# Unity formulas for the coupling constants and the dimensionless physical constants

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**Abstract**

In this paper will be presented the unity formulas for the coupling constants and the dimensionless physical constants. The theoretical value of the strong coupling constant $\alpha_s$=Euler's number/Gelfond's constant is the key that solves many problems of Physics. We will present the recommended theoretical value for the weak coupling constant. It will be presented the formula for the fine-structure constant with the golden angle,relativity factor and the fifth power of the golden mean and the simple expression for the fine-structure constant in terms of the Archimedes constant. The exact mathematical expression for the proton to electron mass ratio using Fibonacci and Lucas numbers and other two exact mathematical expressions for the proton to electron mass ratio. New formulas for the Planck length and the Avogadro's number. The unity formulas that connect the fine-structure constant and the proton to electron mass ratio. We will find the formulas that connect the strong coupling constant and the fine-structure constant. The unity formulas that connect the strong coupling constant,the weak coupling constant and the fine-structure constant. It will be presented the mathematical formulas that connects the strong coupling constant,the weak coupling constant,the proton to electron mass ratio,the fine-structure constant,the ratio of electric force to gravitational force between electron and proton,the Avogadro's number,the gravitational coupling constant for the electron and the gravitational coupling constant of proton. Also we will find the formulas for the Gravitational constant. It will be presented that the gravitational fine structure constant is a simple analogy between atomic physics and cosmology. Finally we will find the expression that connects the gravitational fine structure constant with the four coupling constants. Perhaps the gravitational fine structure constant is the coupling constant for the fifth force. In this work we will assume the theoretical value of the strong coupling constant. This value fits perfectly in the measurement of the strong coupling constant. Also we followed the energy wave theory and the fractal space-time theory.

**Keywords**

Fine-structure constant, Proton to electron mass ratio , Dimensionless physical constants , Coupling constant , Gravitational constant , Avogadro's number , Gravitational constant

## 1. Introduction

In physics,the fundamental interactions,also known as fundamental forces,are the interactions that do not appear to be reducible to more basic interactions. There are four fundamental interactions known to exist:the gravitational and electromagnetic interactions,which produce significant long-range forces whose effects can be seen directly in everyday life,and the strong and weak interactions,which produce forces at minuscule, subatomic distances and govern nuclear interactions. Some scientists hypothesize that a fifth force might exist,but these hypotheses remain speculative. Each of the known fundamental interactions can be described mathematically as a field. The gravitational force is attributed to the curvature of spacetime,described by Einstein's general theory of relativity. The other three are discrete quantum fields,and their interactions are mediated by elementary particles described by the Standard Model of particle physics. A coupling constant is a parameter in field theory,which determines the relative strength of interaction between particles or fields. In the quantum field theory the coupling constants are associated with the vertices of the corresponding Feynman diagrams. Dimensionless parameters are used as coupling constants,as well as the quantities associated with them that characterize the interaction and have dimensions.

Some speculative theories have proposed a fifth force to explain various anomalous observations that do not fit existing theories. The characteristics of this fifth force depend on the hypothesis being advanced. Many postulate a force roughly the strength of gravity with a range of anywhere from less than a millimeter to cosmological scales. Another proposal is a new weak force mediated by W and Z bosons. The search for a fifth force has increased in recent decades due to two discoveries in cosmology which are not explained by current theories. It has been discovered that most of the mass of the universe is accounted for by an unknown form of matter called dark matter. Most physicists believe that dark matter consists of new,undiscovered subatomic particles,but some believe that it could be related to an unknown fundamental force. Second,it has also recently been discovered that the expansion of the universe is accelerating, which has been attributed to a form of energy called dark energy. Some physicists speculate that a form of dark energy called quintessence could be a fifth force.

Euler's number e is an important mathematical constant,which is the base of the natural logarithm. All five of these numbers play important and repetitive roles in mathematics and these five constants appear in a formulation of Euler's identity. Euler's number has many practical uses,especially in higher level mathematics such as calculus,differential equations,trigonometry,complex analysis,statistics,etc. From Euler's identity the following relation of the mathematical constant e can emerge $e=i^{-2i/\pi}$. Euler's number frequently appears in problems related to growth or decay,where the rate of change is determined by the present value of the number being measured. One example is in biology,where bacterial populations are expected to double at reliable intervals. Another case is radiometric dating,where the number of radioactive atoms is expected to decline over the fixed half-life of the element being measured. Gelfond's constant,in mathematics,is the number $e^{\pi}$,e raised to the power π. Like e and π,this constant is a transcendental number. It was named after the Soviet mathematician Aleksandr Gelfond. Gelfond's constant were singled out in Hilbert's 7th problem as an example of numbers whose excess was an open problem. This was first established by Gelfond and may now be considered as an application of the Gelfond–Schneider theorem,noting that $e^{\pi}=(e^{i\pi})^{-i}=(-1)^{-i}=i^{-2i}$.

**2. Strong coupling constant**

In nuclear physics and particle physics,the strong interaction is one of the four known fundamental interactions,with the others being electromagnetism,the weak interaction,and gravitation. Strong force involves the exchange of huge particles and therefore has a very small range. It is clear that strong force is much stronger simply than the fact that the nuclear magnitude (dominant strong force) is about $10^{-15}$ m while the atom (dominant electromagnetic force) has a size of about $10^{-10}$ m. At the range of $10^{-15}$ m,the strong force is approximately 137 times as strong as electromagnetism,$10^{6}$ times as strong as the weak interaction,and $10^{38}$ times as strong as gravitation. The strong coupling constant $\alpha_s$ is one of the fundamental parameters of the typical model of particle physics.

The strong coupling constant $\alpha s$ is one of the fundamental parameters of the typical model of particle physics. The strong nuclear force confines quarks into hadron particles such as the proton and neutron. In addition,the strong force binds these neutrons and protons to create atomic nuclei,where it is called the nuclear force. Most of the mass of a common proton or neutron is the result of the strong force field energy;the individual quarks provide only about 1% of the mass of a proton. The electromagnetic force is infinite in range and obeys the inverse square law,while the strong force involves the exchange of massive particles and it therefore has a very short range.

The last measurement in [17] on 23 November 2.021 of European organization for nuclear research (CERN) is used in a comprehensive QCD analysis at next-to next-to-leading order,which results in significant improvement in the accuracy of the parton distributions in the proton. Simultaneously,the value of the strong coupling constant at the Z boson mass is extracted as:

$$\alpha_{s(mZ)}=0{,}1170\pm0{,}0019$$

For the first time,these data are used in a standard model effective field theory analysis at next to-leading order,where parton distributions and the QCD parameters are extracted simultaneously with imposed constraints on the Wilson coefficient $c_1$ of 4-quark contact interactions. Interaction phenomena in field theory are often defined using perturbation theory,in which the functions in the equations are extended to forces of constant interaction. Usually,for all interactions except the strong one,the coupling constant is much smaller than the unit. This makes the application of perturbation theory effective,as the contribution from the main terms of the extensions decreases rapidly and their calculation becomes redundant. In the case of strong interactions,perturbation theory becomes useless and other calculation methods are required. One of the predictions of quantum field theory is the so-called "floating constants" phenomenon,according to which interaction constants change slowly with the increase of energy transferred during the interaction of particles. Thus,the constant of the electromagnetic interaction increases,and the constant of the strong interaction-decreases with increasing energy. For quarks in quantum chromodynamics,a strong interaction constant is introduced:

$$\alpha_s=g_{qg}^2/4\cdot\pi\cdot\hbar\cdot c=g_{qg}^2\cdot\varepsilon_0\cdot\alpha/q_e^2\alpha_s=\varepsilon_0\cdot g_{qg}^2/q_{p\ell}^2$$

where $g_{qg}$ is the active color charge of a quark that emits virtual gluons to interact with another quark. By reducing the distance between the quarks,which is achieved in high-energy particle collisions,a logarithmic reduction of $\alpha_s$ and a weakening of the strong interaction (the effect of the asymptotic freedom of the quarks) is expected. We will assume the recommended value for the strong coupling constant $\alpha_s$:

$$\alpha_s=\text{Euler's number}/\text{Gelfond's constant}$$

$$\alpha_s=e/e^{\pi} \tag{1}$$

with numerical value:

$$\alpha_s\approx0{,}1174676$$

Also the equivalent expressions for the value of the strong coupling constant are:

$$\alpha_s=e^{1-\pi}=e\cdot e^{-\pi}=e\cdot i^{2i}=i^{-2i/\pi}\cdot i^{2i}=i^{2i-(2i/\pi)}=i^{2i(\pi-1)/\pi} \tag{2}$$

Although it remains a possibility that it is a coincidence this value fits perfectly in the measurement of the strong coupling constant.

### 3. Weak coupling constant

In nuclear physics and particle physics,the weak interaction,which is also often called the weak force or weak nuclear force,is one of the four known fundamental interactions,with the others being electromagnetism,the strong interaction,and gravitation. It is the mechanism of interaction between subatomic particles that is responsible for the radioactive decay of atoms: The weak interaction participates in nuclear fission and nuclear fusion. The theory describing its behavior and effects is sometimes called quantum flavordynamics (QFD),however,the term QFD is rarely used,because the weak force is better understood by electroweak theory (EWT). The effective range of the weak force is limited to subatomic distances,and is less than the diameter of a proton.

The weak interaction has such an incredibly short range that it's strength must be evaluated in a different way than the electromagnetic force. The fact that both the strong force and the weak force initiate decays of particles gives a way to compare their strength. The lifetime of a particle is proportional to the inverse square of the coupling constant of the force which causes the decay. From the example of the decays of the delta and sigma baryons,the weak coupling constant can be related to the strong force coupling constant.

The strong interaction and weak interaction can be compared in a set of particle decays which yield the same final products. The Delta baryons (or Δ baryons, also called Delta resonances) are a family of subatomic particles made of three up or down quarks (u or d quarks). Four closely related Δ baryons exist: $\Delta^{++}$ (constituent quarks: uuu),$\Delta^{+}$ (uud),$\Delta^{0}$ (udd),and $\Delta^{-}$ (ddd), which respectively carry an electric charge of +2 e,+1 e,0 e,and -1 e. The Δ baryons have a mass of about 1232 MeV/$c^2$,a spin of 3/2,and an isospin of 3/2. Ordinary protons and neutrons,by contrast,have a mass of about 939 MeV/$c^2$,a spin of 1/2,and an isospin of 1/2. The $\Delta^{+}$ (uud) and $\Delta^{0}$ (udd) particles are higher-mass excitations of the proton ($N^{+}$,uud) and neutron ($N^{0}$,udd),respectively. However,the $\Delta^{++}$ and $\Delta^{-}$ have no direct nucleon analogues. The decays of the delta baryons:

$$\Delta^{+} \rightarrow p + \pi^{0}$$

The lifetime of the delta baryons is:

$$\tau_{\Delta}\approx 6\times 10^{-24}\ \text{s}$$

The sigma baryons are a family of subatomic hadron particles which have two quarks from the first flavor generation (up or down quarks),and a third quark from a higher flavor generation,in a combination where the wavefunction sign remains constant when any two quark flavors are swapped. They are thus baryons,with total isospin of 1,and can either be neutral or have an elementary charge of +2,+1,0,or -1. They are closely related to the Lambda baryons,which differ only in the wavefunction's behavior upon flavor exchange. The decays of the sigma baryons:

$$\Sigma^{+} \rightarrow p + \pi^{0}$$

The lifetime of the delta baryons is:

$$\tau_{\Sigma}\approx 8\times 10^{-11}\ \text{s}$$

The extraordinary difference of 13 orders of magnitude in the lifetimes comes from the fact that the sigma decay does not conserve strangeness and therefore can proceed only by the weak interaction. The lifetime of a decay is proportional to the inverse square of the coupling constant between the initial and final products,and since the final products are identical, the difference in lifetime must come from the difference in coupling constants. The coupling constant ratio [20] can then be estimated for this situation:

$$\alpha_w/\alpha_s=(\tau_{\Delta}/\tau_{\Sigma})^{1/2}=e\cdot 10^{-7}$$

$$\alpha_w/\alpha_s=e\cdot 10^{-7} \tag{3}$$

From (1) and (3) we can result the value of the weak coupling constant $\alpha_w$:

$$\alpha_w=e\cdot\alpha_s\cdot 10^{-7}$$

$$\alpha_w=(e^2/e^{\pi})\cdot 10^{-7}=e^{2-\pi}\cdot 10^{-7} \quad (4)$$

with numerical value:

$$\alpha_w \approx 3{,}1931\cdot 10^{-6}$$

**4. Fine-structure constant**

One of the most important numbers in physics is the fine-structure constant α which defines the strength of the electro-magnetic field. It is a dimensionless number independent of how we define our units of mass,length,time or electric charge. A change in these units of measurement leaves the dimensionless constant unchanged. The number can be seen as the chance that an electron emits or absorbs a photon. It's a pure number that shapes the universe to an astonishing degree. Paul Dirac considered the origin of the number «the most fundamental unsolved problem of physics». The constant is everywhere because it characterizes the strength of the electromagnetic force affecting charged particles such as electrons and protons. Many eminent physicists and philosophers of science have pondered why α itself has the value that it does,because the value shows up in so many important scenarios and aspects of physics. Nobody has come up with any ideas that are even remotely convincing. The 2.018 CODATA recommended value of α is:

$$\alpha=0.0072973525693(11)$$

With standard uncertainty $0{,}0000000011\times 10^{-3}$ and relative standard uncertainty $1{,}5\times 10^{10}$. We propose in [9] the exact formula for the fine-structure constant α with the golden angle,relativity factor and the fifth power of the golden mean:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5} \quad (5)$$

with numerical value:

$$\alpha^{-1}=137{,}035999164$$

Also we propose in [10] a simple and accurate expression for the fine-structure constant α in terms of the Archimedes constant π:

$$\alpha^{-1}=2\cdot 3\cdot 11\cdot 41\cdot 43^{-1}\cdot \ln 2\cdot\pi \quad (6)$$

with numerical value:

$$\alpha^{-1}=137{,}035999078$$

**5. Gravitational coupling constant**

In physics,the gravitational coupling constant $\alpha_G$ is a constant that characterizes the gravitational pull between a given pair of elementary particles. For the electron pair this constant is denoted by $\alpha_G$. The choice of units of measurement,but only with the choice of particles. The gravitational coupling constant $\alpha_G$ is a scaling ratio that can be used to compare similar unit values from different scaling systems (Planck scale,atomic scale,and cosmological scale). The gravitational coupling constant can be used for comparison of length,range and force values. The gravitational coupling constant $\alpha_G$ is defined as:

$$\alpha_G=G\cdot m_e^2/\hbar\cdot c=(m_e/m_{p\ell})^2$$

There is so far no known way to measure $\alpha_G$ directly. The value of the constant gravitational coupling $\alpha_G$ is only known in four significant digits. The approximate value of the constant gravitational coupling $\alpha_G$ is:

$$\alpha_G \approx 1{,}7518099\times 10^{-45}$$

The gravitational coupling constant $\alpha_{G(p)}$ for the proton is produced similar to the electron,but replaces the mass of electrons with the mass of the protons. The gravitational coupling constant of the proton $\alpha_{G(p)}$ is defined as:

$$\alpha_{G(p)}=G\cdot m_p^2/\hbar\cdot c=m_p^2/m_{p\ell}^2=\mu^2\cdot\alpha_G$$

The approximate value of the constant gravitational coupling of the proton $\alpha_{G(p)}$ is:

$$\alpha_{G(p)} \approx 5{,}9061512\text{x}10^{-39}$$

## 6. Proton to electron mass ratio

In Physics,the ratio of the mass of a proton to an electron is simply the remainder of the mass of the proton divided by that of the electron,from the system of units. Two of the great mysteries of physics are the origin of mass and the mysterious ratio of mass between a proton and an electron. The values of me and mp,and the equilibrium between them,govern nuclear reactions such as the decay of protons and the nuclear synthesis of stars,leading to the formation of basic biochemical elements,including carbon. The space where stars and planets form and support life and molecular structures can appear. The mass ratio of protons to electrons,two constant particles that make up about 95% of the visible Universe,may be related to the total computational value of the Universe. Thus,as pure numbers they are supposed to be associated with prime numbers, entropy, binary and complexity.

The proton-to-electron mass ratio μ is a ratio of like-dimensioned physical quantities,it is a dimensionless quantity,a function of the dimensionless physical constants,and has numerical value independent of the system of units. Two of the great mysteries of physics are the origin of mass and the mysterious mass ratio between the proton and electron. The numerical challenge of the mass ratio of proton to electron in the field of elementary particle physics began with the discovery of the electron by JJ Thomson in 1.897,and with the identification of the point nature of the proton by E. Rutherford in 1.911. These two particles have electric charges that are identical in size but opposite charges.

The 2.018 CODATA recommended value of the Proton to Electron Mass Ratio μ is:

$$\mu=m_p/m_e=1.836,15267343$$

With standard Uncertainty 0,00000011 and relative standard uncertainty $6,0\times10^{-11}$. We propose in [11] the exact equivalent mathematical expression for the proton to electron mass ratio using Fibonacci and Lucas numbers:

$$\mu^{32}=\varphi^{-42}\cdot F_5^{160}\cdot L_5^{47}\cdot L_{19}^{40/19} \tag{7}$$

with numerical value:

$$\mu=1.836,15267343$$

Also we propose in [11] the exact mathematical expressions for the proton to electron mass ratio:

$$\mu^3=7^{-1}\cdot(5\cdot13)^3\cdot[\ln(2\cdot5)]^{11} \tag{8}$$

with numerical value:

$$\mu=1836,152673929$$

Also other exact mathematical expression in [11] for the proton to electron mass ratio is:

$$\mu=6\cdot\pi^5+\pi^{-3}+2\cdot\pi^{-6}+2\cdot\pi^{-8}+2\cdot\pi^{-10}+2\cdot\pi^{-13}+\pi^{-15} \tag{9}$$

with numerical value:

$$\mu=1.836,15267343$$

## 7. The ratio $N_1$ of electric force to gravitational force between electron and proton

The enormous value of the ratio of electric force to gravitational force was first pointed out by Bergen Davis in 1.904. But Weyl and Eddington suggested that the number was about $10^{40}$ and was related to cosmological quantities. The ratio $N_1$ of electric force to gravitational force between electron and proton is defined as:

$$N_1=\alpha/\mu\cdot\alpha_G=\alpha\cdot\mu/\alpha_{G(p)}=\alpha/(\alpha_G\cdot\alpha_{G(p)})^{1/2}=k_e\cdot q_e^2/G\cdot m_e\cdot m_p=\alpha\cdot\hbar\cdot c/G\cdot m_p\cdot m_e\approx2,26866072\times10^{39}$$

The approximate value of the ratio $N_1$ of electric force to gravitational force between electron and proton is:

$$N_1\approx(5/3)\cdot2^{130}=2,26854911\times10^{39}$$

According to current theories $N_1$ should be constant. The ratio $N_2$ of electric force to gravitational force between two electrons is defined as:

$$N_2=\mu \cdot N_1=\alpha/\alpha_G=N_1^2\cdot\alpha_{G(p)}/\alpha_G=N_1^2\cdot\cdot\alpha_{G(p)}/\alpha=k_e\cdot q_e^2/G\cdot m_e^2=\alpha\cdot\hbar\cdot c/G\cdot m_e^2\approx 4{,}16560745\text{x}10^{42}$$

According to current theories $N_2$ should grow with the expansion of the universe.

**8. Avogadro's number $N_A$**

Avogadro's number $N_A$ is defined as the number of carbon-12 atoms in twelve grams of elemental carbon-12 in its standard state. Avogadro's number $N_A$ is the fundamental physical constant that links the macroscopic physical world of objects that we can see and feel with the submicroscopic,invisible world of atoms.The name honors the Italian mathematical physicist Amedeo Avogadro,who proposed that equal volumes of all gasses at the same temperature and pressure contain the same number of molecules. The most accurate definition of the Avogadro's number $N_A$ value involves the change in molecular quantities and,in particular,the change in the value of an elementary charge. The exact value of the Avogadro's number $N_A$ is:

$$N_A=6{,}02214076\times10^{23}$$

The value of the Avogadro's number $N_A$ can also be written in numerical expressions:

$$N_A=84.446.885^3=6{,}02214076\times10^{23}$$

$$N_A\approx45\cdot39^{-1}\cdot\pi\cdot\ln2\cdot\alpha^{-10}=6{,}02214149\times10^{23}$$

$$N_A\approx2^{79}=6{,}04462909\times10^{23}$$

Jeff Yee in [19],the mole and charge are related by deriving Avogadro's number $N_A$ from three constants the Planck length $\ell_{p\ell}$,the Bohr radius $\alpha_0$ and Euler's number e:

$$N_A=\alpha_0/2\cdot e\cdot\ell_{p\ell}$$

The Bohr radius $\alpha_0$ is defined as:

$$\alpha_0=\hbar/\alpha\cdot m_e\cdot c$$

The reduced Planck constant ħ is defined as:

$$\hbar=\alpha\cdot m_e\cdot\alpha_0\cdot c$$

So from these expressions we have:

$$\hbar^2=\alpha^2\cdot m_e^2\cdot\alpha_0^2\cdot c^2$$

$$(\hbar\cdot G/c^3)=\alpha^2\cdot m_e^2\cdot\alpha_0^2\cdot(G\cdot m_e^2/\hbar\cdot c)$$

$$(\hbar\cdot G/c^2)=\alpha^2\cdot\alpha_0^2\cdot(G\cdot m_e^2/\hbar\cdot c)$$

$$\ell_{p\ell}^2=\alpha^2\cdot\alpha_0^2\cdot\alpha_G$$

Therefore the formula for the Planck length $\ell_{p\ell}$ is:

$$\ell_{p\ell}=\alpha\cdot\alpha_0\cdot\alpha_G^{1/2} \quad (10)$$

Also the formula for the Avogadro's number $N_A$ is:

$$N_A=(2\cdot e\cdot\alpha\cdot\alpha_G^{1/2})^{-1} \quad (11)$$

Therefore the formula that connect the fine-structure constant α,the gravitational coupling constant $\alpha_G$ and the Avogadro's number $N_A$ is:

$$2\cdot e\cdot N_A\cdot\alpha\cdot\alpha_G^{1/2}=1 \quad (12)$$

## 9. Unity formula that connect the fine-structure constant and the proton to electron mass ratio

It was explained in [13] that the $\mu \cdot \alpha^{-1}$ is one of the roots of the following trigonometric equation:

$$2 \cdot 10^2 \cdot \cos(\mu \cdot \alpha^{-1})+13^2=0 \quad (13)$$

The exponential form of this equation is:

$$10^2 \cdot (e^{i\mu/\alpha}+e^{-i\mu/\alpha})+13^2=0 \quad (14)$$

This exponential form can also be written with the beautiful form:

$$10^2 \cdot (e^{i\mu/\alpha}+e^{-i\mu/\alpha})=13^2 \cdot e^{i\pi} \quad (15)$$

Also this unity formula can also be written in the form:

$$10 \cdot (e^{i\mu/\alpha}+e^{-i\mu/\alpha})^{1/2}=13 \cdot i \quad (16)$$

So other beautiful formula that connect the fine-structure constant,the proton to electron mass ratio and the fifth power of the golden mean is:

$$5^2 \cdot (5 \cdot \varphi^{-2}+\varphi^{-5})^2 \cdot (e^{i\mu/\alpha}+e^{-i\mu/\alpha})+(5 \cdot \varphi^2-\varphi^{-5})^2=0 \quad (17)$$

The formula that connect the fine-structure constant,the proton to electron mass ratio and the mathematical constants $\pi,\varphi,e,i$ is:

$$10^2 \cdot (e^{i\mu/\alpha}+e^{-i\mu/\alpha})=(5 \cdot \varphi^2-\varphi^{-5})^2 \cdot e^{i\pi} \quad (18)$$

All these equations are simple,elegant and symmetrical in a great physical meaning. Also in [11] we present the exact mathematical expressions that connect the proton to electron mass ratio $\mu$ and the fine-structure constant $\alpha$:

$$9 \cdot \mu-119 \cdot \alpha^{-1}=5 \cdot (\varphi+42) \quad (19)$$

$$\mu-6 \cdot \alpha^{-1}=360 \cdot \varphi-165 \cdot \pi+345 \cdot e+12 \quad (20)$$

$$\mu-182 \cdot \alpha=141 \cdot \varphi+495 \cdot \pi-66 \cdot e+231 \quad (21)$$

$$\mu-807 \cdot \alpha=1.205 \cdot \pi-518 \cdot \varphi-411 \cdot e \quad (22)$$

$$\mu-15 \cdot \alpha^{-1}=-3 \cdot A+9 \cdot S-11 \cdot K-28 \cdot \pi-23 \cdot \varphi+e-30 \quad (23)$$

$$5 \cdot \mu-69 \cdot \alpha^{-1}=52 \cdot Q_A+46 \cdot \pi-72 \cdot \varphi-46 \cdot \pi-111 \cdot e-27 \quad (24)$$

$$\mu-14 \cdot \alpha^{-1}=10 \cdot Q_A+4 \cdot A-5 \cdot S-K-17 \cdot \varphi-12 \cdot \pi-3 \quad (25)$$

where:
K the polygon circumscribing constant with value K=8,7000366252......
S the silver constant with value $S=2+2 \cdot \cos(2 \cdot \pi/7)=3,246979603717$.........
A the Golden Apex with value $A=e^{\pi}-7 \cdot \pi-1=0,14954405765$........
$Q_A$ the Aristotle's Quintessence with value $Q_A=1,0191134319$.........

## 10. Unity formula that connect the strong coupling constant and the fine-structure constant

Jesús Sánchez in the paper [15] explained that the fine-structure constant is one of the roots of the following trigonometric equation:

$$\cos(\alpha^{-1})=e^{-1} \quad (26)$$

Another expression is the following exponential form equation:

$$e^{i/\alpha}-e^{-1}=-e^{-i/\alpha}+e^{-1} \tag{27}$$

From (1) and (26) resulting the beautiful formula that connects the strong coupling constant $\alpha_s$ and the fine-structure constant $\alpha$:

$$e^{\pi}\cdot\alpha_s\cdot\cos(\alpha^{-1})=1 \tag{28}$$

So the formula for strong coupling constant $\alpha_s$ is:

$$\alpha_s=[e^{\pi}\cdot\cos(\alpha^{-1})]^{-1} \tag{29}$$

From (1) and (27) resulting the beautiful formulas that connects the strong coupling constant $\alpha_s$ and the fine-structure constant $\alpha$:

$$e^{i/\alpha}+e^{-i/\alpha}=2\cdot(e^{\pi}\cdot\alpha_s)^{-1} \tag{30}$$

$$e^{i/\alpha}-(e^{\pi}\cdot\alpha_s)^{-1}=-e^{-i/\alpha}+(e^{\pi}\cdot\alpha_s)^{-1} \tag{31}$$

So the formula for strong coupling constant $\alpha_s$ is:

$$\alpha_s=2\cdot[e^{\pi}\cdot(e^{i/\alpha}+e^{-i/\alpha})]^{-1} \tag{32}$$

**11. Unity formula that connect the weak coupling constant and the Fine-structure constant**

From (4) and (26) resulting the beautiful formula that connects the weak coupling constant $\alpha_w$ and the fine-structure constant $\alpha$:

$$e^{\pi-1}\cdot10^{7}\cdot\alpha_w\cdot\cos(\alpha^{-1})=1 \tag{33}$$

So the formula the weak coupling constant $\alpha_w$ is:

$$\alpha_w=[e^{\pi-1}\cdot10^{7}\cdot\cos(\alpha^{-1})]^{-1} \tag{34}$$

From (4) and (27) resulting the beautiful formulas that connects weak coupling constant $\alpha_w$ and the fine-structure constant $\alpha$:

$$e^{i/\alpha}+e^{-i/\alpha}=2\cdot(e^{\pi-1}\cdot10^{7}\cdot\alpha_w)^{-1} \tag{35}$$

$$e^{i/\alpha}-(e^{\pi-1}\cdot10^{7}\cdot\alpha_w)^{-1}=-e^{-i/\alpha}+(e^{\pi-1}\cdot10^{7}\cdot\alpha_w)^{-1} \tag{36}$$

So the formula for the weak coupling constant $\alpha_w$ is:

$$\alpha_w=2\cdot[e^{\pi-1}\cdot10^{7}\cdot(e^{i/\alpha}+e^{-i/\alpha})]^{-1} \tag{37}$$

**12. Unity formula that connect the strong coupling constant,the weak coupling constant and the fine-structure constant**

We will use the expressions (3) and (26) to find the expression that connects the strong coupling constant $\alpha_s$,the weak coupling constant $\alpha_w$ and the fine-structure constant $\alpha$:

$$\alpha_w/\alpha_s=e\cdot10^{-7}$$

$$e^{-1}=\alpha_s/\alpha_w\cdot10^{7}$$

From this expression we have:

$$\cos(\alpha^{-1})=e^{-1}$$

$$\cos(\alpha^{-1})=\alpha_s/\alpha_w\cdot 10^7$$

So the beautiful unity formula that connect the strong coupling constant $\alpha_s$,the weak coupling constant $\alpha_w$ and the fine-structure constant $\alpha$ is:

$$10^7\cdot\alpha_w\cdot\cos(\alpha^{-1})=\alpha_s \quad (38)$$

**13. Unity formula that connect the strong coupling constant,weak coupling constant,the fine-structure constant and the gravitational coupling constant**

Now we will find the equation that connect the four coupling constants. From the expression (3) we have:

$$\alpha_w/\alpha_s=e\cdot 10^{-7}$$

$$e=10^7\cdot\alpha_w/\alpha_s$$

From the expression(12) we have:

$$2\cdot e\cdot\alpha\cdot N_A\cdot\alpha_G^{1/2}=1$$

$$2\cdot(10^7\cdot\alpha_w/\alpha_s)\cdot\alpha\cdot N_A\cdot\alpha_G^{1/2}=1$$

So the beautiful unity formula that connect the strong coupling constant $\alpha_s$,weak coupling constant $\alpha_w$,the fine-structure constant $\alpha$ and the gravitational coupling constant $\alpha_G$ is:

$$\alpha_s\cdot/\alpha_w\cdot\alpha\cdot\alpha_G^{1/2}=2\cdot 10^7\cdot N_A$$

$$\alpha_w\cdot\alpha\cdot\alpha_G^{1/2}\cdot\alpha_s^{-1}=(2\cdot 10^7\cdot N_A)^{-1} \quad (39)$$

Sometimes the gravitational coupling constant for the proton $\alpha_{G(p)}$ is used instead of the gravitational coupling constant $\alpha_G$ for the electron:

$$\alpha_{G(p)}=\mu^2\cdot\alpha_G$$

$$\alpha_G^{1/2}=\alpha_{G(p)}^{1/2}\cdot\mu^{-1}$$

So the beautiful unity formula that connect the strong coupling constant $\alpha_s$,weak coupling constant $\alpha_w$,the fine-structure constant $\alpha$ and the gravitational coupling constant $\alpha_{G(p)}$ for the proton is:

$$\alpha_s\cdot\mu\cdot(\alpha_w\cdot\alpha\cdot\alpha_{G(p)}^{1/2})^{-1}=2\cdot 10^7\cdot N_A$$

$$\alpha_w\cdot\alpha\cdot\alpha_{G(p)}^{1/2}\cdot\alpha_s^{-1}=(2\cdot 10^7\cdot N_A)^{-1}\cdot\mu \quad (40)$$

**14. Mathematical formulas that connects dimensionless physical constants**

In physics,a dimensionless physical constant is a physical constant that is dimensionless,a pure number having no units attached and having a numerical value that is independent of whatever system of units may be used. The term fundamental physical constant is used to refer to some universal dimensionless constants. A long-sought goal of theoretical physics is to find first principles from which all of the fundamental dimensionless constants can be calculated and compared to the measured values.

It was presented in [12] the mathematical formulas that connects the proton to electron mass ratio $\mu$,the fine-structure constant $\alpha$,the ratio $N_1$ of electric force to gravitational force between electron and proton,the Avogadro's number $N_A$,the gravitational coupling constant $\alpha_G$ for the electron and the gravitational coupling constant of proton $\alpha_{G(p)}$:

$$\alpha_{G(p)}=\mu^2\cdot\alpha_G \quad (41)$$

$$\alpha=\mu\cdot N_1\cdot\alpha_G \quad (42)$$

$$\alpha\cdot\mu=N_1\cdot\alpha_{G(p)} \quad (43)$$

$$\alpha^2 = N_1^2 \cdot \alpha_G \cdot \alpha_{G(p)} \tag{44}$$

$$2 \cdot e \cdot \alpha \cdot N_A \cdot \alpha_G^{1/2} = 1 \tag{45}$$

$$\mu \cdot N_1 = 4 \cdot e^2 \cdot \alpha^3 \cdot N_A^2 \tag{46}$$

$$4 \cdot e^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = 1 \tag{47}$$

$$\mu^3 = 4 \cdot e^2 \cdot \alpha \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1 \tag{48}$$

$$\mu = 2 \cdot e \cdot \alpha_G^{1/2} \cdot \alpha_{G(p)} \cdot N_A \cdot N_1 \tag{49}$$

$$\mu = 4 \cdot e^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_{G(p)} \cdot N_A^2 \cdot N_1 \tag{50}$$

From the expression (1) resulting the mathematical formulas that connects the strong coupling constant $\alpha_s$,the proton to electron mass ratio μ,the fine-structure constant α,the ratio $N_1$ of electric force to gravitational force between electron and proton,the Avogadro's number $N_A$,the gravitational coupling constant $\alpha_G$ for the electron and the gravitational coupling constant of proton $\alpha_{G(p)}$:

$$2 \cdot e^{\pi} \cdot \alpha_s \cdot \alpha \cdot N_A \cdot \alpha_G^{1/2} = 1 \tag{51}$$

$$\mu \cdot N_1 = 4 \cdot e^{2\pi} \cdot \alpha_s^2 \cdot \alpha^3 \cdot N_A^2 \tag{52}$$

$$4 \cdot e^{2\pi} \cdot \alpha_s^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = 1 \tag{53}$$

$$\mu^3 = 4 \cdot e^{2\pi} \cdot \alpha_s^2 \cdot \alpha \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1 \tag{54}$$

$$\mu = 2 \cdot e^{\pi} \cdot \alpha_s \cdot \alpha_G^{1/2} \cdot \alpha_{G(p)} \cdot N_A \cdot N_1 \tag{55}$$

$$\mu = 4 \cdot e^{2\pi} \cdot \alpha_s^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_{G(p)} \cdot N_A^2 \cdot N_1 \tag{56}$$

From the expression (4) resulting the mathematical formulas that connects the weak coupling constant $\alpha_w$,the proton to electron mass ratio μ,the fine-structure constant α,the ratio $N_1$ of electric force to gravitational force between electron and proton,the Avogadro's number $N_A$,the gravitational coupling constant $\alpha_G$ for the electron and the gravitational coupling constant of proton $\alpha_{G(p)}$:

$$2 \cdot e^{\pi-1} \cdot 10^7 \cdot \alpha_w \cdot \alpha \cdot N_A \cdot \alpha_G^{1/2} = 1 \tag{57}$$

$$\mu \cdot N_1 = (2 \cdot e^{\pi-1} \cdot 10^7)^2 \cdot \alpha_w^2 \cdot \alpha^3 \cdot N_A^2 \tag{58}$$

$$(2 \cdot e^{\pi-1} \cdot 10^7)^2 \cdot \alpha_w^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = 1 \tag{59}$$

$$\mu^3 = (2 \cdot e^{\pi-1} \cdot 10^7)^2 \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1 \tag{60}$$

$$\mu = 2 \cdot e^{\pi-1} \cdot 10^7 \cdot \alpha_w \cdot \alpha_G^{1/2} \cdot \alpha_{G(p)} \cdot N_A \cdot N_1 \tag{61}$$

$$\mu = (2 \cdot e^{\pi-1} \cdot 10^7)^2 \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_{G(p)} \cdot N_A^2 \cdot N_1 \tag{62}$$

From the expression (3) resulting the mathematical formulas that connects the strong coupling constant $\alpha_s$,the weak coupling constant $\alpha_w$,the proton to electron mass ratio μ,the fine-structure constant α,the ratio $N_1$ of electric force to gravitational force between electron and proton,the Avogadro's number $N_A$,the gravitational coupling constant $\alpha_G$ for the electron and the gravitational coupling constant of proton $\alpha_{G(p)}$:

$$\alpha_s = 2 \cdot 10^7 \cdot \alpha_w \cdot \alpha \cdot N_A \cdot \alpha_G^{1/2} \tag{63}$$

$$\mu \cdot N_1 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^3 \cdot N_A^2 \tag{64}$$

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 \tag{65}$$

$$\mu^3 \cdot \alpha_s^2 = 4\cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1 \quad (66)$$

$$\mu \cdot \alpha_s = 2\cdot 10^7 \cdot \alpha_w \cdot \alpha_G^{1/2} \cdot \alpha_{G(p)} \cdot N_A \cdot N_1 \quad (67)$$

$$\mu \cdot \alpha_s^2 = 4\cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_{G(p)} \cdot N_A^2 \cdot N_1 \quad (68)$$

**15. Gravitational constant G**

The gravitational constant G is an empirical physical constant that participates in the calculation of gravitational force between two bodies and is denoted by the letter G. It usually appears in Isaac Newton's law of universal gravitation and Albert Einstein's general theory of relativity. The physicist Sir Isaac Newton in 1.687 published his book "Philosophiae Naturalis Principia Mathematica" where he presented the law of universal gravity to describe and calculate the mutual attraction of particles and huge objects in the universe. In this paper,Isaac Newton concluded that the attraction between two bodies is proportional to the product of their masses and inversely proportional to the square of the distance separating them. However,these must be adjusted by introducing the gravity constant G. The 2.018 CODATA recommended value of gravitational constant G is:

$$G = 6{,}67430\times 10^{-11}\ m^3/kg\cdot s^2$$

With standard uncertainty $0{,}00015\times 10^{-11}\ m^3/kg\cdot s^2$ and relative standard uncertainty $2{,}2\text{x}10^{-5}$. The gravitational coupling constant $\alpha_G$ is defined as:

$$\alpha_G = G\cdot m_e^2/\hbar\cdot c = (m_e/m_{p\ell})^2$$

From (12) the gravitational coupling constant $\alpha_G$ can be written in the form:

$$2\cdot e\cdot N_A\cdot \alpha\cdot \alpha_G^{1/2} = 1$$

$$\alpha_G = (4\cdot e^2\cdot \alpha^2\cdot N_A^2)^{-1}$$

Therefore the formula for the gravitational constant G is:

$$G = \alpha_G\cdot \hbar\cdot c/m_e^2$$

$$G = (4\cdot e^2\cdot \alpha^2\cdot N_A^2)^{-1}\cdot (\hbar\cdot c/m_e^2) \quad (69)$$

From (51) resulting the formula for the gravitational constant G:

$$G = (2\cdot e^{\pi}\cdot \alpha_s\cdot \alpha\cdot N_A)^{-2}\cdot (\hbar\cdot c/m_e^2) \quad (70)$$

From this expression resulting the mathematical formula for the strong coupling constant $\alpha_s$:

$$\alpha_s = (2\cdot e^{\pi}\cdot \alpha\cdot N_A)^{-1}\cdot (\hbar\cdot c/G\cdot m_e^2)^{1/2} \quad (71)$$

From (57) resulting the formula for the gravitational constant G:

$$G = (2\cdot e^{\pi-1}\cdot 10^7\cdot \alpha_w\cdot \alpha\cdot N_A)^{-2}\cdot (\hbar\cdot c/m_e^2)$$

From (63) resulting the formula for the gravitational constant G:

$$G = \alpha_s^2\cdot (2\cdot 10^7\cdot \alpha_w\cdot \alpha\cdot N_A)^{-2}\cdot (\hbar\cdot c/m_e^2) \quad (72)$$

**16. Gravitational fine-structure constant**

The cosmological constant Λ is presumably an enigmatic form of matter or energy that acts in opposition to gravity and is considered by many physicists to be equivalent to dark energy. Nobody really knows what the cosmological constant is exactly,but it is required in cosmological equations in order to reconcile theory with our observations of the universe. One potential explanation for the cosmological constant lies in the realm of modern particle physics. Experiments have verified that empty space is permeated by countless virtual particles constantly popping in and out of existence. This ceaseless action creates what is known as a "vacuum energy,"or a force arising from empty space,inherent in the fabric of space-time that could drive apart the universe.

Laurent Nottale in [16] assumed that the cosmological constant Λ is the sum of a general-relativistic term and of the quantum, scale-varying, gravitational self-energy of virtual pairs. A renormalization group approach is used to describe its scale-dependence. We argue that the large scale value of Λ is reached at the classical electron scale. This reasoning provides with a large-number relation:

$$\alpha\cdot(m_{p\ell}/m_e)=(\ell_{p\ell}\cdot\Lambda^{1/2})^{-1/3} \tag{73}$$

This unity formula is a simple analogy between atomic physics and cosmology. The gravitational fine structure constant $\alpha_g$ is defined as:

$$\alpha_g=\alpha_G^{3/2}/\alpha^3=(\alpha_G/\alpha^2)^{3/2} \tag{74}$$

$$\alpha_g^2\cdot\alpha^6=\alpha_G^3 \tag{75}$$

The gravitational fine structure constant $\alpha_g$ also equals:

$$\alpha_g=\ell_{p\ell}\cdot\Lambda^{1/2}=(G\cdot\hbar\cdot\Lambda/c^3)^{1/2} \tag{76}$$

$$\alpha_g=\ell_{p\ell}^3/r_e^3 \tag{77}$$

$$\alpha_g=(2\cdot e\cdot\alpha^2\cdot N_A)^{-3} \tag{78}$$

From (39) and (74) resulting the unity formula that connect the gravitational fine structure constant $\alpha_g$,the strong coupling constant $\alpha_s$,the weak coupling constant $\alpha_w$,the fine-structure constant α and the gravitational coupling constant $\alpha_G$:

$$\alpha_g=(\alpha_w\cdot\alpha_G^{1/2}/\alpha_s\cdot\alpha)^3\cdot e^{-3}\cdot 10^{21}$$

$$\alpha_g=(10^7\cdot\alpha_w\cdot\alpha_G^{1/2}/e\cdot\alpha_s\cdot\alpha)^3 \tag{79}$$

This expression connects the gravitational fine structure constant $\alpha_g$ with the four coupling constants. Perhaps the gravitational fine structure constant $\alpha_g$ is the coupling constant for the fifth force.

**17. Conclusions**

In this work we found a total of 73 new expressions for the coupling constants and the dimensionless physical constants. We assumed that the theoretical value of the strong coupling constant is $\alpha_s$=Euler's number/Gelfond's constant. Although it remains a possibility of being a coincidence,this value fits perfectly in the measurement of the strong coupling constant. Also we followed the energy wave theory and the fractal space-time theory. The recommended theoretical value for the strong coupling constant $\alpha_s$ is:

$$\alpha_s=\text{Euler's number/Gelfond's constant}=e/e^\pi=e^{1-\pi}$$

The formula that connects the strong coupling constant $\alpha_s$ and the weak coupling constant $\alpha_w$ is:

$$\alpha_w/\alpha_s=e\cdot10^{-7}$$

The recommended theoretical value for the weak coupling constant $\alpha_w$ is:

$$\alpha_w=(e^2/e^\pi)\cdot10^{-7}=e^{2-\pi}\cdot10^{-7}$$

The formula for the Fine-structure constant α with the Golden Angle,Relativity factor and the Fifth Power of the Golden Mean is:

$$\alpha^{-1}=360\cdot\varphi^{-2}-2\cdot\varphi^{-3}+(3\cdot\varphi)^{-5}$$

The simple expression for the fine-structure constant α in terms of the Archimedes constant π is:

$$\alpha^{-1}=2\cdot3\cdot11\cdot41\cdot43^{-1}\cdot\ln2\cdot\pi$$

The exact mathematical expression for the proton to electron mass ratio using Fibonacci and Lucas numbers is:

$$\mu^{32}=\varphi^{-42}\cdot F_5^{160}\cdot L_5^{47}\cdot L_{19}^{40/19}$$

The exact mathematical expressions for the proton to electron mass ratio are:

$$\mu^{3}=7^{-1}\cdot(5\cdot13)^{3}\cdot[\ln(2\cdot5)]^{11}$$

$$\mu=6\cdot\pi^{5}+\pi^{-3}+2\cdot\pi^{-6}+2\cdot\pi^{-8}+2\cdot\pi^{-10}+2\cdot\pi^{-13}+\pi^{-15}$$

The formula for the Planck length $\ell_{p\ell}$ is:

$$\ell_{p\ell}=\alpha\cdot a_0\cdot\alpha_G^{1/2}$$

The formula for the Avogadro's number $N_A$ is:

$$N_A=(2\cdot e\cdot\alpha\cdot\alpha_G^{1/2})^{-1}$$

The unity formulas that connect the fine-structure constant and the proton to electron mass ratio are:

$$2\cdot10^{2}\cdot\cos(\mu\cdot\alpha^{-1})+13^{2}=0$$

$$10^{2}\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})+13^{2}=0$$

$$10^{2}\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})=13^{2}\cdot e^{i\pi}$$

$$10\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})^{1/2}=13\cdot i$$

$$5^{2}\cdot(5\cdot\varphi^{-2}+\varphi^{-5})^{2}\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})+(5\cdot\varphi^{2}-\varphi^{-5})^{2}=0$$

$$10^{2}\cdot(e^{i\mu/\alpha}+e^{-i\mu/\alpha})=(5\cdot\varphi^{2}-\varphi^{-5})^{2}\cdot e^{i\pi}$$

The exact mathematical expressions that connect the proton to electron mass ratio $\mu$ and the fine-structure constant $\alpha$ are:

$$9\cdot\mu-119\cdot\alpha^{-1}=5\cdot(\varphi+42)$$

$$\mu-6\cdot\alpha^{-1}=360\cdot\varphi-165\cdot\pi+345\cdot e+12$$

$$\mu-182\cdot\alpha=141\cdot\varphi+495\cdot\pi-66\cdot e+231$$

$$\mu-807\cdot\alpha=1.205\cdot\pi-518\cdot\varphi-411\cdot e$$

$$\mu-15\cdot\alpha^{-1}=-3\cdot A+9\cdot S-11\cdot K-28\cdot\pi-23\cdot\varphi+e-30$$

$$5\cdot\mu-69\cdot\alpha^{-1}=52\cdot Q_A+46\cdot\pi-72\cdot\varphi-46\cdot\pi-111\cdot e-27$$

$$\mu-14\cdot\alpha^{-1}=10\cdot Q_A+4\cdot A-5\cdot S-K-17\cdot\varphi-12\cdot\pi-3$$

The unity formulas that connect the strong coupling constant $\alpha_s$ and the fine-structure constant $\alpha$ are:

$$e^{\pi}\cdot\alpha_s\cdot\cos(\alpha^{-1})=1$$

$$e^{i/\alpha}+e^{-i/\alpha}=2\cdot(e^{\pi}\cdot\alpha_s)^{-1}$$

$$e^{i/\alpha}-(e^{\pi}\cdot\alpha_s)^{-1}=-e^{-i/\alpha}+(e^{\pi}\cdot\alpha_s)^{-1}$$

The unity formulas that connect the weak coupling constant $\alpha_w$ and the fine-structure constant $\alpha$ are:

$$e^{\pi-1}\cdot 10^7\cdot\alpha_w\cdot\cos(\alpha^{-1})=1$$

$$\alpha_w=[e^{\pi-1}\cdot 10^7\cdot\cos(\alpha^{-1})]^{-1}$$

$$e^{i/\alpha}+e^{-i/\alpha}=2\cdot(e^{\pi-1}\cdot 10^7\cdot\alpha_w)^{-1}$$

$$e^{i/\alpha}-(e^{\pi-1}\cdot 10^7\cdot\alpha_w)^{-1}=-e^{-i/\alpha}+(e^{\pi-1}\cdot 10^7\cdot\alpha_w)^{-1}$$

$$\alpha_w=2\cdot[e^{\pi-1}\cdot 10^7\cdot(e^{i/\alpha}+e^{-i/\alpha})]^{-1}$$

The unity formula that connect the strong coupling constant $\alpha_s$,the weak coupling constant $\alpha_w$ and the fine-structure constant $\alpha$ is:

$$10^7\cdot\alpha_w\cdot\cos(\alpha^{-1})=\alpha_s$$

The unity formula that connect the strong coupling constant $\alpha_s$,weak coupling constant $\alpha_w$,the fine-structure constant $\alpha$ and the gravitational coupling constant $\alpha_G$ is:

$$\alpha_w\cdot\alpha\cdot\alpha_G^{1/2}\cdot\alpha_s^{-1}=(2\cdot 10^7\cdot N_A)^{-1}$$

The unity formula that connect the strong coupling constant $\alpha_s$,weak coupling constant $\alpha_w$,the fine-structure constant $\alpha$ and the gravitational coupling constant $\alpha_{G(p)}$ for the proton is:

$$\alpha_w\cdot\alpha\cdot\alpha_{G(p)}^{1/2}\cdot\alpha_s^{-1}=(2\cdot 10^7\cdot N_A)^{-1}\cdot\mu$$

The mathematical formulas that connects the $\mu$,$\alpha$,$N_1$,$N_A$,$\alpha_G$ and $\alpha_{G(p)}$ are:

$$\alpha_{G(p)}=\mu^2\cdot\alpha_G$$

$$\alpha=\mu\cdot N_1\cdot\alpha_G$$

$$\alpha\cdot\mu=N_1\cdot\alpha_{G(p)}$$

$$\alpha^2=N_1^2\cdot\alpha_G\cdot\alpha_{G(p)}$$

$$2\cdot e\cdot\alpha\cdot N_A\cdot\alpha_G^{1/2}=1$$

$$\mu\cdot N_1=4\cdot e^2\cdot\alpha^3\cdot N_A^2$$

$$4\cdot e^2\cdot\alpha\cdot\mu\cdot\alpha_G^2\cdot N_A^2\cdot N_1=1$$

$$\mu^3=4\cdot e^2\cdot\alpha\cdot\alpha_{G(p)}^2\cdot N_A^2\cdot N_1$$

$$\mu=2\cdot e\cdot\alpha_G^{1/2}\cdot\alpha_{G(p)}\cdot N_A\cdot N_1$$

$$\mu=4\cdot e^2\cdot\alpha\cdot\alpha_G\cdot\alpha_{G(p)}\cdot N_A^2\cdot N_1$$

The mathematical formulas that connects the $\alpha_s$,$\mu$,$\alpha$,$N_1$,$N_A$,$\alpha_G$ and $\alpha_{G(p)}$ are:

$$2\cdot e^{\pi}\cdot\alpha_s\cdot\alpha\cdot N_A\cdot\alpha_G^{1/2}=1$$

$$\mu\cdot N_1=4\cdot e^{2\pi}\cdot\alpha_s^2\cdot\alpha^3\cdot N_A^2$$

$$4 \cdot e^{2\pi} \cdot \alpha_s^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = 1$$

$$\mu^3 = 4 \cdot e^{2\pi} \cdot \alpha_s^2 \cdot \alpha \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1$$

$$\mu = 2 \cdot e^{\pi} \cdot \alpha_s \cdot \alpha_G^{1/2} \cdot \alpha_{G(p)} \cdot N_A \cdot N_1$$

$$\mu = 4 \cdot e^{2\pi} \cdot \alpha_s^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_{G(p)} \cdot N_A^2 \cdot N_1$$

The mathematical formulas that connects the $\alpha_w$,$\mu$,$\alpha$,$N_1$,$N_A$,$\alpha_G$ and $\alpha_{G(p)}$ are:

$$2 \cdot e^{\pi-1} \cdot 10^7 \cdot \alpha_w \cdot \alpha \cdot N_A \cdot \alpha_G^{1/2} = 1$$

$$\mu \cdot N_1 = (2 \cdot e^{\pi-1} \cdot 10^7)^2 \cdot \alpha_w^2 \cdot \alpha^3 \cdot N_A^2$$

$$(2 \cdot e^{\pi-1} \cdot 10^7)^2 \cdot \alpha_w^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1 = 1$$

$$\mu^3 = (2 \cdot e^{\pi-1} \cdot 10^7)^2 \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1$$

$$\mu = 2 \cdot e^{\pi-1} \cdot 10^7 \cdot \alpha_w \cdot \alpha_G^{1/2} \cdot \alpha_{G(p)} \cdot N_A \cdot N_1$$

$$\mu = (2 \cdot e^{\pi-1} \cdot 10^7)^2 \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_{G(p)} \cdot N_A^2 \cdot N_1$$

The mathematical formulas that connects the $\alpha_s$,$\alpha_w$,$\mu$,$\alpha$,$N_1$,$N_A$,$\alpha_G$ and $\alpha_{G(p)}$ are:

$$\alpha_s = 2 \cdot 10^7 \cdot \alpha_w \cdot \alpha \cdot N_A \cdot \alpha_G^{1/2}$$

$$\mu \cdot N_1 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha^3 \cdot N_A^2$$

$$\alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \mu \cdot \alpha_G^2 \cdot N_A^2 \cdot N_1$$

$$\mu^3 \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_{G(p)}^2 \cdot N_A^2 \cdot N_1$$

$$\mu \cdot \alpha_s = 2 \cdot 10^7 \cdot \alpha_w \cdot \alpha_G^{1/2} \cdot \alpha_{G(p)} \cdot N_A \cdot N_1$$

$$\mu \cdot \alpha_s^2 = 4 \cdot 10^{14} \cdot \alpha_w^2 \cdot \alpha \cdot \alpha_G \cdot \alpha_{G(p)} \cdot N_A^2 \cdot N_1$$

The formulas for the gravitational constant G are:

$$G = (2 \cdot e^{\pi} \cdot \alpha_s \cdot \alpha \cdot N_A)^{-2} \cdot (\hbar \cdot c/m_e^2)$$

$$G = (2 \cdot e^{\pi-1} \cdot 10^7 \cdot \alpha_w \cdot \alpha \cdot N_A)^{-2} \cdot (\hbar \cdot c/m_e^2)$$

$$G = \alpha_s^2 \cdot (2 \cdot 10^7 \cdot \alpha_w \cdot \alpha \cdot N_A)^{-2} \cdot (\hbar \cdot c/m_e^2)$$

The gravitational fine structure constant $\alpha_g$ is defined as:

$$\alpha_g = \alpha_G^{3/2}/\alpha^3 = (\alpha_G/\alpha^2)^{3/2}$$

$$\alpha_g^2 \cdot \alpha^6 = \alpha_G^3$$

The gravitational fine structure constant $\alpha_g$ also equals:

$$\alpha_g = \ell_{p\ell} \cdot \Lambda^{1/2} = (G \cdot \hbar \cdot \Lambda/c^3)^{1/2}$$

$$\alpha_g = \ell_{p\ell}^3/r_e^3$$

$$α_g=(2·e·α^2·N_A)^{-3}$$

The unity formula that connect the gravitational fine structure constant $α_g$,the strong coupling constant $α_s$,the weak coupling constant $α_w$,the fine-structure constant α and the gravitational coupling constant $α_G$ is:

$$α_g=(10^7 · α_w · α_G^{1/2}/e · α_s · α)^3$$

Maybe this expression is the unity formula that connects the five coupling constants.

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