

# In quest of higher dimensions – Superstring Theory and the Calabi – Yau manifolds

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Higher dimensions are impossible to visualize as the size of dimension varies inversely proportional to its level. The more the dimension ranges, the least its size. We are a set of points living in a particular point of space and a particular frame of time. i.e., we live in space-time. The space has more dimensions that meets the human eye. We are living in a world of hyperspace. Our world being a smaller dimension is floating in higher dimensions. The quest for the visually of higher dimensions has been a fantasy to mankind but this aspect of nature is completely locked. We can transform dimensions i.e., from higher to lower dimensions, or from lower to higher dimensions, but only through mathematics. The relative notion of mathematics helps us to do the thing, which is perhaps impossible in the experimental part of physical reality. Humans being an element of 3 Dimensions – length, breath, height can only perceive one higher dimensions, that is space-time. but beyond that the notion of dimension itself changes. The dimensions got curled up in every intersection of the coordinates of space in such a way that the higher dimensions remain stable to us. But in reality it is highly unstable. In the higher dimensions, above 4, the space is tearing apart and joining again spontaneously, but the tearing portion itself covered by 2 dimensional Branes which acts as a stabilizer for the unstable dimensions. Dimensions will get smaller and smaller with the space-time interwoven in it. But at Planks length that is  $10^{-33}$  meter, the notion of space-time itself breaks down thereby making impossible for the higher dimensions to coexist along with space. Without space, there will be no identity of any dimension. The space itself is the fabric for the milestone of residing higher dimensions. Imagine our room, which is 3 dimensional. But what is there inside the room. The space and of course the time. Space-time being a totally separate entity is not quite separate when compared with other dimensions because it makes the residing place for the higher dimensions or the hyperspace itself. We all are confined within a lower dimensional world within a randomness of higher dimensions. Time being alike like space is an arrow which has the capability of slicing space into different forms. Thereby taking a snapshot of our every nanosecond we vibrate within space-time. As each slice of time represents each slice of space, similarly each slice of space represents each slice of time. The nature of space-time is beyond human consciousness. It is the identity by which we breathe, we play, we survive. It is the whole localization of species that encompasses itself with space thereby making space-time a relative quantity depending upon the reference frame. The only thing that can encompass space-time or even change the relative definition of space-time is the speed, the speed far beyond the speed of light. The more the speed, the less the array of time flows. Space-time being an invisible entity makes the other dimensions visible residing in it only into the level of 3, that is l, b, h. After that there is a infamous structure formed by the curling of higher dimensions called CALABI-YAU manifold. This manifold depicts the usual nature of the dimensional quadrants of the higher order by containing a number of small spherical spheres inside it. The mathematics of string theory is still unable to solve the genus and the containing spheres of the manifold which can be the ultimate quest for the hidden dimensions. Hidden, as, the higher dimensions are hidden from human perspective of macro level but if we probe deeper into the fabric of the space-time of General Relativity then we will find the 5<sup>th</sup> dimension according to the Kaluza-Klein theory. And if we probe even deeper into it at the perspective of string theory we will be amazed to see the real nature of quantum world. They are so marvelously beautiful, they contain so many forms of higher dimensions ranging from 6 to 10. And even many more of that, but we are still not sure about it where they may exist in a ghost state. After all, the quantum nature is far more beautiful that one can even imagine with a full faze of weirdness.

**KEYWORDS** – Hypersurface – Conifold Singularity – Higher Dimensions – Glome – Torus – Calabi-Yau Manifolds – M-Theory – Duality – Symmetry – Particles – Resonance – Multiverse – Coordinates – No Ghost Theorem – Mobius Strip – Klein Bottle – Topology – D(p)-Branes

**What is a dimension?** – A dimension is an attribute by which the property of an object can be defined. Co-Dimension is an advanced concept which takes into account the observable dimensions minus the unobservable one. In mathematics dimensions are established by means of coordinates, A single

dimension is measured by a single coordinate, whereas many dimensions are measured by many coordinate. In normal we use 'x', 'y', 'z', 'ct' – To measure the length, breadth, height and space-time respectively. Sometimes for the sake of simplicity 'x', 'y', 'z' are compactified within a single spatial dimensions



axis  $X, Y, Z, U$ . The Projection is stereographic or point to point projection and denotes that holographic Principle holds true for such projection. I cannot say on the basis of my thinking that GLOME is totally an hologram. Rather I would say that universe has no surface just like the sphere has no length. The universe If is surface-less then there must be a huge gap between the two consecutive ellipsoid and this denotes the dark energy or the expansion of the universe. A sphere which is 3D normally is not visualisable at its centre. But to drop a dimension and then try to visualize the centre of a circle is easy. In the same way the universe being a GLOME or a hypersphere when projected via stereographic point to point projection then it generally shows that the centre of the universe is visible. As I have said that universe is homogeneous and isotopic, the centre of the universe is everywhere. From every location, it seems that you are at the centre of the universe. Although universe is finite but relativistically universe is infinite due to its accelerated expansion. The bridging point between the ellipsoid or where 2 ellipsoid meets in the GLOME can act as a patch of high dark matter disk. When the sun revolves around the black hole at the centre of the galaxy it wiggles up and down due to the gravity of the dark matter which is tremendously heavy in this regions. The baryonic matter is less compared to the non baryonic matter and this leads to the increased or fluctuating gravity which triggers the comets from the Oort cloud which have an impact on the mass extinctions. Scientists showed that the dark matter disk or the patches of dark matter along the path of solar revolution (Approx 250 million years) can be the potential cause for the wiping out of the dinosaurs due to meteor shower. However, I try to link this theory with the help of the hyperspherical coordinates of the GLOME. The GLOME has a volume and one possible fact is that everybody is trapped outside the GLOME. The hypersphere has both the properties of a topologically curved spaces as well as the 4<sup>th</sup> Dimension. The 5<sup>th</sup> dimension might be time. The extra – 6,7,8,9,10 are the spatial curled up hidden dimensions. If we consider our universe as a Dp(5)-Brane then our universe is floating in the bulk of 10D. Colliding universes may exist and this can alter the shape of the coordinates. Generally a hypersphere having no surface area has a positive curvature. But with the special alignment of the other 4 axis – the hypersphere can be squeezed down into a flat disk and we humans are the 1D objects than relatively compared with the flat 2D disk of the hypersphere but this actually never happens. Our universe can also have a chaotic shape like an ameba. Or two amebas joined together with a tiny tissue. In that case we cannot rule out the effect of hypersphere. We have to think of this universe as a truncation point between the D-7 Branes where all the matter curve exists and their junction may give rise to a U(1) Abelian Gauge Bosons. But that D-7 Brane or the 7D is a part of a higher dimensional Calabi-Yau 4-Fold which is too infinitesimal to notice. We can say for sure that our universe is not a hypercube or a tesseract. But we can say that our universe might be a hypersphere where the extra dimensions are hidden from us and what we see is their projection on the 3D-Dp-Branes. Our universe is floating like a bubble in a multiverse. What if the coordinates of the universe is also changing with the floating speed? That means our universe squeezes from its normal value but that doesn't happens. The curvature can only be either positive or

zero, it can't get negative. Negatively curved universe is not our universe as because the coordinates will never align in such a way that the curvature goes below zero. This is mathematically impossible. Therefore, as it's hard to find the proper shape of the universe, it is also hard to determine what probabilistic shape the universe holds that's unknown to us. Holographic Principle is a very nice idea. Moreover the Conformal mapping of the stereographic projection is angle preserved and the CFT/AdS correspondence holds true for the geometry of the universe.

Well, our universe is a bubble floating among multiple universe or multiverse. But the concept of multiverse is still an oblivion to the mathematician. There can't even be an infinitesimal limit that this conjecture can be proved. So, better left aside the concept of multiverse and proceed towards the topological geometric shape of our universe or perhaps many such universe, if an identical universe just as ours is existent somewhere in the multiverse. Perelman proved the Poincare Conjecture there by showing that how a spherical universe can be splitted into two different universes by the formation of singularity in between them. Let's consider our universe as a sphere, then it will elongate from the both right or left, thereby expanding it longitudinally like a cylinder. Now after a certain amount of expansion, the sphere will take the shape of a barbell, with the two opposite sides inflating and in between contracting thereby creating a singularity. This singularity will then diminish its size and then thereby ultimately disappears by creating two different universes. But in reality our universe is not at all so smooth like a sphere. It doesn't have a spherical symmetry either. Then what exactly is the probabilistic shape of our universe? Well, mathematics has the answer. Nobody knows exactly the shape of the universe but instead we have known the size of the so called observable universe from the CMBR or Cosmic Microwave Background radiation. Our universe is not at all a sphere because it has certain properties which the spherical symmetry doesn't always support. But from the CMBR and the red shifting of the distant supernovas it has been calculated that at any three points from a certain region of the universe will provide a Triangle with slight more than 180 degrees. This states that our universe is somewhat flat with a little positive curvature. But that's not at all accurate with a 100% accuracy, maybe it is hyperbolic or maybe it is elliptical or maybe it is flat. Now, the geometry of the universe is somewhat complicated, it is actually not the geometry of the universe rather it is the geometry of the observable universe. And there are some definite features,

- [1] If one goes from one end of the universe, then he will come to its initial position upside down.
- [2] If one travels from the outside of the universe, then he will eventually get to the inside of the universe and again comes to its surface thereby making the notion that outside meets inside. But that is physically impossible.
- [3] If we see one star in a sky, then we can see its duplicate or another star in the sky, that means for every star there is its duplicate star. Why? Maybe its because the light from one star goes around the edges of the universe and travels back in a final position carrying the reflection of this original star. Maybe its poss-

ible if our universe has a positive curvature like that of a cylinder or sphere.

Now if our Universe is a hypercube or simply a cube, although its not, its actually of a higher dimension, but still if our universe is a cube for the sake of simplicity then if we draw two parallel lines then they will never intersect with each other as because there are no connecting points. Its all open. The lines will go forever and thereby providing a notion that our universe may be infinite. But on the other hand if our universe is some sort of a sphere or any geometrical object of positive curvature then if we draw two straight lines or specifically two geodesics then they will meet with each other after a certain amount of time. This may provide us a notion that our universe is finite with a connecting point. Now, this concludes that we are living in an infinite portion of a finite universe. That means from micro point of view the universe is finite but from macro point of view our universe is infinite. Whatever is infinite is finite and whatever is finite is actually a finite.

Actually whatever above is happening is true in respect to our observable universe but not from unobservable universe. This is because there are still some light waves that haven't come yet to the earth. Actually, whenever we are looking at the sky, we see only the past. For example, the sunlight is 8 minutes older, the farthest stars are million light years older and so on, so on.

Take a longitudinal piece of paper. Paint one side green and the other side red. Now twist the paper and glue to two ends. You will get a strip called MOBIUS STRIP. It is non-orientable topology and has only one edge and if an ant travels from the outside he will comes its initial position as its mirror image without even crossing any edge. That means MOBIUS STRIP represents a mirror image of an object if he travels from one point and return to the other. Actually, one thing you will notice that, in the MOBIUS STRIP which you had made, the green side joins the red side. And the number of twist is 1. So, the next twist will not be a MOBIUS STRIP anymore. Because the green side will join the green side and the red side will join the red side. But in the next turn again a MOBIUS STRIP is created. So to create a MOBIUS STRIP, just before gluing the both open ends of the paper twist it in ODD turns like, 1, 3, 5, 7, 9 etc.... And in even turn this notion is completely absent. Any object travelling through the MOBIUS STRIP will seem to be on the same side although he has travelled through its both side. And a mirror image is produced. That means the right handed man will seem left handed and vice versa. But before flipping you will actually be upside down. And after you flip then your exact mirror image will be produced. MOBIUS STRIP is a surface with only one side any one boundary. Its boundary is a simple closed curve that is homeomorphic to a circle. One of its important property of depicting it as a single boundary is that, if one draws a line on the one side, it will join to the line on the other side without even crossing its edge. If you cut the strip into a scissor, then one will get a long strip with two full twists in it rather than two separate strips. This happens as the strip has only one edge that is twice as long as the strip. Well, what actually is homeomorphic. It depends on the genus but not on the struc-

ture. A coffee mug is homeomorphic to a torus or doughnut as because the torus got one genus that is a puncture in its middle just like the same way a coffee mug got one genus in its handle attaching the mug body. So, although the MOBIUS STRIP is homeomorphic to a circle its boundary is not at all a true circle. But it is possible to embed the MOBIUS STRIP in 3-Dimensions so that the boundary acts like a closed space or a perfect circle under certain circumstances. And, by this way we can get a KLEIN's BOTTLE, when two MOBIUS STRIPS are joined together. A KLEIN'S BOTTLE is an object of 4th Dimension and it has no genus but it's the surface where the outside meets the inside and one travelling through its surface will experiences no edge or boundary and will ultimately return to its starting point upside down. So, it's time to explore a Klein's bottle now.

A KLEIN'S BOTTLE is a mathematical extended part of the MOBIUS STRIP in 3-Dimensions (For simplicity 3D model is taken although it is of 4D) Non-Orientable 2-Manifold against which the notion of normal vector can't be continuously determined. It is a Null-Sided surface without any boundary through which a traveler returns to its starting point upside down. If you watch the KLEIN BOTTLE properly then you will see that the bottle itself intersects the bottle thereby making a 2-Dimensional Circular cross section. But, that is not real. For the visual clarity this model has been adopted. But, for the 4th Dimension there is no notion of intersection into its own body, rather the tube will enter in its own body without any circular 2D Cross-section. KLEIN'S BOTTLE is the only bottle in topology that is self-intersecting. Actually, regarding the portion of the cylindrical self-intersection, there is an abstract 4D analogy – Let's consider that the number 8 is self-intersecting at the middle. But if we higher up one line in the intersecting position then the two lines will never intersect. The same thought experiment can be applied in the KLEIN'S BOTTLE analogy of self-intersection portions. The cylinder or the bottle is actually growing but is not piercing its existence as it is of 4th dimension. Well, its quite complicated stuff to understand. KLEIN'S BOTTLE is closed with connecting points and it has a positive curvature. It is homeomorphic to the sum of two projective planes and also homeomorphic to a sphere. If you watch the rail lines from the distant then you will notice that the two parallel lines of the single tract intersects at infinity. This is just an optical illusion and called as a projective plane. The traditional KLEIN BOTTLE image is ACHIRAL that means it is not supposed to form MIRROR SYMMETRY or it is indistinguishable from its original image when projected to a Mirror Plane. If the traditional KLEIN'S BOTTLE is to be cut, then it deconstructs into 2 oppositely Chiral MOBIUS STRIPS.

Now, what will happen if the KLEIN'S BOTTLE don't self-intersect into itself rather it becomes a continuous closed boundary-less loop, then it will take the shape of a TORUS or DONUT. And this is the actual shape of our OBSERVABLE UNIVERSE according to CMBR.

A TORUS can be get by revolving a CIRCLE in 3-Dimensional Space about an axis coplanar with the CIRCLE. When the axis of revolution does not touch the surface then

one can get a RING SHAPED STRUCTURE OR A TOROIDAL REVOLUTION. In Geometry the set of points in space are coplanar if there exists a plane that contains all the points into its surface. A RING TORUS is homeomorphic to the Cartesian product of two circles. These circles when rotates generates a 3d Euclidean Space. A TORUS consists of an infinite alignment of 2 Types of circles, the longitudinal circle or the 'BIG CIRCLE', the latitudinal circle or the 'SMALL CIRCLE'. It can also be called as MAGOR & MINOR CIRCLE. The Family of 3-Tori makes up a compactified 6D objects in String Theory called the CALABI-YAU MANIFOLD. The TORUS being a product of two interlocking Circles of  $S1 * S2$ , is the key element to be taken into consideration while unrevealing the shape of the Universe. The circles in a torus is closed path. A simple manifestation of the TORUS is by joining the two ends of the cylinder together. Our universe can be called as a HYPERTORUS or N-Dimensional Torus. Now, there is an amazing property of Torus. If you take a flat paper of 2D and draw two lines longitudinal & latitudinal then makes up a cylinder and glues up the open ends of the cylinder to make a torus you will find that although the two lines are of same length but when the torus is formed the circles created by the two lines are varying in size. One becomes larger than the other.

Now, if the exact shape of our universe is a HORN TORUS. In HORN TORUS the MAJOR RADIUS is equal to the MINOR RADIUS or  $R = r$ . And in this sub-group of TORUS, it has actually no hole in the middle. The middle point of the TORUS acts as the Singularity which creates the BIG BANG. Matter curves SPACE and SPACE tells MATTER how to move according to the GENERAL THEORY OF RELATIVITY OF THE EINSTEIN. So, the Space itself is just like a 'Bunked Race Track' where anything can move without accelerating or decelerating or changing its direction. So, as the curvature is must steeper in upper and lower sections so when BIG BANG occurs matter erupts from the White HOLE with a very giant speed called inflation which occurs about  $10^{-36}$  seconds after the singularity and lasted for  $10^{-32}$  to  $10^{-33}$  seconds. And then as the matter spreads around its curvature so, its speed becomes much less and the curvature is very less inclined and so the universe takes on an expansion rate rather than an Inflation rate. Now whatever the geometry is, we need to understand what makes up the universe (our universe) and this develops a question why matter dominates anti matter? There are two possible reasons,

- [1] Matter is created more than the Anti-Matter and when the Matter, Anti-Matter Annihilates then the excess matter which is left has become our stars and planets.
- [2] The matter and Anti-Matter has been created at an equal amount but they are still far away to come in contact with each other. So, no question of any annihilation comes.

**Strings and dimensions** – There are many versions of the Superstring Theory but M-Theory unifies those versions into a single structural framework. M in M-Theory stands for 'Magic', 'Mystery', or 'Membrane'. Some physicists sarcastically called it as a 'Monstrous theory'. M-Theory is actually the mother of all strings. The most crucial problem in modern

physics is the understanding of the Quantum Gravity. Newtonian Gravity is superseded by the Einstein's Theory of Gravity in General Relativity. However, quantum mechanics explains the other Non-Gravitational Forces like the Electromagnetic Force, Strong & Weak Nuclear force. So, a quantum theory of gravity is needed to merge the Relativistic Gravity to the Quantum Mechanics and M-Theory is the only way to achieve this.

String theory assumes that the fundamental constituents of the nature are not point particles rather they are 1-Dimensional strings. String duality is a notion of string theory which assumes that the theories of strings are linked to each other by means of T-duality & S-Duality. Apart from the Bosonic string theory there assumed to be five versions of String Theory. Let's discuss them below,

- [1] Bosonic – Only Boson, no Fermions, consists of both open and closed strings and a hypothetical particle of imaginary mass called Tachyon travelling faster than light is believed to be existent in this theory. It takes into account 26 Space-time Dimensions.
- [2] Type I – The concept of supersymmetry is there and is consistent of both open and closed strings in a symmetry of SO (32). It consists of 10-Dimensions.
- [3] Type IIA – The concept of supersymmetry is there between Boson & Fermion and a further extension of the strings called D-Brane of 2 dimensions are there in which open strings are attached. Closed strings are also a part of this theory.
- [4] Type IIB – Supersymmetry is there between Bosons & Fermions and consists of both open and closed strings attached to D-Branes which takes into account a maximum dimension of 10.
- [5] Type Heterotic SO (32) – Non-Tachyonic supersymmetry string theory takes into account of 10 dimensions consists of both closed strings and the maximum space-time dimensions being 10 with different notions between right and left moving strings.
- [6] Heterotic  $E8 * E8$  – supersymmetry between forces and matter with closed strings there and takes a maximum dimension of 10 with a symmetry group of  $E8 * E8$  (E stands for exception & 8 stands for dimensions).

The dimensions of D-Branes with regard to IIA is ODD that is, 1,3,5,7,9 while the dimensions of D-Branes with regard to IIB is 0,2,6,4,8 which is EVEN. The closed strings can move freely while the open strings are attached to the D-Branes with their two open ends. Apart from the Bosonic theory there were actually 5 string theories which can't be regarded as the candidate for the THEORY OF EVERYTHING. This theory accounts for 10 Space-Time dimensions where the rest 6-dimensions are compactified to a very small curled up single dimension with the structure of a CALABI-YAU MANIFOLD. These 5 String theories can be called as a Supersymmetric string theories because of the notation of supersymmetry which again can be unified by a single unified theory called the 'M'-Theory taking into account the Supergravity along with 11 spatial Dimensions. Two theories when are completely different, then they can be linked with the concept

of T & S duality thereby making the different theories into a single unified framework by the option of one theory being DUAL to the other by the incorporation of the DUALISTIC Principles. In String theories there are 10 Space-Time dimensions which means there are 9-Dimensions of Space along with 1-dimensions of time. Now if 1-dimensions can be assumed as the circumference of the Circle then one can return from the point where they started. A string when travelling around the circle then there exists different states of momentum and energy and in addition the strings wrapped around thereby forming a tension force of stretching and this wrapping modules are called the winding number. Now, the momentum of the strings in a curled up dimensions of radius R is similar to the winding number of the strings in a circle of radius  $1/R$  and vice versa, which makes the strings inter-related to each other by the notion of this Duality Principle. The large distance of momentum varies with the small distance of winding number and the small distance of winding number varies with the large distance of momentum. In any way the 5 different string theories can be interrelated by a maximum of 2 to 3 theories. T-duality relates Type IIA string Theory with Type IIB String Theory and compactify their dimensions curled up in a circle and if a worldsheet is considered then they get compactified into a cylinder where the large radius is inherently linked with small radius and the momentum is related with the winding number. The notion of large and the notion of small are became one and same, their properties inseparable from one another. They got wind up in higher dimensions. A string can be either left moving or right moving with the center of the string momenta is considered as the sum of the right and left fields whereas the stretch in the middle is considered as the difference. Now, there comes a second concept of S-Duality which takes into account a particular type of aspect called COUPLING CONSTANT which means the ability of the string to emit or absorb other strings within itself. Strings carries mass and electric charge and they may do the process of emitting and absorbing other strings or attached and decay to other strings by a method called PERTUBATION THEORY. String theories have a coupling constant which follows the phenomena of perturbation theory which depends upon the mode of the oscillation of the strings having the coupling constant small or large. If two strings are linked by S-Duality, then a string with weak coupling constant is linked with strong coupling constant and by this process the two string theories can be linked with each other. By means of string theory Type I is related to Heterotic SO (32) and Type IIB theory with itself. Further type IIA is of strong coupling constant and behaves as a 11-Dimensional theory with 10 Space-Time dimensions and 1 Time dimension which incorporates the 11-Dimensional M-theory. In case of T-Duality the momenta takes on discrete values that means if the energy of one string is 1-Joule and the other string is 3-Joule then there can't be no values in between them. The values are always discrete in nature. The existence of these dualities are very important in superstring theories which helps to unify the theories into a single structural framework of M-Theory. In case of the DUALITY, the geometry of the Space-Time breaks into Plank's scale Physics of  $1.6 * 10^{-35}$  meter & T-Duality is closely related to another concept of symmetry called as

MIRROR SYMMETRY which is a part of the ENUMERATIVE ALGEBRIC GEOMETRY.

The concept of duality makes the two different theories to be dual in a nontrivial way such that the properties of one are dual to the other. In Particle physics, Particles are of 0-Dimensional points which can be further extended to 1-Dimensional strings. The mathematics of the string theory can be studied in various higher dimensions apart from the Three Spatial dimensions of Length, Breadth, Height that is Up/Down, left/Right, Backward/Forward... indeed it takes 1 more Dimensions of Now/Then called the Time-dimensions along with the other 6-Dimensions compactified & curled up in circles. The idea of this curled up circles comes from the Kaluza-Klein theory of 5th Dimensions. If one watches a wire from its near, then the thin wire appears to be of a cylindrical object. But, if he moves away further and further from that wire then the wire will become like a 1-Dimensional String or rod rather than 3-Dimensional wire. So, an ant walking over the wire will see the wire as a straight line from its point of view but from your point of view the wire is of 3-dimensional. But the ant can't recognize it. This is because of the fact that the radius of the wire is getting smaller as it is moving farther from you and ultimately got curled up into a 1-Dimensional object but from the view point of the Ant, the wire is just like flat. So, the higher dimensions are so curled up that its impossible for us to notice stat just like the ant if we stayed on that wire.

Winding number is a number by which a curve circles a plane from a given point counter-clockwise. If each string is closed without any end points, then the winding number is the number of turns or curves the string actually makes. If the counter-clockwise rotation is 8 times and clock wise rotation is 4 times, then the winding number is equal to  $8-4$  that is 4 turns. The counter-clockwise turns behave as negative while the clockwise turns behave as positive. Their difference will result in each winding number. The momenta of the strings are quantized and the theories can be simultaneously replaced by the changing of momentum and winding number. The duality of the strings are the equivalence principle with the total energy remains unchanged.

If string theory takes into account 10-Dimensions, then the 4 are the spatial dimensions whereas the other 6-Dimensions are compactified into a single curled up dimensions forming a structure called the CALABI-YAU manifold. The strings propagate over these manifolds and vibrates in a chaotic order with ends either open or closed. It was eventually found that a single CALABI-YAU manifold doesn't determines the property of the string analogy rather two distinct CALABI-YAU manifolds represents the same analogy by a process called as the Mirror Symmetry. This is an important duality in string theory as because they are mirror to one another and helps to solve many complicated problems in string analogy. If the CALABI-YAU manifold is dissected into two pieces, then also the Mirror Symmetry can be observed in accordance with T-Duality. A torus can be called as the simplest manifestation of the CALABI-YAU manifold which can be treated as the product of two circles. The circles are organized within the

torus space and the torus itself acts as a space between the circles. In this case the mirror symmetry can be viewed as the T-Duality acting itself on the longitudinal circles of the torus. T-Duality is called TOPOLOGICAL DUALITY whereas S-Duality is called as STRONG-WEAK DUALITY.

In String theory, the 0-dimensional particles are replaced by 1-dimensional vibrating strings having mass and charge and they propagate throughout Space-Time freely without any restrictions, in fact they are very chaotic in nature, this strings can be open as a curved line or closed as a loop but when the strings are viewed from the macro perspective they can be represented by particles which are several in nature depends upon the distinct vibrational pattern. Two small strings combine to form a large strings and one large strings can get splits up into two small strings.

String theory needs extra dimensions of about 10 for mathematical simplicity and the extra dimensions are compacted to form a comparatively curled up lower dimensions and incorporates an idea of SUPERSYMMETRY. According to the SUPERSYMMETRY, every BOSON has a FERMEONIC Partner and every FERMEON has a Bosonic partner. Their masses are same but their spins are different. It is the spin which makes the BOSONIC STRINGS identical to the FERMEONIC STRINGS. Some Examples Are, The SUPERSYMMETRIC partner of PHOTONS are PHOTINOS, GLUONS are GLUINOS, FERMEON are SFERMEON, ELECTRON are SELECTRON, HIGGS are HIGGSINO. The most important is the SUPERGRAVITY which incorporates GRAVITY as a closed string of spin 2 with GRAVITINO of half integer spin 3/2 behaving as the FERMEONS.

String models have been trying to develop in which STRINGS represent the HIGH ENERGY Physics and so for this the extra dimensions must be compactified to lower dimensions for the simplicity. Or in other ways, the extra dimensions can be restricted. When CALABI-YAU manifolds have become a notion for compactifying the extra dimensions then two versions of the string theory can be compactified into two distinct features of the CALABI-YAU manifolds of Type IIA & Type IIB thereby producing a Mirror Symmetry. Mirror Symmetry states that two different models called A-Model & B-Model are equivalent in the same way such that there is a duality in between them. Actually these two CALABI-YAU manifold gives rise to the same physics by incorporating The Mirror Symmetry. The application of mirror symmetry belongs to the branch of mathematics called Enumerative Geometry which raises the questions of counting the number of solutions to a geometric question. Calculations of the mirror Symmetry of the B-Models is much easier than the A-Models. Gauge theory is also related to this type of Symmetry.

When the notion of STRINGS is expressed in higher Dimensions then there comes the notion of P-Branes, P being a variable incorporate many dimensions. The world Brane comes from the “membrane” which represents 2-dimensional Branes. D-Brane is an important part of the Brain that arises when one considers the open strings whose end points are attached to the Branes. Whereas Closed Strings can move freely from one

brane to another or one dimension to another. Just as Graviton is an example of Closed Strings, the photons are examples of Open Strings, so, as the end point of photons are attached to the Branes, they can only vibrate within the boundary area of the Brane but not away from it. The letter ‘D’ in D-Brane refers to Dirichlet Boundary Conditions. Sub manifolds are manifolds which rests within a manifolds. A sub manifolds is a surface embedded inside the CALABI-YAU manifolds and D-Branes are just like the sub manifolds but the end point of the strings contains the charges. In A-model D Branes can also be viewed as sub manifolds within the CALABI – YAU manifolds but their length, breadth and height are minimized to half. A torus is made up of infinitely many circles which are capable of decomposing the Torus. These circles PARAMETRIZE the circles when they are decomposed meaning that there is a correspondence with the circles and the points. A Torus is actually the union of these two circles and the circles lay down one after the other to form the Torus. This auxiliary space plays a very important role in SYZ or STROMINGER-YAU—ZASLOV CONJECTURE. The CALABI-YAU manifold has 6-Dimensions. They can be divided into 3-Tori which is a topological manifold of 3-Genus or holes and a 3-Sphere. In a normal sphere or a 2 Sphere there is a 2-dimensional boundary over a 3-dimensional sphere but in a 3-Sphere, there is a 3-dimensional boundary of a 4-dimensional hypersphere. If a Torus represents a Space-Time, then the strings can propagate through the Space-Time. According to the T-Duality a string when propagating through Space-Time can have momentum as well as winding number. If the Torus can be split into many circles and the T-Duality can be applied then, a new Torus will be formed which is the mirror symmetry of the CALABI-YAU Manifold.

SUPERGRAVITY is an important aspect of M-Theory which combines the notion of SUPERSYMMETRY along with General Relativity. It develops the idea of Gravitons, the mediator of the Gravity. SUPERGRAVITY is said to be of 4-Dimensional theory which can be further extended to 5 Dimensions that being referred as Kaluza-Klein Theory as Kaluza-Klein constructed a 5-dimensional gravitational theory that when dimensionally reduced on a circle, the electromagnetism can be coupled as Gravity. Therefore, it can be said as a unified Field Theory of Physics.

The CALABI-YAU Manifolds are important in Superstring Theory because of the unseen 6-dimensions of the CALABI-YAU Space. They are so small that they are impossible to detect experimentally. The small higher dimensions are an example of the CALABI-YAU Manifold. But there is a controversy regarding the nature of the higher dimensions. In the “Braneworld” model, the CALABI-YAU are actually large and we live in the subset of the intersection of the CALABI-YAU with the D Brane. Different strings vibrate differently and each of their vibrational pattern is associated with the holes of the CALABI-YAU Manifold or Space. The low energy string vibrations correspond to the known particles whereas the high energy vibrations correspond to the rare elements found in nature. The String vibrations effect the curled up CALABI-YAU Space and the intersection of different holes correspond to the masses of the particles.

ANTI-de SITTER SPACE is an important aspect of the Superstring theory. It consists of a space or a Lorentzian manifold with a negative curvature just similar to a HYPERBOLIC Plane. Just as MINKOWSKI Space is the analogue of EUCLIDEAN Space, ANTI-de SITTER SPACE is the analogue of the SPHERICAL Space. The ANTI-De SITTER SPACE corresponds to the CFT or the Conformal Field Theory of the Superstrings. The ANTI-De SITTER SPACE has a negative energy density and a positive vacuum. De-SITTER Space is related to the Theory of General Relativity relating to a cosmological constant comprising of 3+1 Dimensions which is analogous to which we live in, differs largely from the ANTI-De SITTER SPACE which embedded in the 5-Dimensional SPACE-TIME. The CFT doesn't corresponds to the Gravitational Force rather it takes into account the other 3 types of forces, the Electromagnetic Force, The Strong & Weak Nuclear force in 4 Spatial Dimensions and if String Theory is included then it takes into account 1 extra Dimensions that is of 5-Dimensions. It is quite different from the constant Scalar curvature where the Space-Time remains same everywhere and from the Negative Curvature which is like a horse saddle surface which is quite opposite to the surface of a sphere having the Positive Curvature. A CFT describes force that works on short distances rather than large distances that's why it ignored GRAVITATIONAL FORCES. A CFT is usually a QFT or Quantum Field Theories that is scalar invariant that means the notion will not change with the change in Length, Breadth, Energy and other variables. Space-Time in General Relativity is Energy-Mass equivalent and is connected with Space, Time & Gravity. Curvature in space is considered in presence of the matter which produces a resultant Gravitational Force. The Relativistic equation is predicted by Einstein in 1916 as  $E=mc^2$ . In GR both the large and small masses produces the curvature. Gravity is usually treated as the 3-dimensional superspace invokes a 2-Dimensional Space-Time. But geometrically Gravity is the 5-Dimensional superspace of the 4-Dimensional world. In GR Gravity bends time as well as Space and formed an equilibrium between the Gravity & Acceleration. Thus it differs from the Newtonian Gravity. In the ANTI-de SITTER SPACE, Space-Time is slightly curved like a hyperbolic plane and when the projection is done then it's called the CFT. ANTI-de SITTER SPACE in ST or String Theory takes into account the 5 dimensions where all the physical structure of the space can be determined. There exists a negative cosmological Constant. The extra 5th dimension is the TIMELIKE Dimension rather than LIGHTLIKE or SPACELIKE in a LIGHT CONE.

**Dimensional analogy and further splitting** – Why we write  $H_2O$  but not  $OH^2$ ... Well, its because although they look similar but still there is a lack of symmetry between them. By  $H_2O$  we mean  $(H_2)O$  & by  $OH_2$ , we mean  $(OH)_2$ . This is because our nature is asymmetrical. It lacks symmetry as because it is stable and the average temperature of the universe is  $-270^\circ C$ . So, one is water & other is hydroxide. But let's leave the oxygen molecule & concentrate on Hydrogen because it is the primal element upon which other elements are rendered. It contains protons, neutrons & electrons. We can broadly classify it into 2 classes – The Quarks & The Leptons. Here pro-

tons & neutrons are disassembled into quarks and the electron is indivisible and so it leads to leptons group. Now there are 3 Gauge fields penetrating the two groups – Quarks & Leptons. One is the Gauge boson field having the Electromagnetic particle like photons which excited electrons emit. Now there is strong nuclear force like Gluonic force that prevents the quarks to move out freely and there is a Higgs field by which all the particles interact with it and acquire masses. The heaviest of which is the Higgs Boson.

Now, let's look at the proton. It will decay, yes although the decay rate is too small but still it will decay like 1 in  $10^{30}$  years and this is governed by the weak nuclear forces. Now, the electrons are held together in the orbit by the electromagnetic forces. But that's not all, there is a Gravitational force – That is pervading every object and interacts with them even it interacts with the photon too. Let's see how?

Now break the quarks, you will get strings... Both open and closed strings. Open strings represent photons & closed strings represent graviton. But what about the charge? Well, the strings are electrically charged and have only 1 dimension which is vibrating through space-time at a regular period of time. This specific vibration property are accessed by a special kind of dynamical object of 2 dimensions or even 3,4,5,6,7,... Dimensions called Branes or membranes or D-Branes. These branes are also electrically charged having the open ended strings attached to its ends which prevents the particles or waves from escaping to higher dimensions. There is 2 things that is a constant factor with strings,

- [1] The string tension.
- [2] The coupling constant.

The string tension is the tension caused by the strings to vibrate and the coupling constant is the T-Duality which helps in the wounding of strings and its momentum number & the other is S-Duality or the strong-weak duality which helps in the splitting & joining of strings. The interactions of the strings are called as the fundamental concept being expressed in terms of the Perturbations theory.

Now General Relativity assumes a dynamical space. A space evolving over time due to gravity. But in case of string theory, the dynamical space has been switched off due to the effect of gravity on strings – The gravity when switched off, the strings behaves as a quantized number with a non-zero vacuum energy. The cosmological constant needed for the strings turned out to be negative or very close to zero. This is because of the negatively curvature spaces within which the extra dimensions of strings operate. But its not possible to take  $i$  to account to all those dimensions, so a 3 dimensional analogue of the strings has to be taken into account by a process of mathematics called conformal field theory.

In case of strings we can see that the gauge theory gets accumulated with the quantized strings and hence forms the loop quantum gravity. In this theory the gravity is expressed as the flux lines of the superconductor with little or no resistance to take into effect the events of causality occurring in aspect with special relativity. Here the notion of space is absent... Instead



there is a notion of spin foam or events that takes places at different times and clumped together to form the background of the gravity to operate.

But imagine the quantum world as a grid. A cross section of different lengths of latitude and longitude. And in each grid there lies a higher dimensional object. If it is a sphere then the geometry becomes easy, if its a torus then the geometry becomes complicated as it is a torsion of two circles. But what about the more complicated dimensions like Calabi-Yau manifold. Well, it is also there. And more precisely the D-Branes also takes a major part in compactifying higher dimensions. If the Brane is of 10D then the 9D left open and the other 1D got encompassed within the other 6D Calabi-Yau spaces and thereby wraps the space from being tearing out.

The analogy of higher dimensions is a necessary fact of string theory and it can be used to create the maximally black holes with temperature, radiation and entropy similar to the black holes. The nature is very complex in string theory but still the SUPERSYMMETRY has not achieved its place in the standard model of particle physics and the gauge theory fails to cooperate with the string theory.

It is quite easy to understand the mass of a particle according to the notion of string theory. The loops in string theory vibrate in a resonance pattern similar to those of vibrating strings of the violin. More frantic vibrational pattern has more energy and less frantic vibrational pattern have less energy. The energy of a vibrating string depends on its energy, the corresponding amplitude, the distance measured between the wavelengths of crests and troughs. If the energy is more then the wavelength will also increase and vice versa. And greater energy of string will correspond to a particle of greater mass. Thus, the mass of the elementary particle is dependent on the vibrational pattern of its internal strings. The different vibrational pattern gives rise to different notions of mass. The electric charge, the weak charge and the strong charge are carried on by the strings, the different ways it vibrates. Just as photons are the existing results of the string vibration, the graviton is also the same.

Different elementary particle has different charge. Just like electron have negative charge and protons have positive charge. But string theory changes this notion of difference as the underlying principles of the elementary particles is that they are composed of the same strings. This is in a way of unification which is the main aim of the string theory. This universe being composed of a different number of vibrating strings is the resultant of a cosmic symphony just like the individual strings of the violin makes up its symphony.

The string has corresponding tension on its loops. Two vibrating strings having the same tension will not have the same energy. The string on higher tension will have more energy than the string with lower tension. And it is also proportional to the number of crests and troughs in the corresponding vibrating pattern.

**Effects of dark energy on dimensions** – Are we living on a space of three spatial dimensions, Length, Breadth, Height? Absolutely not. In fact, we are living on a 4th Dimension of Space-Time where the Space itself interwoven with a temporal dimension The Time. But that does not represent the true nature of our Universe. In a matter of fact there are actually 11-Dimensions in the Universe with 4 Spatial Dimensions, 6 Higher Dimensions and another 1 Dimension are used to incorporate the notion of SUPERGRAVITY into the string Theory. Now these 6 Dimensions are again compactified into a curled up Dimensions which is too small to notice and takes the shape of a CALABI-YAU manifold inside which various sub-manifolds are existent. Now the real nature of the universe is something more complicated, it is being dominated by the dark energy which comprises of 68-73 % of the total matter-energy proportion. This dark energy is a derived concept from the Einstein's Cosmological Constant which Einstein takes into account to make the universe static rather than collapsing in its own Gravity to make his Field Equations Stable. But Hubble's Observation later proves that the universe is not static rather it is expanding. So, the Cosmological Constant or The Lambda of the relativity is used to make an Anti-Gravity Effect in order to make the vacuum energy density stable. Actually the Universe is in an Expansion Mode. But, still, the Baryonic masses are not moving away from each other rather in between the Baryonic Mass more and more Dark Energy has been created over Time to Time in order to make the energy density of the Universe stable. Our Universe has already been dominated by the Dark Energy and the final conclusion will be that, One Day almost all the masses of the Universe will fade away and dark energy will rule the entire universe. Dark energy gives a positive acceleration and a relativistic negative pressure which in turns creates a positive pressure thereby helps in the expansion of our Universe forever and the Expansion rate or Anti-Gravity effect is much more than the Gravitational Pull of the clusters of matter taken together to make up this Universe. But our universe is not a single Universe. There are multiverse or an infinite alignment of the Multiple Universe floating side by side just like a soap bubble and each universe has its self-contained notion of space and Time frame to make one Universe differs from the other. But where does this Universe rests. Well, there comes the notion of the String Theory. A Zeroth-Dimensional Particles has been replaced by the 1-Dimensional strings which can again be extended to form the P-Branes and this P being a variable number makes up any dimensions of 1, 2, 3, 4, 5 & So On. But our Universe is actually a 3-dimensional Brane Floating inside a higher dimensional Bulk or Hyperspace. This infinite Branes are floating in the Bulk and are interacting with each other but it's just only a prediction. There is no experimental resemblance to it. If Our Universe is a confined 3D brane and if we are floating in higher dimensions, then there can also be a possibility of the existence of other Branes more than 3-dimensions with a maximum of 5-Dimensions. Every particle we observe are just the vibrational strings inside the Branes. These strings can be Open or Closed and each vibrational pattern will give rise to each unique Particles. The Photon is an example of Open Strings whereas the Graviton is an example of a tiny Closed string. The Ends of Open strings are attached with the Branes or Membranes but the Closed strings are free

to move from one Brane to the other thereby the GRAVITON has the capability of escaping from One Dimension to the Other Dimension. If the Gravitational force is considered on a larger scale inside the Brane Cosmological model, then the force is very weak but if the Gravity is considered on a smaller scale then its force is rather very strong. Moreover, the other three Fundamental forces, The ELECTROMAGNETIC Force, STRONG & WEAK Nuclear Force are very dominant on the Branes Scenario. As a matter of fact, the Weak Force is  $10^{32}$  Times Stronger than the Gravitational force. This raises the HIERARCHY Problem. Fermi's Constant is used to denote the constant of the weak Force whereas Newtonian Constant is used to denote the constant of the Gravitational Force. Now due to RENORMALISATION or THE ELIMINATION OF THE INFINITIES of The EQUATIONS, the fundamental value is always less than the effective value. So, in a same way, the Higgs mass is considered to be very less than the Plank's Mass. Now, the cosmological constant or the LAMBDA can be both Positive or Negative where the Universe can get expansion forever or the Universe may seem to stop expansion and eventually Shrink in Size. But the Current Cosmological Constant provides a useful means of Anti-gravity if the Anthropic Principle is excluded from this concept. So, we are actually living in a lower dimensional Brane floating inside a Higher Dimensional Brane. And the Collision of each Brane gives rise to a new universe by creating a BIG BANG. But in reality, the Relativistic Model is different from the Brane scenario.

**KKLT Conifold singularity** – The flux tubes, the Calabi-Yau Spaces with a million holes, the Anti-D Brane that allows a universal repulsion like our expanding universe with a positive cosmological constant with other D1, D2, D3, D4, D5 Branes compactified in a Torus and giving the remaining 3-Branes, The D3 Branes residing in the landscape gives rise to a Conifold Singularity with many Vacua. The M theory considers the Strings as membranes with each membrane being compactified in 6 directions resulting in expanding another directions similar to a D-5 Brane where the Gravity is being tossed over. Thats 11 Dimensional Supergravity. Whats the difference between M theory & String theory ? The ends of D-2 Membranes when gets closed together attaches with each other to form a closed string of Graviton similar to Type IIA & Type IIB Theory where IIA has D0, D2, D4, D6, D8 Branes and IIB has D1, D3, D5, D7, D9 Branes. Each branes are compactified along 6 directions and our world represent a 3-Sphere or 3-Torus... more specifically a 3-Torus as the local dimension is 3 but overall its a 4 Dimensional and if you try to lasso a D1 Brane or string in a torus – It won't slip off. But if you try to lasso a D1 Brane in the equator of a 3-Sphere it will not lasso up – It will slip off to the either of the edges. Now replace D1 with D3.... Its our world flexible to move in 4th dimension time and 7 compactified dimensions including 1 of SUPER-SYMMETRIC Gravity.

**Gravity as a Junctional Truncation in a Differential Cohomological Manifold in F-Theory** – Dimensions – 4,5,6,7 are internal dimensions, 8,9 are complex dimensional plane, 10,11 are elliptic fibres And Gauge bundle in vector spaces, Adding up all this 4,5,6,7,8,9,10,11 – We get a 8-D Calabi-

Yau manifold where the 8,9 Dimensions are orthogonal truncation and if we consider this as a D7 (p,q) Brane of cycle  $pa+qb$  where a & b are the wrap number of the 2-Toric manifold then the other 5-D Brane goes away at the Plank-Brane and the intersecting cycles of Winding contribution gives a Coupling singularity of  $1/G_s$  where 3 F1 Branes intersect... We get gauge Bosons at the junction point. If the complex Tau goes mapping between the Branes as,

$$\frac{a(\tau)+b(\tau)}{c(\tau)+d(\tau)}$$

what we get is an infinitely stretched Plank Brane from the M2 Brane (TeV) Brane and we at the Truncation gets Gauge Bosons called Gravity with a Lie Group  $SL(2,Z)$  where S-Duality is Dual to Type IIB Strings. A Non-Perturbed lighter Gravity can be decomposed from F-Theory by the Truncation of K3 4-Folds in RANDALL-SUNDRUM Model.

Calabi-Yau Manifold when would be a Kähler manifold with the following metrics,  $\Omega = dx^1 dx^2 * \text{Log}(\det g) = 0 = \text{Ricci curvature}(R_{ij})$  then this satisfies the vacuum solutions of Einstein field equations.

**Why string theory is a reliable theory?** – Many people say that string theory is a mathematical fantasy or to some extreme it is a mathematical garbage?

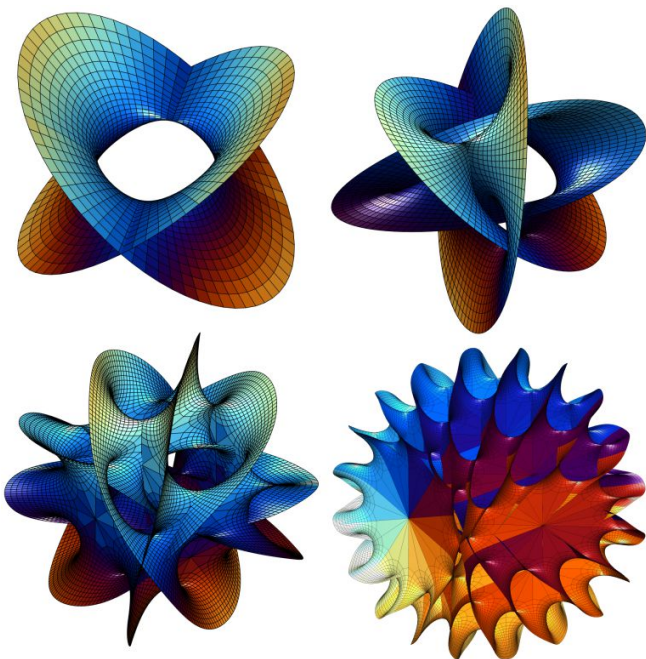
We will provide you 8 examples to show why string theory is a valid theory!

- [1] Heisenbergs Uncertainty Principle –  $\langle \Psi | [\Delta X, \Delta P] | \Psi \rangle \sim \hbar$ . Position and momentum never commutes. So, if the Position is 0 Dimension as a Point Particle then its Momentum needs to be infinity. But, to provide the necessary means of cut off from infinity we need to increase the Position from 0D Particle to 1D strings.
- [2] Mesons – A meson is held by Quarks and anti quarks together. But what's that glue substance that having opposite charges stretches them infinitely apart. This might be strings.
- [3] Condensate & Supersymmetry – Gravitons  $> 125$  GeV. Higgs are detected around 125GeV. But to increase further into Planks mass which is just a particle of dust, we need to increase the energy. The more the energy, the more the frequency. But there is a gap between the GeV's and TeV's. Something must be present in between them. They are the SUSY Particles which extends into strings for further amplification of energy. After that there is Black hole.
- [4] To provide an accelerator greater than that of LHC we need to make a particle accelerator 8 light minutes that is about 8,000,000,000,0 miles. After that we need to achieve the Planks mass into the size of the billionth of a billionth of a Photon that is  $1/1000000000000000000$  of Photon. It's impossible for humans to discover strings. (Measurements are approx.)

- [5] We are living in a Brane cosmology with 10 Spatial and 1 temporal dimensions. If SUSY is true then, there must be a closed string that penetrates through every dimensions especially the curled up compactified ones. Ours is a 4D Brane Scenario.
- [6] String theory provides a pixelated theory of universe in which the AdS is a holographic spacetime near the Black hole. It provides the information loss paradox of Black hole where information conservation is protected.
- [7] Every particle is elementary but not fundamental. So, there must be some unique elements that gives rise to the variety of particles. And that particles are extended into 1D strings where every type of resonance is giving a distinguishing properties of particles.
- [8] Strings are high energetic ultraviolet Branes where 2 strings collide to form a black hole. On the other hand nuclear physics is dealt with Infrared Branes where 2 particles collide to form quark gluon plasma. Both UV - BRANES at high energies is symmetrical to IR - BRANES at low energies.

**Vibrational patterns of the sectors of strings** – Now the vibration states of the string corresponds to the particle nature as,

- Left node vibration of  $|R\rangle$  sector with left node vibration of  $|NS\rangle$  sector give rise to Bosons.
- Right node vibration of  $|R\rangle$  Sector with right node vibration of  $|NS\rangle$  sector gives rise to Bosons.
- Right node vibration of  $|R\rangle$  sector with left node vibration of  $|NS\rangle$  sector give rise to fermions.
- Right node vibration of  $|NS\rangle$  sector with left node vibration of  $|R\rangle$  sector give rise to fermions.



**Fig:** Calabi-Yau surface for  $N=2$ ,  $N=3$ ,  $N=5$  and  $N=8$ . (Courtesy: <https://www.pinterest.com/pin/235524255499591585/>)

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