the new x and run this new odd again:

(3 ({ 3 + 8x }) + 1) / 2
(9 + 24x + 1) / 2
(10 + 24x) / 2
5 + 12x
1 + 4 + 12x
1+ 4 (1 + 3x) or { 1 + 4x since 1+3x is actually an 'x' after applying 3n+1 }

And this continues uninterupted until you get to the very first equation which is the even numbers:

(3 ({ 1 + 4x }) + 1) / 2
(3 + 12x + 1) / 2
(4 + 12x) / 2
2 + 6x
2 (1 + 3x)
2 (1 + 3x) or { 0 + 2x since 1+3x is actually an 'x' after applying 3n+1 }

Now this is an even number which can be divided at least once more by 2. Continully dividing by additional 2's will give us another odd number eventually. This odd number will fall into an upper equation but we have no way of knowing which one...we can not predetermine as far as I can tell. This will cause another uninterupted cascade down to the { 0 + 2x }. All cascades behave in this fashion and since the tree is nothing but cascades, the entire tree one giant cascade.

Section 5 – Observations from Cascading

This a good place to point out an obvious fact. Starting at any level equation, it must then continually and directly cascade to the first level { 0 + 2n }. So for each number in a given level it cascades directly to level { 0 + 2n } through it's very own path. This implies that the same number of entries in the preceding cascade are acounted for. So if { 7 + 16x} has a finite number of say 8 entries; and the preceding level { 3 + 8x } has twice as many to start; 16; then 8 of those are automatically accounted for. If level { 1 + 4x } has double that again; 32; and 8 of those are accounted for; leaving 24. And so on and so forth. But remember that all entries in the { 3 + 8x } also cascade uninterupted to first level...so only half of the prior levels entries are left in play... meaning that at level 0 { 0 + 2x } only half of the half remain in play (that means ¼ of the entire natural counting numbers set). The rest fall on some predetermined path from higher levels.

{ 0 + 2x } **2**, 4, 6, **8**, 10, 12, 14, 16, 18, 20, 22, 24, **26**, …
{ 1 + 4x } **1**, **5**, 9, 13, **17**, 21, 25, 29, …
{ 3 + 8x } **3**, **11**, 19, 27, **35** …
{ 7 + 16x } **7**, **23**, 39, …

So for the above all 3 number shown in subset for { 7 + 16x } cascade through each prior level consuming one number each in that level. And there's a pattern formed. Taking 7; it translates to (7*3+1)/2 = 11. 23 translates to 35 in the prior level. So the first entry (smallest) ends up tranlating to the second entry in the prior level. The next translates to the third item past 11 in the prior level – 35; and the next to three items past 35; as so on. If we start in the prior level with that first item 3; it translates to 5 in the prior level...11 translates to to three past 5 or 17...and so on. When jumping to the first evens level it does not translate to the second but the first...so 1 translates to 2 which is the first item in { 0 + 2x }. But each additional item hits 3 items higher after that; 5 translates to 8 – 9 translates to 14.

It may not be so obvious at this point but all the odd entries (all odd number in the natural number set) are accounted for. All the entries are already accounted for in all levels above { 0 + 2n }. That implies that any of the evens when divided by the appropriate number of 2s will spill to an odd number in a higher level that has already been acounted for. So without taking a leap of faith we can be confident that each and every natural number set is included in the tree. Right?

Section 6 – Trivial Loop jumps Out

This is a good place to point out the trivial loop and how it comes into being:

{ 0 + 2x } **2**, **4**, 6, 8, 10, …
{ 1 + 4x } **1**, 5, 9, 13, …

See how this happens? With these equations it jumps right out the page.

Section 7 – Putting it all Together

The next leap comes when you can only accept that no matter how much bouncing around it does, this process will eventually lead down to the trivial loop { 1 – 2 – 4 }. But I don't expect you to accept this blindly. If every third item in { 0 + 2x } is accounted for; that is 2, 8, 14, 20, 26, … let's do some quick number crunching... 2 reduces to the trivial; 8 also reduces to the trivial loop; infact all powers of 2 which are included in this subset will do just that. I call these the 'backbone collapse to trivial'. This is the obvious part. The final not so obvious is in the tree structure itself as I have drawn it. The power of 2s backbone is across the very top and the only possible direction in that row is left to '1' by dividing by 2 over and over. The next level down is where any possible backbone entry less 1 is divisible by 3...example (5*3)+1 = '16'. Now 5 can grow to the right by multiplying by 2 consecutively – 10, 20, 40, … or it can go up if multiplied by 3 and one added.

```
1 – 2 – 4 – 8 – 16 – …
                 |
                05 – 10 – 20 – 40 – ...
                 |              |
                 |             13 – 26 – 52 – ...
                 |                        |
                 |                       17 – 34 – 68 – ...
                 |                              |
                03 – 06 – 12 – …                11 – 22 – 44 – ...
                                                      |
                                                     07 – 14 – 28 – ...
                                                               |
                                                              09 – 18 – …
```

Once the tree is built it should be obvious that you can only proceed left and up. Going left and up will eventually lead to the backbone. Right? So the rest of those evens that are not exact powers of 2 will be found somewhere else in this tree structure where it can only go up or left and approach the backbone. Maybe someone has a better way to explain this. I sure hope that made sense!

I'm not too worried about the rest of the structure because I am ultimately trying to show there is at least one case where this cascade will be infinite and hence unending (or continually growing). This is the only case where the tree can grow forever...it has be an infinite cascade. So how does this play into it?

As one approaches infinity the ultimate number of steps in the cascade I discussed above approaches infinity as well. At infinity the process breaks. Infinity will enter an infinite number of steps in this cascade. So is this not a counter example? To disprove the conjecture?

I am thinking not. Since this happens at the very endpoint we can likely use this to show that the only case where it can grow infinitely is at that endpoint of infinity and since we can never get to the endpoint of infinity; there are no other situations where it is possible so long as { n < infinity }. All numbers from 1 up to but not including infinity will reduce to the ultimate loop { 4 – 2 – 1 }.

Section 8 – Exploring the negative numbers in the sequence { 3n-1 ; n/2 }

I found it interesting in that if one uses the negative natural counting numbers from -1 to -infinity in the { 3n-1 ; n/2 } instead of the above Collatz { 3n+1 ; n/2 } one gets the exact same tree as outlined above...except it contains nothing but negative numbers; and instead of going left and up as seen in Collatz it goes right and up. It changes direction which is expected. The magnitude remains the same. The same trivial loop occur except it is { -1 – -2 – -4 }.

My special set of equations are slightly different but the same rules apply (Negatized).

- -0 + 2x { -0 + (2^1)x } { -(((2^1) / 2) -1) + (2^1)x }
- -1 + 4x { -1 + (2^2)x } { -(((2^2) / 2) -1) + (2^2)x }
- -3 + 8x { -3 + (2^3)x } { -(((2^3) / 2) -1) + (2^3)x }
- -7 + 16x { -7 + (2^4)x } { -(((2^4) / 2) -1) + (2^4)x }
- …
- -(((2^y) / 2) -1) + (2^y)x
- …
- -(((2^infinity) / 2) -1) + (2^infinity)x

{ -0 + 2x } -2, -4, -6, -8, -10, -12, -14, -16, -18, -20, -22, -24, -26, …
{ -1 + 4x } -1, -5, -9, -13, -17, -21, -25, -29, …
{ -3 + 8x } -3, -11, -19, -27, -35 …
{ -7 + 16x } -7, -23, -39, …

See the same trivial loop { -1 – -2 – -4 } and it jumps out as well. The rest of the argument is exacly the same for the negative natural counting numbers in the sequence { 3n-1 ; n/2 }.

Do my formulas show a convergence as well:

(3 ({ -7 + 16x }) - 1) / 2
(-21 + 48x - 1) / 2
(-22 + 48x) / 2
-11 + 24x
-3 - 8 + 24x
-3 + 8 (-1 + 3x) or { 3 + 8x since -1+3x is actually an 'x' after applying 3n-1 }

And this is the case for all these equations.

Let's try the cascade to { 0 + 2x }:

(3 ({ -1 + 4x }) - 1) / 2
(-3 + 12x - 1) / 2
(-4 + 12x) / 2
-2 + 6x
-0 - 2 + 6x
-0 + 2 (-1 + 3x) or { 0 + 2x since -1+3x is actually an 'x' after applying 3n-1 }

They behave exactly the same way as the positives. So I will not bore you by showing more of them in detail. Once was quite enough to prove the point.

But, lets consider if 4x+1 holds true in this tree. -1*4 + 1 = -3. So no, it would appear we would have to negatize the process to 4x-1. That works just like 4x+1 did in Colatz. We will likely see the same thing occur in the following sections where 3 trees become possible.

Section 9 – Exploring the negative numbers in the Collatz sequence { 3n+1 ; n/2 }

Placing the negative numbers in my original equations (they have been negatized) yeilds the following:

- -0 + 2x { -0 + (2^1)x } { -(((2^1) / 2) -1) + (2^1)x }
- -1 + 4x { -1 + (2^2)x } { -(((2^2) / 2) -1) + (2^2)x }
- -3 + 8x { -3 + (2^3)x } { -(((2^3) / 2) -1) + (2^3)x }
- -7 + 16x { -7 + (2^4)x } { -(((2^4) / 2) -1) + (2^4)x }
- …
- -(((2^y) / 2) -1) + (2^y)x
- …
- -(((2^infinity) / 2) -1) + (2^infinity)x

{ -0 + 2x } -2, -4, -6, -8, -10, -12, -14, -16, -18, -20, -22, -24, -26, …
{ -1 + 4x } -1, -5, -9, -13, -17, -21, ...
{ -3 + 8x } -3, -11, -19, -27, -35 …
{ -7 + 16x } -7, -23, -39, …

I had to make a slight change to my equations to cover all the negative natural numbers but for all intents and purpose the same levels pop out valid.

Now what does the tree structure look like:

```
-1 – -2 – -4 – -8 – -16 – -32 – …
                |         |
                |        -11 – -22 – -44 – -88 – -176 – …
```

```
      |                         |             |
      |                         |           -59 – -118 – -236 – ...
      |                         |                          |
      |                         |                        -79 – -158 – ...
      |                         |
      |                       -15 – -30 – -60 – ...
     -3 – -6 – -12 – ...
```

This first tree has the loop at the very top left before any branching begins. The loop is { -1 – -2 }. Keep that in mind for the following two loops. Seems this tree does not include -5 so lets start a new tree with -5 as part of the loop:

```
-5 – -10 – -20 – -40 – -80 – -160 – …
              |            |
              |          -27 – -54 – -108 – ...
              |
              |
            -7 – -14 – -28 – -56 – ...
                   |
                  -5
```

Seems this loop is { -5 – -14 – -7 – -20 – -10 }. Also note that this being a loop for the second tree does in fact start at the top left and works it way down the first possible branch

And finally there is yet a third tree with it's own loop that covers the remainder of (1/3) of the natural counting numbers. And I'm taking an educated guess that it is -16-1 = -17 because the last loop was -4-1 = -5 and the very first loop was just -1. So my thinking was -(0)-1 = -1 is the { -1 – -2 – -4 } loop; -(2^2)-1 = -5 is the { -5 – -14 – -7 – -20 – -10 } loop; -(2^4)-1 = -17 is the next loop. Interesting, ehh? Also note that this loop as well begins at the upper left and proceeds down the second possible branch. I have not been able to show why this is the case but an educated guess would indicate it definitely has something to do with the 0; 2^2 and 2^4. It's also interesting that all three start numbers for each of the trees originates from { -1 + 4x } = **-1**, **-5**, -9, -13, **-17**, -21, -25, …And it is not a coincidence. Another way to look at it is simply 1+0; 1+4; 1+16 or 1+0; 1+2^2; 1+2^4. Powers of 2 still play an important role. It's going to take more work to determine exactly what is happening... the joy of number theory!

The following discussion is a fitting guess on what is happening and how these powers of 2 play into it. Directly following this I get into how to divide the natural counting numbers into 3 sets because 3n in the 3n+1 dictates that much. It takes a little leap to notice that in Collatz a power of three comes into play at two critical jump points (to new separate trees). Here is a table layout of the odd numbers applied to both 3n+1;n/2 & 3n-1;n/2:

Note that I have highlighted the odd numbers that can potentially jump off into their own tree which of course are given by 1; 1+2^2; 1+2^4 – 1, 5, 17. See above. And because we are dealing with multiples of 3 and three groupings/sets where we have { multiples of 3 }; { multiples of 3 + 1}; { multiples of 3 +2 }. Seems 3 plays a critical role.

So in Collatz we see what happens when we look at the three jump points 1, 5 and 17. 1 starts the natural loop { 1 – 2 – 4 }. At 5 we have the potential to jump off to a new tree but because 5 goes to 5 + 3 = 8 it stays in the original tree. It's also interesting that 8 = 2^3. Anything other than the addition of a power of 3 would have caused it to form it's own tree. Now with 17 we can see that again it goes to 17 + 3*3 = 26. Now again there was the potential of jumping off to a new tree had this number been created using a power of 3. The power of 3 kept it in the original loop. So in the case of Collatz and 1, 5, 17 all three stay in the same 1 – 2 – 4 loop.

Now see what happens when we look at the jump points 1, 5, 17 in the 3n-1;n/2 sequence. { 1 – 2 – 1 } is the natural first base loop. In the case of 5 it gives 5 + 2 = 7. This is adding a power of 2...not three. So 5 can break clean of the original loop because it has no way (needed to add a power of 3 to fall into the original loop) of entering the { 1 – 2 } loop.

The same thing happens with 17 in the 3n-1;n/2 sequence. Instead of adding a multiple of 3 to enable it access to the original loop it has a multiple of 2 (specifically 2 ^ 3 = 8). Note as well that 3 = 2+1 and 3*3 = 2*2*2+1. I point this out because we are actually dealing with 3n-1; so I would expect that at these jump points to see a number that is one less than what it would've been in Collatz. Now I suspect that the jump points 5 and 7 are the only two points where we can have 3^? = 2^? +1. I've seen this at play elsewhere I think in the a^x = b^y + 1; where x <> y (not equal).

How's that for some obscure reasoning?

Odd number	3n+1	n/2	3n-1	n/2
1	4	2	2	1
3	10	5	8	4
5	16	**8 (5+'3')**	14	**7 (5+'2')**
7	22	11	20	10
9	28	14	26	13
11	34	17 (11+3*2')	32	16 (11+'5')
13	40	20	38	19
15	46	23	44	22
17	52	**26 (17+'3*3')**	50	**25 (17+'2*2*2')**
19	58	29	56	28
21	64	32	62	31

Another interesting observation is that the set of all natural counting numbers can be subdivided into three distinct groupings. This provides ammunition and goes hand in hand with what I was dicussing above regarding only three possible trees.

Lets look at the number line and logically break into three groups. This will make more sense as we look at it in detail.

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, …

Starting at 0; add 3 consecutively to isolate all the multiples of 3. This is one third of the entire set:

0, 3, 6, 9, 12, 15, 18, 21, 24, … and leaves:

1, 2, 4, 5, 7, 8, 10, 11, 13, 14, 16, 17, 19, 20, 22, 23, …

Next, starting at 1, add 3 consecutively and strip out that third. This is the subset that is any multiple of 3 plus 1.

1, 4, 7, 10, 13, 16, 19, 22, … and leaves the final sub group:

2, 5, 8, 11, 14, 17, 20, 23, …

So starting at 2 and adding 3 consecutively gives us all the remaining numbers of the final sub-group. This final sub-group is simply a multiple of 3 plus 2! There are no more multiples of 3 plus anything that will result in a fourth sub-grouping.

The three sub-groups are:

{ 1, 4 ,7, 10, 13, 16, 19, 22, … }
{ 2, 5, 8, 11, 14, 17, 20, 23, … }
{ 3, 6, 9, 12, 15, 18, 21, 24, … }

This shows the three evenly distributed groupings that contain exactly 1/3 of the original natural counting numbers set. It also shows that even deeper than that, half of each of these 3 sub-groupings is even numbers. These 3 sub-groupings are integral in the Colatz tree as well. **3n**(+1) dictates that. Right?

I wonder if there is a conection to my original group of equations:

{ 0 + 2x } 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, …
{ 1 + 4x } 1, 5, 9, 13, 17, 21, 25, 29, …
{ 3 + 8x } 3, 11, 19, 27, 35 …
{ 7 + 16x } 7, 23, 39, …

And there is! Let's start with { 0 + 2x }:

{ 1, **4** ,7, **10**, 13, **16**, 19, **22**, … }
{ **2**, 5, **8**, 11, **14**, 17, **20**, 23, … }
{ 3, **6**, 9, **12**, 15, **18**, 21, **24**, … }

What about { 1 + 4x }:

{ **1**, 4 ,7, 10, **13**, 16, 19, 22, … }
{ 2, **5**, 8, 11, 14, **17**, 20, 23, … }
{ 3, 6, **9**, 12, 15, 18, **21**, 24, … }

And { 3 + 8x }:

{ 1, 4 ,7, 10, 13, 16, **19**, 22, 25, 28, 31, 34, 37, … }
{ 2, 5, 8, **11**, 14, 17, 20, 23, 26, 29, 32, **35**, 38, … }
{ **3**, 6, 9, 12, 15, 18, 21, 24, **27**, 30, 33, 36, 39, … }

And last for us to make point { 7 + 16x }:

{ 1, 4 ,**7**, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, … }
{ 2, 5, 8, 11, 14, 17, 20, **23**, 26, 29, 32, 35, 38, … }
{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, **39**, … }

My original equations hit each of these three sub-groupings evenly and in a defined pattern. Notice that in { 0 + 2x } each entry in the subgroup is separated by 6; 4 + 6 = 10 + 6 = 16, … In the next { 1 + 4x } it is a separation of 12; 1 + 12 = 13 + 12 = 25, … And if I wasa betting man I would wage to guess that the next { 3 + 8x } is sepatated by 24; 11 + 24 = 35,... with each equation we multiply the difference by an additional 2 (double it)... 6, 12, 24, 48, 96, … The original 6 in that sequence is the result of 3 * 2. Hmmm, multiples of 3 and powers of 2! 3 * 2^1; 3 * 2^2; 3 * 2^3; ...

So the third loop looks like this:

```
-17 – -34 – -68 – -136 – -272
              |              |
              |            -91 – -182 – -364 – ...
              |                    |
              |                  -61 – -122 – -244 – ...
              |                           |
            -23 – -46 – -92 – …         -41 – -82 – -164 – ...
                          |                          |
                        -31 – -62 – -124 – …       -55 – -110 – ...
                                |                          |
                              -21 – -42 – -84 – …        -37 – -74 – ...
                                                                 |
                                                               -25 – -50 – -100 – ...
                                                                       |
                                                                     -17
```

This loop is a little more involved: { -17 – -50 – -25 – -74 – -37 – -110 – -55 – -164 – -82 – -41 – -122 – -61 – -182 – -91 – -272 – -136 – -68 – -34 }

Also of some interest is the length of these loops and how they appear to relate to the jump points they start from:

{ -1 - -2 } - two steps
{ -5 – -14 – -7 – -20 – -10 } - five steps
{ -17 – -50 – -25 – -74 – -37 – -110 – -55 – -164 – -82 – -41 – -122 – -61 – -182 – -91 – -272 – -136 – -68 – -34 } - eighteen steps

The first loop begins at -1; but you need at least two steps to form a loop so voila you have a two step loop. The second loop starting with -5 requires exactly five steps. And the third loop starting -17 requires exactly eighteen steps. Now remember the way these trees work, powers of 2 and branching. The first and the third loops require one more step than the starting numbers. The second loop only requires the original five steps. This seems very coincidental, doesn't it? Too convenient! Now if I considered that in this case we are dealing with negative numbers (treat the negative sign as direction only; the actual magnitude of the numbers are same no matter the sign) then instead of adding '1' to the step count for the first and third loops I should've indicated that we are actually adding '-1'. -1 + -1 = -2; -17 + -1 = -18.

Generally, I would say since 'three' is prominent in the way this sequence works, we will only find the three separate trees with their own single loop. And I would expect that the numbers are distributed evenly among the three; with half of that third evenly split between even and odd.

Someone else has already done the statistics that show this to be the case; there are only the three trees and they each contain a 1/3 of the entire natural number set. So I'm not going to rehash that here and simply accept it.

Do my formulas show a cascading convergence as well:

(3 ({ -7 + 16x }) + 1) / 2
(-21 + 48x + 1) / 2
(-20 + 48x) / 2
-10 + 24x
-3 + 1 - 8 + 24x
1 - 3 - 8 + 24x
1 - 3 + 8 (-1 + 3x)
-3 + 8 (-1 + 3x) +1

It does cascade to an odd number in the prior level but has 1 added to make it even (or it ultimately jumps to { 0 + 2x }).

It's a little difficult to explain. Suffice to say we do infact cascade back to the prior level but instead of the number remaining odd it has one added to make it even again and thus divisible once more by 2...but this actually brings us directly back to the first level { 0 + 2x }. This is holding true for the first tree that has the loop { 1 – 2 – 4 }. But it does not appear to be the case in other two trees with the other two loops? I'm going to have to investigate this further to see if I can determine what is happening there and explain it in mathematical terms.

So, No, they break down and can not show a step by step cascade! In the case of the first tree with the { 1 – 2 – 4 } loop the cascade is directly to level { -0 + 2x }. The other two trees do the same thing at least mathematically as we have shown by working these equations through 3n-1.

I Think we need to look specifically at what is happening at { -1 + 4x } level. It's likely buried but doing the same cascade to { 0 + 2n } level.

(3 ({ -1 + 4x }) + 1) / 2
(-3 + 12x + 1) / 2
(-2 + 12x) / 2
-1 + 6x
1 - 2 + 6x
1 + 2 (-1 + 3x)
-0 + 2 (-1 + 3x) +1

It is doing the same thing. There is a hidden cascade to the prior level but it gets lost in translation and is overidden to first even level { -0 + 2x }. So what this is ultimately saying is that all levels over { -0 + 2x } have all their elements cascade directly to level { -0 + 2x }. Luckily there are enough elements in { -0 + 2x} for a one-to-one match with all the elements combined from upper levels. Right?

So we can likely build on that fact like we did before. In this case all levels cascade directly to { -0 + 2n }. So yes, all odd numbers will be accounted for and as a result all evens. Likewise, if magically have three evenly (1/3) distributed trees; that is 1/3 of all the natural number set falls in each of trees. The same odd and even as shown above will hold in each of these three trees as well.

Needless to say it is much easier to show with these three smaller trees that as n approaches infinity it is not creating a multi-level cascade that could reach infinity in steps...but instead have only a single cascade directly to level { 0 +2n }. So, there is NO situation where this sequence can grow indefinately and no quasi-counter to use to prove by contradiction like we did above in earlier discussion. I don't think we need to.

It is easily shown after all this that there is one and only one loop for each of the three individual trees. The structure dictates that.

The Collatz trees each hold the 4x+1 rule we've seen in the above discussion. It is without doubt that in the next section it will be 4x-1. -3*4+1 = -11; -23*4+1 = -91.

Let's go back to these three subsets outlined above:

{ 1, 4 ,7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, … }
{ 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, … }
{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, … }

The three loops occur and contain only numbers from the first two subsets... the ones that are not a multiple of 3 (the third subset). So the first two subsets only. The { -1 – -2 } loop:

{ **1**, 4 ,7, 10, 13, 16, 19, 22, 25, 28, 31, 34, 37, … }
{ **2,** 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, … }
{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, … }

And the { -5 – -14 – -7 – -20 – -10 } loop:

{ 1, 4 ,**7**, **10**, 13, 16, 19, 22, 25, 28, 31, 34, 37, … }

{ 2, **5**, 8, 11, **14**, 17, **20**, 23, 26, 29, 32, 35, 38, … }
{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, … }

And the { -17 – -50 – -25 – -74 – -37 – -110 – -55 – -164 – -82 – -41 – -122 – -61 – -182 – -91 – -272 – -136 – -68 – -34 } loop:

{ 1, 4 ,7, 10, 13, 16, 19, 22, **25**, 28, 31, **34**, **37**, 40, 43, 46, 49, 52, **55**, 58, **61**, 64, 67, 70, 73, … }
{ 2, 5, 8, 11, 14, **17**, 20, 23, 26, 29, 32, 35, 38, **41**, 44, 47, **50**, 53, 56, 59, 62, 65, **68**, 71, **74**, … }
{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63, 66, 69, 72, 75, … }

This makes sense since any row in the Collatz tree that starts with a multiple of 3 is a dead end row that can't spawn new branches so the loop items must not venture into that subset.

I wonder if there's a pattern here that me might pick up on if we overlay the three loops each in a different color:

{ **1**, 4 ,7, 10, 13, 16, 19, 22, **25**, 28, 31, **34**, **37**, 40, 43, 46, 49, 52, **55**, 58, **61**, 64, 67, 70, 73, … }
{ **2**, 5, 8, 11, 14, **17**, 20, 23, 26, 29, 32, 35, 38, **41**, 44, 47, **50**, 53, 56, 59, 62, 65, **68**, 71, **74**, … }
{ 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63, 66, 69, 72, 75, … }

I wonder what these loops look like in my equations:

{ 0 + 2x } – **2**, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, **34**, 36, 38, 40, 42, 44, 46, ...
{ 1 + 4x } – **1**, 5, 9, 13, **17**, 21, **25**, 29, 33, **37**, **41**, 45, 49, 53, 57, **61**, 65, 69, 73, 77, 81, 85, 89, ...
{ 3 + 8x } – 3, 11, 19, 27, 35, 43, 51, 59, 67, 75, 83, **91**, 99, 107, 115, 123, 131, 139, 147, 155, ...
{ 7 + 16x } – 7, 23, 39, **55**, 71, 87, 103, 119, 135, 151, 167, 183, 199, 215, …
{ 15 + 32x } – 15, 47, 79, 111, 143, 175, 207, 239, 271, 303, 335, 367, ...
{ 31 + 64x } – 31, 95, 159, 223, 287, 351, 415, 479, ...
{ 63 + 128x } – 63, 191, 319, 447, 575, 703, 831, ...
{ 127 + 256x } – 127, 383, 639, 895, 1151, 1407, 1663, ...
{ 255 + 512x } – 255, 767, 1279, 1791, 2303, ...
{ 511 + 1024x } – 511, 1535, 2559, 3583, ...

That's interesting but doesn't tell us much except that the loops are confined to elements from { 0 + 2x }, { 1 + 4x }, { 3 + 8x } and { 7 + 16x } only; with each loop starting on an element in { 1 + 4x } ONLY.

Section 10 – Exploring the Positive numbers in the sequence { 3n-1 ; n/2 }

Much like the previous section, placing the positive numbers in the { 3n-1 ; n/2 } sequence will generate the exact same three loops only in this case all the numbers are positive and the direction of travel is left and up instead of right and up.

We would use the original set of equation that have not been negatized.

- 0 + 2x   { 0 + (2^1)x } { (((2^1) / 2) -1) + (2^1)x }
- 1 + 4x   { 1 + (2^2)x } { (((2^2) / 2) -1) + (2^2)x }
- 3 + 8x   { 3 + (2^3)x } { (((2^3) / 2) -1) + (2^3)x }
- 7 + 16x   { 7 + (2^4)x } { (((2^4) / 2) -1) + (2^4)x }
- …
- (((2^y) / 2) -1) + (2^y)x
- …
- (((2^infinity) / 2) -1) + (2^infinity)x

{ 0 + 2x } 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, …
{ 1 + 4x } 1, 5, 9, 13, 17, 21, ...
{ 3 + 8x } 3, 11, 19, 27, 35 …
{ 7 + 16x } 7, 23, 39, …

Now what does the tree structure look like:

```
1 – 2 – 4 – 8 – 16 – 32 – …
            |           |
            |         11 – 22 – 44 – 88 – 176 – …
            |                     |           |
            |                     |         59 – 118 – 236 – ...
            |                     |                       |
            |                     |                     79 – 158 – ...
            |                     |
            |                    15 – 30 – 60 – ...
            3 – 6 – 12 – ...
```

This first tree has the loop at the very top left before any branching begins. Keep that in mind for the following two loops. Seems this tree does not include 5 so lets start a new tree with 5 as part of the loop:

```
5 – 10 – 20 – 40 – 80 – 160 – …
           |          |
           |        27 – 54 – 108 – ...
           |
           |
           7 – 14 – 28 – 56 – ...
                |
                5
```

Seems this loop is { 5 – 14 – 7 – 20 – 10 }. Also note that this being a loop for the second tree does in fact start at the top left and works it way down the first possible branch

And finally there is yet a third tree with it's own loop that covers the remainder of (1/3) of the natural counting numbers. And I'm taking an educated guess that it is 16+1 = 17 because the last loop was 4+1 = 5 and the very first loop was just 1. So my thinking was 0+1 = 1 is the { 1 – 2 – 4 } loop; 2^2+1 = 5 is the { 5 – 14 – 7 – 20 – 10 } loop; 2^4+1 = 17 is the next loop. Interesting, ehh? Also note that this loop as well begins at the upper left and proceeds down the second possible branch. I have not been able to show why this is the case but an educated guess would indicate it definitely has something to do with the 0; 2^2 and 2^4. See above for further observations on this coincidence.

So the third loop looks like this:

```
17 – 34 – 68 – 136 – 272
          |          |
          |        91 – 182 – 364 – ...
          |              |
          |            61 – 122 – 244 – ...
          |                   |
        23 – 46 – 92 – …     41 – 82 – 164 – ...
                  |                   |
                31 – 62 – 124 – …    55 – 110 – ...
                      |                    |
                    21 – 42 – 84 – …     37 – 74 – ...
                                               |
                                             25 – 50 – 100 – ...
                                                    |
                                                    17
```

This loop is a little more involved: { 17 – 50 – 25 – 74 – 37 – 110 – 55 – 164 – 82 – 41 – 122 – 61 – 182 – 91 – 272 – 136 – 68 – 34 }

Generally, I would say since 'three' is prominent in the way this sequence works, we will only find the three separate trees with their own single loop. And I would expect that the numbers are distributed evenly among the three; with half of that third evenly split between even and odd.

Again, someone else has already done the statistics that show this to be the case; there are only the three trees and they each contain a 1/3 of the entire natural number set. So I'm not going to rehash that here and accept it.

All of the exact same discussion remain true here to for proof as we have shown above in earlier sections.

Lets try a couple of the equations to make sure:

$(3 (\{ 7 + 16x \}) - 1) / 2$
$(21 + 48x - 1) / 2$
$(20 + 48x) / 2$
$10 + 24x$
$3 - 1 + 8 + 24x$
$-1 + 3 + 8 + 24x$

-1 + 3 + 8 (1 + 3x)
3 + 8 (1 + 3x) -1

and:

(3 ({ 1 + 4x }) - 1) / 2
(3 + 12x - 1) / 2
(2 + 12x) / 2
1 + 6x
-1 + 2 + 6x
-1 + 2 (1 + 3x)
0 + 2 (1 + 3x) -1

As expected, instead of adding one to get even we subtract 1 to get even and back to level { 0 + 2x }. The mechanics are the same.

Also, the 4x-1 rule does hold here as expected. 3*4-1 = 11; 15*4-1 = 59.

Section 11 – Understanding the 'NOT so Random' jumps within the Collatz Tree

What appears to be random jumps is actually constrained. Let's explore what is happening at each of my equations starting with { 0 + 2x }.

{ 0 + 2x } 2, **4**, 6, **8**, 10, **12**, 14, **16**, 18, **20**, 22, **24**, 26, **28**, 30, **32**, 34, **36**, 38, **40**, 42, **44**, 46, **48**, 50, ...

In the above illustration I have highlighted in different colors the sequences where you take the first x=2 and multiply by 2 successively.. 2, 4, 8, 16, 32,... I left the very first number in this sequence un-hilighted which will come in play later. The next available number is x=6 giving 6, 12, 24, 48, 96, … The next available number is x=10 giving 10, 20, 40, 80, 160,... And the next is x=14 giving 14, 28, 56, … Then it's x=18 giving 18, 36, 72, … Obviously there is a distinct pattern here and that is after rooting out all numbers that are multiples of '2' of a prior lower number we end up having every second number starting at 6 available for this operation... 6, 10, 14, 18, 22, …. So obviously, every number in this equation will end up in the Collatz Tree. Where it is in that tree is unimportant. Half of this set is divisible by at least 4. The other half is only divisible by 2 leading to an odd number that will fall somewhere else in the tree. I hope you can accept that.

Let me show the next few equations expanded out:
{ 1 + 4x } **1**, **5**, **9**, 13, 17, 21, 25, 29, 33, 37, …
{ 3 + 8x } **3**, **11**, 19, 27, 35, 43, 51, 59, 67, 75, …
{ 7 + 16x } **7**, 23, 39, 55, …
{ 15 + 32x } 15, 47, 79, …
{ 31 + 64x } 31, 95, ...

There is a pattern to how every second base even number in { 0 + 2x } jumps to upper level equations. So for the sequence 2, 6, 10, 14, 18, 22, 26, 30, 34, 38, 42, … do the division by 2 and you get 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, … Obviously 1, 5, 9, 13, 17, 21, … of this list all fall in the { 1 + 4x } equation. Note that this list is formed by adding 4 consecutively; 1+4=5+4=0+4=13... I'll be willing to bet that starting at 3 and adding 8 consecutively will give us a list that in the { 3 + 8x }... 3+8=11+8=19+8=27... 3, 11, 19, 27, … Then if we take 7 which is the next available starting sequence you would add 16 consecutively giving 7, 23, 39, 55, … which is the { 7 + 16x } equation. The pattern should now be obvious.

Let's explore the cascading level effect starting with the { 3 + 8x } equation. If you pick 3 you will pass through to the prior level { 1 + 4x } and that is so. 3*3+1=10/2=5. The same happens to 11...3*11+1=34/2=17. And the next 19 does it as well 3*19+1=58/2=29. And it just so happens 5, 17, 29, … are separated by 12 (3 * 4 or 3 * 2 * 2). This covers every number in { 3 + 8x }. The exact same thing happens if we investigate { 7 + 16x }... 3*7+1=22/2=11; 3*23+1=70/2=35; 3*39+1=118/2=59; or 11, 35, 59, … separated by 24 (3 * 8 or 3* 2 * 2 * 2). Looking at { 15 + 32x } we see similar 3*15+1=46/2=23; 3*47+1=142/2=71; 3*79+1=238/2=119; 23, 71, 119 are separated by 48 (3 * 16 or 3 * 2 * 2 * 2 * 2). Pattern has been established. Finally let's look at what happens with level { 1 + 4x }. We can see from the above that only 5, 17, 29, .. are pass through from upper levels. All other points in this equation remain untouched from upper levels leaving 1, 9, 13, 21, 25, 33, 37, … Note that all those that are passed through from upper levels reduce to an odd number that is smaller than it started at. 5 reduces to 1; 17 reduces to 13; 29 reduces to 11; 41 reduces to31; 53 reduces to 5; 65 reduces to 49, and so on. This is good because we can prove that given all numbers up to k are proven, then k+1 = 5 ends in a number that is less than 5 (actually 1) and this is the case for all of these.

Let's continue on with this trend of thought. 1, 13, 25, 37, … is another sequence separated by 12 in { 1 + 4x } that has not been touched from pass through from upper levels. These behave the same way as the pass throughs seen above. They all reduce to a number smaller than the starting number; 1 reduces to 1 (trivial); 13 reduces to 5; 25 reduces to 19; 37 reduces to 7; 49 reduces to 37; 61 reduces to 23. So with the same assumption that for k all lower assume true; k+1 = some number from this list results in a number smaller than k that has already been proven.

This leaves the final multiple of 3 sequence (again separated by 12) 9, 21, 33, 45, 57, 69, … And once again for the same agruement above all these reduce to numbers smaller than the original. 9 reduces to 7; 21 reduces to 1; 33 reduces to 25; 45 reduces to 17; 57 reduces to 43; 69 reduces to 13; so if up to k assumed true; it is obvious that k+1 ends up smaller than k so it is true as well.

This may be an ackward way to prove all numbers are included and reduce to the trivial loop in the Collatz tree. It does seem to work though.

Section 12 – Putting It All Together

Expanding upon the first few sections in this report, I will show where my set of equations originated and this is an important observation in showing that all the natural numbers are contained in the union of these subsets.

	2	4	6	8	10	12	14	16	18	20	22	24	26
1		5		9		13		17		21		25	
	3				11				19				
			7								23		
							15						

Let me draw the above in a format that is a little clearer to follow noting that I will skip over all the even number not formed by successively adding 6 to 2...

2	8	14	20	26	32	38	44	50	56	62	68	74	80
1	5	9	13	17	21	25	29	33	37	41	45	49	53
	3			11			19			27			35
				7									23
													15

There is a very unique pattern that makes it very easy to see that 'ALL' the natural numbers will be included. I've taken away any even number that does not grow 'stacks' back to smaller upper level members. This shows the cascading effect I've tried to explain above in other sections. Note how each upper level injects it's first member onto the stack resulting from the second member of the previous level. It's next member is is injected onto the thrid stack from the previous level...so each new member skips two prior level stacks before being injected. That is why I dropped two even numbers before creating stacks. It really jumps out now! Once you accept this you can see where my sequence of equations originated. And just in case you don't realize it, the lowest number in a stack multiplied by 3 and add 1 then divide by 2 give the next up... continue (*3+1)/2 to next level up and so on. These are the basic rules for odd/even numbers in Collatz conjecture.

This realization also brought me to the idea that if this goes on toward infinity there should be '1' stack approaching infinity! Right? The farther right one goes the longer the stacks can grow. But no prior stack less than infinity can be in the same state...the next closest one is one level smaller half way back from infinity (infinity/2). Think about that for a moment. Remember that each upper level equation has half the members the previous one did... hence my halving infinity. This should be enough to show all numbers are infact included; it's a complete set.

The very first row of even numbers is 50% of the total natural number set. The second row is an additional 25% (¼) of the natural number set... the third is 12.5% (1/8) and so on and so forth. You can also see several patterns when written in this fashion. Each set contains only half as many members as the previous set. You will also note that starting at row 1; the first available odd numbers missed in prior levels (even numbers row) start those sets. So the second row uses 1 as its starting number with successive members formed by adding 4 over and over. The third row would begin with 3 since it was not already used in the two prior rows...and it's members are given by adding 8 successively over and over. The next row begins with 7 and it's members are separated by 16. And this continues on. As you can see every number will be used and only ONCE. I'm also going to point out that if you pick the first member of any row greater than 1 (the even numbers row) and apply the (3n+1)/2 rules you will go up one row and to the right! For example (1*3+1)/2=2. The next row is (3*3+1)/2 =5. The next row starting with 7... (7*3+1)/2=11; (11*3+1)/2=17. The next is (15*3+1)/2=23; (23*3+1)/2=35; (35*3+1)/2=53. That was the important stuff to take forward...

I've shown above that only 3 loops can occur in the negative counting numbers under 3x+1; x/2 and only 1 loop using the positive counting numbers under 3x+1; x/2. So the existence of a second loop is not possible if following the original conjecture using only positive counting numbers under 3x+1; x/2. Two additional loops become possible only when using the negative counting numbers under 3x+1; x/2. There are two breakaway points, one at -5 and an additonal one at -17. The reasonaing as shown above plays with the -3+1=-2 & -3*3+1=-2*2*2 observation. The original loop as unstated would be -1*3+1=-2. As you can obviously see this gives rise to -1*3+1=-1*2 & -1*3*3+1=-1*2*2*2. I probably did a better job of showing this above. Needless to say the 3 jumping points (or three loops) start at -1; -5; and -17. You'll also note that -1+-2*2=-5 and -1+-2*2*2*2=-17 or -1+-2*2=-3*2+1 and -1+-2*2*2*2=-3*3*2+1. This special state can not occur in the positive counting numbers so there is only one loop starting at 1. No other loops can exist. So part one of the proof is confirmed...only the main loop exists.

Now I can build the other part of the proof from above observations. I noted that these counting numbers can be created using an infinite set of sequences; 0+2x; 1+4x; 3+8x; 7+16x; … The first sequence forms all the even numbers. The second sequence has half as many members all of which are odd and seperated by 4. The third sequence has half as many members as the second sequence with these being separated by 8...and so on and so forth.

I also noted that any number you start at would fit in one of the sequences and that as you apply the rules you end in the previous sequence stepping through each all the way back to the first. So if you started in the 7^{th} sequence you end up in the 6^{th}, then the 5^{th}, 4^{th}, 3^{rd}, 2^{nd}, and finally1^{st}. But the 1^{st} may not and usually does not end there and this brings up to another sequence greater than or equal to 1! And that process continues until one reaches the main loop 4-2-1. And this observation is VERY important. No matter what the starting number it will cascade down through the second sequence (1+4x) a number of times on it's way to the first sequence (0 +2x) where it'll make another jump wherever.

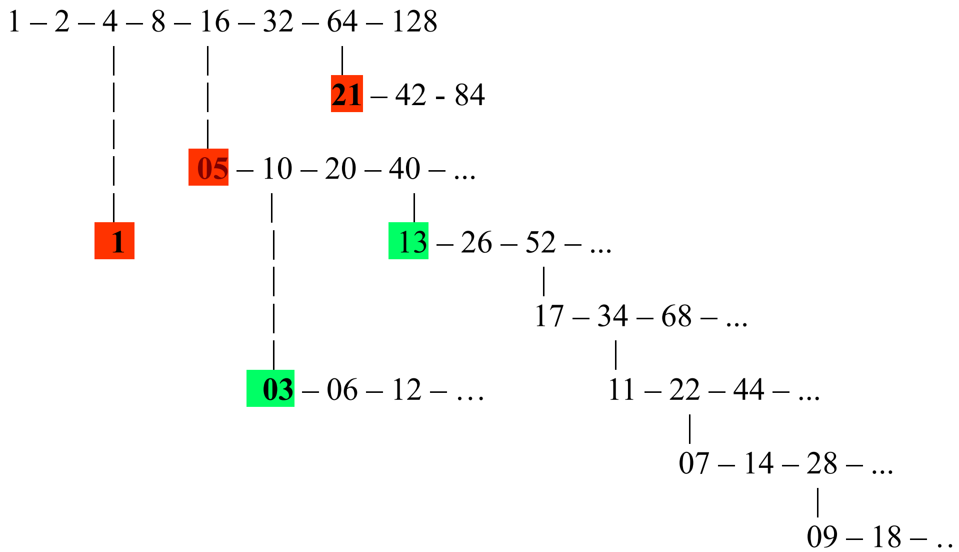
Lets take a closer look at just the even numbers. We know that there's a pattern here too. Check out the following:

2 – 1
4 – 2 – 1
6 – 3
8 – 4 – 2 – 1
10 – 5
12 – 6 – 3
14 – 7
16 – 8 – 4 – 2 – 1
18 – 9
20 – 10 – 5
22 – 11
24 – 12 – 6 – 3
26 – 13
28 – 14 – 7
30 – 15
32 – 16 – 8 – 4 – 2 – 1

34 – 17
36 – 18 – 9
38 – 19
40 – 20 – 10 – 5
42 – 21
44 – 22 – 11
46 – 23
48 – 24 – 12 – 6 – 3
50 – 25
52 – 26 – 13
54 – 27
56 – 28 – 14 – 7
58 – 29
60 – 30 – 15
62 – 31
64 – 32 – 16 – 8 – 4 – 2 – 1
66 – 33
68 – 34 – 17
70 – 35
72 – 36 – 18 – 9
74 – 37
76 – 38 – 19
78 – 39
80 – 40 – 20 – 10 – 5
82 – 41
84 – 42 – 21
86 – 43
88 – 44 – 22 – 11
90 – 45
92 – 46 – 23
94 – 47
96 – 48 – 24 – 12 – 6 – 3
98 – 49
100 – 50 – 25

As clearly seen above only powers of 2 'even' numbers can reduce directly to 1. example 2^1, 2^2; 2^3;... or 2, 4, 8, 16, 32, 64, … Now looking at the remainder of this may be critical if you are playing the stats game. As you can see from the way I have it drawn... half the even numbers are divisble by 2 only once. That's 50% of them. Of the 50% that remain a further 50% of them are divisible by an additional 2. So 25% of the total natural even numbers are divisible by 4 (2*2). And if you take the remaining 25%, half of them are divisble by another 2... 12.5% are divisible by 8 (2*2*2). 6.25% are divisible by 16; and so on and so forth. I do not need any of this even number stuff for my proof though.

Because of the way the the Collatz Tree forms I noted that the starting odds on the successive limbs of any backbone branch are formed by applying (odd*4)+1 to each upper limb. Example 1, 5, 21, … and another 3, 13, 53,...



So this leads to the obvious next step that I overlooked in my original report and that is that any odd number where you can substract 1 and have it evenly divisible by 4 is automatically collapsable to the 4-2-1 loop. For example; 21 – 1 = 20 /4 =5. Note that you may be able to continue subtracting another 1 and still have it divisible by a further 4. But this is not the norm. So if we know 1 to x (assumed) are true, then x+1 being an odd number where x+1 subtract 1 is evenly divisible by 4 is also true. So the 25% of natural numbers in the 1+4x series are all true as well. Like the even numbers; if we know 1 to x (are assumed to be true) then x+1 as long as it is even is also true because (x+1)/2 is in the set we already assumed true...that's 1 to x.

So, we can easily show that all even numbers can reduced to the main 4-2-1 loop knowing that if you have already proven 1 to x; then x+1 if it happens to be even has the rule x/2 applied and the result is a proven x! We can now bring the above discussion (for odd numbers) about what happens if you can subtract 1 and have it divisible by 4...and that this will result in a number that falls in the 1 to x already proven. And this is good because the original sequences I used to create the counting number sets has a special feature. The second sequence 1+4x has all of it's elements being evenly divisible by 4 after subtracting 1. For example ((1+4x)-1)/4 =x That is the set 1, 5, 9, 13, 17, 21, … None of the other sequences will ever have an element that can do this. So the fact that we cascade through all sequences on the way down to the first sequence means we will go through the 1+4x sequence...and all elements in that set will automatically bring one to a number that is in the proven 1 to x! But this is only true if you start in 1+4x sequence. If you cascade from a higher level through 1+4x you are by no means proven. In some cases you may have a number that is smaller than the starting number and in the assume 1 to x true set, but this is not the norm.

Now, any odd number that falls in (is a member of) 1+4x sequence means that it starts proven. So we have been able to prove all even numbers (50%) & all odd numbers where x-1 is evenly divisible by 4 (25%) are Proven. That's 75% total.

If we take the third sequence 3+8x we can show that when it cascades into 1+4x it is close enough that it will be automatically proven.

$(3(3+8x)+1)/2$
$(10+24x)/2$
$(4+6+24x)/2$
$(4+6(1+4x))/2$
$2+3(1+4x)$ now see if it is evenly divible by 4 after subtracting 1...
$(2+3(1+4x)-1)/4$
$(1+3(1+4x))/4$
$(4+12x)/4$
$1+3x$.

So any odd number that falls in 3+8x sequence will automatically be smaller or in the 1 to x assumed. 1+3x is smaller than the original 3+8x.

So as seen above any number that falls in 3+8x sequence (level 3) will cascade directly to level 2 (1+4x) where it automatically becomes true! The resulting number is smaller than the starting odd and in the 1 to x assumed. So that's an additional 12.5% which gives us a 87.5% of natural numbers proven.

Let's try doing the same thing to the next two sequences to see if they are close enough as well. That's the 7+16x and 15+32x. I'm going to try 31+64x as well because I know that's where it begins to fail. The math shows they both are... however 31+64x is not! Nor are any above that.

$(3(7+16x)+1)/2$
$(22+48x)/2$
$11+24x$
$(3(11+24x)+1)/2$
$(34+72x)/2$
$17+36x$
$(17+36x-1)/4$
$(16+36x)/4$
$4+9x$
$4+9x<7+16x!$

$(3(15+32x)+1)/2$
$(46+96x)/2$
$23+48x$
$(3(23+48x)+1)/2$
$(70+144x)/2$
$35+72x$
$(3(35+72x)+1)/2$
$(106+216x)/2$
$53+108x$
$(53+108x-1)/4$
$(52+108x)/4$
$13+27x$
$13+27x<15+32x!$

$(3(31+64x)+1)/2$
$(94+192x)/2$
$47+96x$
$(3(47+96x)+1)/2$
$(142+288x)/2$
$71+144x$
$(3(71+144x)+1)/2$
$(214+432x)/2$
$107+216x$
$(3(107+216x)+1)/2$
$(322+648x)/2$
$161+324x$
$(161+324x-1)/4$
$(160+324x)/4$
$40+81x$
$40+81x>31+64x!$

I'm going to apply a twist to all levels greater than the third (3+8x). Let's go in the opposite direction. First let's look at something special that occurs with a number of the upper level sequences...

Level 1 (0+2x) starts with 2 (even numbers)
Level 2 (1+4x) starts with 1
Level 3 (3+8x) starts with 3 (multiple of 3!)
Level 4 (7+16x) starts with 7
Level 5 (15+32x) starts with 15 (multiple of 3!)

Level 6 (31+64x) starts with 31
Level 6 (63+128x) starts with 63 (multiple of 3!)
Level 8 (127+256x) starts with 127

Level 9 (255+512x) starts with 255 (multiple of 3!)
Level 10 (511+1024x) starts with 511

Any backbone row starting with an odd number that is divisible by 3 (multiple of 3) can not spawn new backbones. That exactly half of the remaining levels which is clearly the case as seen above. These levels will be skipped over until I find a way to handle even multiples of 3.

Let's start with an odd number from the sequence 7+16x...say 23! Now let's multiply it by 2 and see if the result subtract 1 is evenly divisible by 3. If it is, the number is proven because it falls in the 1 to x assumed proven and is smaller than the original.

So the sequence starting with 7 has an even division of members into three groups; one where after you multiply by 2 you can subtract 1 and have it evenly divisble by 3; one where you must multiply by 4 then subtract 1 and it will be evenly divisible by 3 (but the resulting number is not smaller than the starting! It is however evenly divisible by 4 after subtracting 1! This then makes it smaller than the starting.); and a final group that is evenly divisible by 3 (a multiple of 3 – dead end backbone) which I can not handle at this time. So each group is exacly 1/3 (33%). I can prove 2 of these subgroups meaning 66% are provable.

7 – 14 – 28	(28 – 1 = 27 / 3 = 9) Remember special I pointed out 9 – 1 = 8 / 4 = 2!
23 – 46	(46 – 1 = 45 / 3 = 15!)
39 – 78 – 156	(Multiple of 3; I can't do anything with this yet)
55 – 110 – 220	(220 – 1 =219 / 3 = 73) 73 – 1 = 72 / 4 = 18!
71 – 142	(142 – 1 = 141 / 3 = 47!)
87 – 174 – 348	(Multiple of 3!)

The other levels that are not autmatically multiples of 3 do the exact same thing. They behave in the same fashion. Let's look at the level starting with 127.

127 – 254 – 508	(508 – 1 / 3 = 169) 169 – 1 / 4 = 42!
383 – 766	(766 – 1 / 3 = 255!
639 – 1278 – 2556	(Multiple of 3)
895 – 1790 – 3580	(3580 – 1 / 3 = 1139) 1139 – 1 / 4 = 298!
1151 – 2302	(2302 – 1 / 3 = 767!)
1407 – 2814 – 5628	(Multiple of 3)

So like I mentioned above 7+16x and 15+32x can be proven in the same fashion as 3+8x because they are within a distance that will allow for it. I do however use both those sequences above to show what happens in all upper levels and how three distinct groupings/sets become possible. The numbers are smaller to deal with to show this point. Looking at sequence 127+256x you can see how quickly the numbers grow.

So as stated above we've shown that 66% of the members in every other upper level sequences (

half of them because the other half are multiples of 3) by simply applying the rules as shown above; one third are simply multiplied by 2 then divisible by by 3 after subtracting 1; another third by multiplying by 4 then divisble by 3 after subtracting 1...but can be further reduced by subtracting 1 and have it divisible by 4; the remaining third are multiples of 3 and no proof yet.

Doing the math we have 12.5% remaining to cover off the upper levels but remember that as we go up levels the members included are halved. So the levels have the following associated percentages:

Level 1 – 50% (100% provable)
Level 2 – 25% (100% provable)
Level 3 – 12.5% (100% provable)
Level 4 – 6.25% (100% provable)
Level 5 – 3.125% (100% provable)
Level 6 – 1.5625% (66% provable = 1.0417%)
Level 7 – 0.78125% (Multiple of 3 – Excluded)
Level 8 – 0.390625% (66% provable = 0.2604167%)
and so on ...

So continuing on with the math we can prove 50% + 25% + 12.5% + 6.25%+ 3.125% + 1.0417% + 0.2604167% giving a grand total of 98.177% . So I am able to prove slightly more than 98% of all the natural counting numbers set are provable.

My quandry now is that I can not fashion a method to handle those multiples of 3 instances (the remainder and only case yet to be proven) which account for only 2%. Wow, that's close. I wonder if anyone else has come this close?

I believe this is the better part of what is required for a proof! Now to put it in a proofy format.

Section 13 - Conclusion

With all the above discussion I have concluded that the original conjecture holds true for all positive counting number; 98% of them proven. This leaves a subset that containtain nothing but multiples of 3 starting deadend backbones). That multiple of 3 must be important somehow? These backbones can not spawn any further limbs.

I am not a mathematician so my technical terminology leaves a lot to be desired. But I hope I have succesfully made my case.

Maybe someone can formalize this into an actual presentable proof that the community as a whole will accept.

It has been a joy working on this 'unsolvable' problem.