Prime Generation and primality test using 2x+1 and Summation of a constant

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Abstract:

We introduce another way to enumerate primes up to N using 2x+1 and the summation of a constant. By which can also be used for primality test of a given integer.

Definition:

For all n integer, we can write n as $(\frac{n-1}{2})$; thus $(\frac{n-1}{2}) \Leftrightarrow 2x+1$. And the multiples of a prime can be written as $(\frac{p-1}{2})$ adding to the summation of a constant c, where c is a prime p; multiplied by 2 adding 1 thus $((\frac{p-1}{2})+\sum_{i=1}^{n} p)\times 2+1$; $((\frac{p-1}{2})+\sum_{i=1}^{n} p)\times 2+1 \equiv 0 \, mod \, p$.

By definition above we're gonna use the formulae:

$$2x+1 \text{ and } \sum_{i=1}^{n} c$$

then we'll gonna substitute:

$$2x+1 \Rightarrow 2a_n+1$$

$$\sum_{i=1}^{n} c \Rightarrow a_n + \sum_{i=1}^{n} c$$

where :

$$c = 2a_n+1$$

n is a tuple (sequence)where:

$$(((a_n + \sum_{i=1}^{n} c) \times 2)+1) \leq a$$

On prime generation(prime sieve**)**

Let say we are given an integer a:

a= 10 ; since we need to find all primes less than a well use the list **$\{a_1,a_2,\ldots a_{n-1}\}$**; thus {1,2,3,4,5,6,7,8,9}

let:

$$c=2x+1 \quad = \quad c=2a_n+1$$

and we're gonna use to find the multiple of c

$$a_n+\sum_{i=1}^{n} c$$; to a limit where the function if multiplied by 2 adding 1 should be less than **a**

c		3	5	7	---------	---------	---------	---------	---------	---------	---------
a_n		1	2	3	4	5	6	7	8	9	10
$a_n+\sum_{i=1}^{n} c$	n=1	4	Here we skip since the function will give 7 7(2)+1=15 which is > a	Here we skip since the function will give10 10(2)+1=21 which is > a	Here we skip since 4 is on the a_1	Here we gonna skip since the function will add up if multiplied by 2 then added 1 is greater than a meaning instead of listing all $\{a_{1,\ldots}a_{n-1}\}$ we can list only up $\frac{a_n}{2}$ since $(((\frac{a_n}{2})\times 2)+1)>a_n$					
	n=2 so on	Here we skip since the function will give 7 7(2)+1=15 which is > a									

Checking if $a_n \neq \left(a_{n-(n-1)}+\sum_{i=1}^{n} c\right)$; if true c is prime

<u>**On primality test (**trial division using the prime generation**)**</u>

<u>example:</u> given integer a
a=100

we'll gonna use the method from prime generation above but we'll gonna use the limit since we know that the largest factor of a number is the squareroot; so $(a=\sqrt{a})\Rightarrow(a=10)$

Checking if $a \bmod c$; if true **a** not is prime

This method need to check first if a is even or odd by dividing 2.

As you can see above we started generating primes from 3 because:

<u>if we consider 1 as prime:</u>

$1\Rightarrow\frac{1-1}{2}\Rightarrow\frac{0}{2}$	thus the x above will start at 0 then if we feed 0 to the $x+\sum_{i=1}^{n} p$ where x is 0 and p is 1.
$0+\sum_{i=1}^{n} 1$	this set will produce an integer a where 2a+1 will produce all odd and even integer. So we can say this function is the prime of primes for all odd integer.

<u>if we consider 2 as prime:</u>

$2\Rightarrow\frac{2-1}{2}\Rightarrow\frac{1}{2}$	thus the x above will start at $\frac{1}{2}$ then if we feed $\frac{1}{2}$ to the $x+\sum_{i=1}^{n} p$ where x is $\frac{1}{2}$ and p is 2.
$\frac{1}{2}+\sum_{i=1}^{n} 2$	this set will produce a where 2a+1,we'll produce all even integer that if divide by 2 is equal to all odd integer. Which be written as $2\times\left(0+\sum_{i=1}^{n} 1\right)$,where n is only odd integer(including primes). So we can say this function is the prime of primes for all even integer. So if we don't consider 1 as prime then so is 2 we can't consider as prime.

note:

and the gaps of primes is bounded by how many multiple of primes between 2 given primes

example:
89,97 gap is 8
(89-1)/2=44
(97-1)/2=48 44,{45,46,47}48 ; thus 3 is the gap

Now to calculate the gaps; the formula is:

$$2x+2$$

Where x is equals **(a-b)-1**;where a is the larger prime and b is the smaller prime.