The Speed of Light On The Earth And In The Gravity-Free Space

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Abstract The metric for describing the spacetime geometry of a charged and rotating source in general relativity is the Kerr-Newman metrics, and it is an appropriate one to discuss the movement on the Earth. It is explicit that the velocity of light we measure is dependent on the rotation and net charges of the Earth, and usually the measurements are less than it in the really free space, at infinity or the gravity-free space, as long as the net charges of the Earth are less than $1.4\text{x}10^{19}$ C. According to this, the velocity of light in the really gravity-free space should be corrected. When we adopt the coordinate time at infinity as the unified time at each measurement placement on the Earth, it is 0.2085 *m/s* slightly larger than what we identify in vacuum on the Earth, $2.9979458\text{x}10^{8}$ *m/s*. The calculations also show the tiny deviation on the speed of light along the longitudinal direction between two poles and the equator, and it also exists deviation between the left-handed circularly light and the right-handed circularly one in the equator. We also discuss the difference on the speed of light in the local reference frame by using the proper time.

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I. Introduction

As we know, the speed of light is not a constant when it passes through some materials like air, fiber, and glass [1]. We can define a refraction index for some material to denote the relation between the speed of light in the material and the speed of light c in vacuum. Due to the existence of a lot of different materials, the measured speed of light is not a constant in those different materials. The gravity is also another thing that affects the speed of light we measure. In astronomy, for example, the gravitational-lens effect is a famous phenomenon when light passes through the vicinity of a strong gravitational source. The spacetime structure causes the deflection of light and the change of its velocity. Actually, the speed of light identified on the Earth is measured in vacuum, but not in really free space because the gravity exists. Although the gravity is very weak, the speed of light c measured in vacuum is still not a correct value c_0 in the really free space.

In this research, we study the correction term on the speed of light by considering the effects from mass, rotation, and net charges of the gravitational source. The correct term can tell us the difference of the light speed between in vacuum on the Earth and in the really free space. It also gives the difference of the light speed between two poles and the equator on the Earth.

II. Light In The Charged And Rotating Gravity

In 1915, Einstein proposed the equation of gravity

$$G_{ij} = R_{ij} - \frac{1}{2} g_{ij} R = 8\pi G T_{ij}, \tag{1}$$

where G_{ij} is the energy-momentum tensor, R_{ij} is the Ricci tensor, g_{ij} is the metric tensor, R is the Ricci scalar, G is the gravitational constant, and T_{ij} is the stress-energy tensor [2-8,11]. Several main metrics for discussing the Einstein's spacetime structure have been revealed many years, such as the Schwarzschild metric [2-8,11], the Kerr metric [2-4,8-10], and the Kerr-Newman metric [4,8,12-15]. For example, the Schwarzschild metric in the coordinates (t, r, θ, ϕ) is

$$\mathrm{d}s^2 = \left(1 - \frac{R_S}{r}\right)\mathrm{d}t^2 - \left(1 - \frac{R_S}{r}\right)^{-1}\mathrm{d}r^2 - \mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\phi^2 \tag{2}$$

where ds is the invariant interval and R_S=$2GM/c_0^2$ is the Schwarzschild radius [2-8,11]. General relativity tells us that the light path is along a null geodesic on the spacetime geometry [2-8,11], so the light path or a photon path is along ds^2=0. It has been used to derive the velocity of light in the Schwarzschild metric [2,3,5-8,11] and Kerr metric [2-4,8]. The velocity of light v_r along the radial direction in the Schwarzschild field is

$$v_r^2 = c_0^2\left(1 - \frac{R_S}{r}\right)^2, \tag{3}$$

The Schwarzschild solution considers very simple case that the gravitational source is non-rotating and charged-free.

However, as we know, almost all stellar bodies are rotating even charged. Like the Earth, it rotates once per 24 hours. When we discuss the velocity of light on the Earth, we should consider this rotating effect. The charged effect of the Earth is unknown and should also be considered in discussions. In order to consider mass, rotation, and charge in the gravitational calculation, we have to use the Kerr-Newman metric [4,8,12-15]. The Kerr-Newman metric [4,8] is the one simultaneously including these three related parameters, and they are R_S related to the total mass M, a related to the total angular momentum J, and R_Q related to the totally net charge Q. The expression for the Kerr-Newman metric in the coordinate (t, r, θ, ϕ) [4,8,12-15] is

$$\begin{aligned}\mathrm{d}s^2 &= -c_0^2\mathrm{d}\tau^2 \\ &= \left(\frac{\mathrm{d}r^2}{\Delta} + \mathrm{d}\theta^2\right)\rho^2 - (c_0\mathrm{d}t - a\sin^2\theta\mathrm{d}\phi)^2\frac{\Delta}{\rho^2} \\ &+ \left((r^2 + a^2)\mathrm{d}\phi - ac_0\mathrm{d}t\right)^2\frac{\sin^2\theta}{\rho^2},\end{aligned} \tag{4}$$

where c_0 is the speed of light in the really free space, τ is the proper time, t is the coordinate time,

$$\rho^2 = r^2 + a^2\cos^2\theta, \tag{5}$$

$$\Delta = r^2 - rR_S + a^2 + R_Q^2, \tag{6}$$

$a = J/Mc_0$, $R_Q^2 = KQ^2G/c_0^4$, and K is the Coulomb's constant. The coordinate time in a gravitational field is the time read by the clock stationed at infinity because the proper time and coordinate time becomes identical [5]. As mentioned before, the propagation of light is along $\mathrm{d}s^2=0$, then Eq. (4) can give an equation to describe three velocity components of light ($\mathrm{d}r/\mathrm{d}t$, $r\mathrm{d}\theta/\mathrm{d}t$, $r\sin\theta\mathrm{d}\phi/\mathrm{d}t$)

$$\frac{\rho^4}{\Delta(\Delta - a^2\sin^2\theta)}\left(\frac{\mathrm{d}r}{dt}\right)^2 + \frac{\rho^4}{r^2(\Delta - a^2\sin^2\theta)}\left(r\frac{\mathrm{d}\theta}{\mathrm{d}t}\right)^2$$
$$-\frac{[\Delta a^2\sin^2\theta - (r^2+a^2)^2]}{r^2(\Delta - a^2\sin^2\theta)}\left(r\sin\theta\frac{d\phi}{\mathrm{d}t}\right)^2 - \frac{2ac_0[(r^2+a^2)-\Delta]\sin\theta}{r(\Delta - a^2\sin^2\theta)}\left(r\sin\theta\frac{\mathrm{d}\phi}{\mathrm{d}t}\right)$$
$$= c_0^2. \tag{7}$$

It shows that each velocity component in the coordinates ($r,\ \theta,\ \phi,\ t$) has something to do with mass, rotation, and charges, and each velocity component must be real and finite by observation. Eq. (7) allows us to measure the speed of light using the same coordinate time everywhere. On the other hand, when an observer rests in a local reference frame, Eq. (4) gives the time relationship between the proper time and the coordinate time

$$\mathrm{d}\tau^2 = \frac{(\Delta - a^2\sin^2\theta)}{\rho^2}\mathrm{d}t^2. \tag{8}$$

When we measure the speed of light in the local reference frame, we adopt the proper time in Eq. (8) to calculate the speed of light.

III. The Speed Of Light Along The Longitudinal Direction Measured On The Earth

When we measure the velocity of light on the Earth, the light path is almost a tangential line on the Earth's surface. We can assume that the propagation of light is along the longitudinal direction so the velocity of light is only the term $r\mathrm{d}\theta/\mathrm{d}t$, that is,

$$\left(r\frac{d\theta}{dt}\right)^2 = c_0^2\frac{r^2(\Delta - a^2\sin^2\theta)}{\rho^4} = c_0^2\frac{r^2\left(r^2 - rR_S + a^2\cos^2\theta + R_Q^2\right)}{(r^2+a^2\cos^2\theta)^2}. \tag{9}$$

This equation tells us that this velocity component has relation with not only the mass of the Earth, but also the rotational speed and the totally net charges of the Earth. Using the mass $M_\oplus$=5.97x10^{24} *kg* [16], the radius $R_\oplus$=6.378x10^6 *m* [16], and the rotational angular velocity $\omega_\oplus$=7.292x10^{-5} rad/s [16], the parameters R_S and a for the Earth are

$$R_S = \frac{2GM_\oplus}{c_0^2} \approx 8.87 \times 10^{-3}\ m, \tag{10}$$

and

$$a = \frac{J}{M_\oplus c_0} = \frac{\frac{2}{5}M_\oplus r_\oplus^2 \omega_\oplus}{M_\oplus c_0} = \frac{2}{5}\frac{r_\oplus^2 \omega_\oplus}{c_0} \approx 3.9579. \tag{11}$$

Here the speed of light in the really free space is approximated to the value in vacuum on the Earth, so in Eqs. (10) and (11) the speed of light still use the value 2.99782458x10^{8} m/s. In the following, we will show this approximation good enough. Substituting Eqs. (10) and (11) into Eq. (8) and using $r=R_\oplus$, the ratio of $R_\oplus R_S$ to a^2 is

$$\frac{R_\oplus R_S}{a^2} = 3.612 \times 10^3. \tag{12}$$

It means that the rotational effect comparing to the R_S term on the speed of light is much small. When we consider the speed of light in the equator or $\cos\theta=0$, then Eq. (8) reduces to

$$\left(R_\oplus \frac{d\theta}{dt}\right)^2 = c_0^2\left(\frac{R_\oplus^2 - R_\oplus R_S + R_Q^2}{R_\oplus^2}\right). \tag{13}$$

Next, according to Eq. (13), we can estimate the effects of R_S and R_Q on the speed of light. If the measurement of the speed of light on the Earth were the same as that in the really free space, it means

$$R_\oplus R_S = R_Q^2. \tag{14}$$

Substituting the definitions of R_S and R_Q in Eq. (14) and further arranging it then gives

$$2M_\oplus c_0^2 R_\oplus \approx KQ^2, \tag{15}$$

or

$$Q \approx 2.761 \times 10^{19}\ \ C. \tag{16}$$

Eq. (16) gives the upper charge limit to make sure the longitudinal speed of light measured on the Earth less than it in the really free space. When the charges of the Earth is more than 2.761x10^{19} C, the measured speed of light is higher than it in the really free space. Usually, the net charges on the Earth is much less than this value because charging to so high value needs a lot of energy. The possibly maximal charges Q_{max} of the Earth can be estimated. Considering the Coulomb's self-energy of Q_{max} approximately equal to the equivalent energy of the Earth, then it gives

$$Q_{max} \approx \left(\frac{GM_{\oplus}^2}{K}\right)^{1/2} = 5.15 \times 10^{14}\ C. \tag{17}$$

Theoretically speaking, the charges of the Earth should be much less than the value in Eq. (17) in common case so the longitudinal speed of light measured on the Earth is less than it in the really free space.

Finally, the upper limit speed of light in the really free space can be calculated by letting R_Q=0 in Eq. (13), then we have

$$c^2 = \left(r\frac{d\theta}{dt}\right)^2 \approx c_0^2\left(1 - \frac{R_S}{R_{\oplus}}\right) = c_0^2 - \frac{2GM_{\oplus}}{R_{\oplus}}. \tag{18}$$

It gives the maximal increase of the square speed is 1.25x10^8 m^2/s^2 and the upper limit speed of light in the really free space is

$$c_{0,max1} = \sqrt{c^2 + \frac{2GM_{\oplus}}{r_{\oplus}}} \approx c + 0.2085\ m/s. \tag{19}$$

It means that the maximal speed of light in the really free space is 0.2085 *m/s* slightly more than the value measured along the longitudinal direction in the equator when the net charge Q is zero.

The other special value is at two poles where the rotational term is appeared and maximal due to cosθ=1. The speed of light is expressed as

$$c = \left(r\frac{d\theta}{dt}\right)\bigg|_{r=R_{\oplus}} = \frac{R_{\oplus}\left(R_{\oplus}^2 + a^2 - R_{\oplus}R_S + R_Q^2\right)^{1/2}}{R_{\oplus}^2 + a^2} c_0. \tag{20}$$

Similar to Eq. (19), we can also find the speed of light in the really free space more than the value measured at two poles. Adopting R_Q=0, then we have

$$c_{0,max2} = \frac{R_{\oplus}^2 + a^2}{R_{\oplus}\left(R_{\oplus}^2 + a^2 - R_{\oplus}R_S\right)^{1/2}} c \approx (1.0 + 6.956 \times 10^{-10})c$$
$$\approx c + 0.2085\ m/s. \tag{21}$$

The result is almost the same as the increase in the equator, which is also 0.2085 *m/s* at two poles when the net charge Q is zero. Actually, the difference between Eqs. (19) and (21) is very tiny, and it is only 5.768x10^{-5} *m/s* between the two cases. It might be detected by the interference technology. When we consider the small net charge case, the difference is almost the same. As a result, the difference also tells us that the speed of light is not homogeneous on the Earth due to the rotation and the net charges of the Earth. It is only homogeneous in the really free space.

Then we consider the difference on the speed of light between the equator and two poles varying with R_Q. The difference is

$$\Delta c = \left[\left(\frac{R_\oplus^2 - R_\oplus R_S + R_Q^2}{R_\oplus^2}\right)^{1/2} - \frac{R_\oplus\left(R_\oplus^2 + a^2 - R_\oplus R_S + R_Q^2\right)^{1/2}}{R_\oplus^2 + a^2}\right] c_0. \quad (22)$$

This difference almost keeps a constant even the net charges on the Earth is as large as 10^{10} *C*, so it reveals that the rotating term is the main fact causing the difference of the light speed between in vacuum on the Earth and the really free space. By the way, the explicit difference might appear at the white dwarf star or neutron star where R_S and R_Q are much large values comparable to those of the Earth.

When we measure the speed of light in the local reference frame, the proper time is used to calculate the speed of light. Then we have the expression of the speed for light

$$\left(r\frac{d\theta}{d\tau}\right)^2 = c_0^2\frac{r^2}{\rho^2} = c_0^2\frac{r^2}{r^2 + a^2\cos^2\theta}. \quad (23)$$

This equation tells us that the speed of light at two poles is the same as the speed in the gravity-free space. Furthermore, the difference on the speed of light between the equator and two poles is

$$\Delta c = \left[1 - \frac{R_\oplus^2}{R_\oplus^2 + a^2}\right] c_0$$
$$\approx 1.15 \times 10^{-4}\ m/s. \quad (24)$$

It is a very small term and possibly detected by the interference technology.

IV. The Speed Of Light Along The Latitudinal Direction Measured On The Earth

Then we calculate the difference on the light speed between it along the latitudinal direction on the Earth and in the really free space. The velocity of light along the latitude on the Earth obeys the following equation

$$-\frac{(\Delta a^2\sin^2\theta - (r^2 + a^2)^2)}{r^2(\Delta - a^2\sin^2\theta)}\left(r\sin\theta\frac{\mathrm{d}\phi}{dt}\right)^2 - \frac{2ac_0\left(-\Delta + (r^2 + a^2)\right)\sin\theta}{r(\Delta - a^2\sin^2\theta)}\left(r\sin\theta\frac{\mathrm{d}\phi}{dt}\right) = c_0^2. \quad (25)$$

To solve Eq. (25), we substitute hc_0 for $r\sin\theta(\mathrm{d}\phi/\mathrm{d}t)$ where h is real. Then the equation becomes

$$-\frac{(\Delta a^2\sin^2\theta-(r^2+a^2)^2)}{r^2(\Delta-a^2\sin^2\theta)}h^2-\frac{2a\big(-\Delta+(r^2+a^2)\big)\sin\theta}{r(\Delta-a^2\sin^2\theta)}h=1. \qquad (26)$$

In the following, we solve Eq. (26) directly to obtain two real solutions of h_+ and h_-, which are

$$\begin{aligned} h_\pm &= \frac{1}{c_0}\left(r\sin\theta\frac{\mathrm{d}\phi}{\mathrm{d}t}\right)_\pm \\ &= \frac{-\dfrac{2a(-\Delta+(r^2+a^2))\sin\theta}{r(\Delta-a^2\sin^2\theta)}\mp\dfrac{2(r^2+a^2-rR_S+R_Q^2)^{1/2}(r^2+a^2\cos^2\theta)}{r(\Delta-a^2\sin^2\theta)}}{2\dfrac{(\Delta a^2\sin^2\theta-(r^2+a^2)^2)}{r^2(\Delta-a^2\sin^2\theta)}} \\ &= \frac{ra(rR_S-R_Q^2)\sin\theta\pm r\,(r^2+a^2-rR_S+R_Q^2)^{1/2}(r^2+a^2cos^2\theta)}{(r^2+a^2)(r^2+a^2\cos^2\theta)+(rR_S-R_Q^2)a^2\sin^2\theta}. \end{aligned} \qquad (27)$$

It can be further expressed as

$$c_\pm=\left(r\sin\theta\frac{d\phi}{dt}\right)_\pm=c_0h_\pm=\pm\frac{r\big(r^2+a^2-rR_S+R_Q^2\big)^{1/2}}{r^2+a^2}\,c_0$$

$$+r\big(rR_S-R_Q^2\big)a\sin\theta\,\frac{1\mp\big(r^2+a^2-rR_S+R_Q^2\big)^{1/2}(a\sin\theta)/(r^2+a^2)}{(r^2+a^2)(r^2+a^2\cos^2\theta)+\big(rR_S-R_Q^2\big)a^2\sin^2\theta}c_0. \quad (28)$$

Since the only unknown quantity is R_Q, so the speed of light is a function of R_Q. On the surface of the Earth, it becomes

$$\begin{aligned} c_\pm(R_Q)\big|_{r=R_\oplus} &= \pm\frac{R_\oplus\big(R_\oplus^2+a^2-R_\oplus R_S+R_Q^2\big)^{1/2}}{R_\oplus^2+a^2}c_0 \\ &\quad +R_\oplus\big(R_\oplus R_S-R_Q^2\big)a\sin\theta \\ &\quad \times\frac{1\mp\big(R_\oplus^2+a^2-R_\oplus R_S+R_Q^2\big)^{1/2}a\sin\theta/\big(R_\oplus^2+a^2\big)}{\big(R_\oplus^2+a^2\big)^2+\big(-R_\oplus^2-a^2+R_\oplus R_S-R_Q^2\big)a^2\sin^2\theta}c_0. \end{aligned} \qquad (29)$$

There are two solutions for $\theta\neq 0$. Respect to the rotating axis of the Earth, one moves left-handed circularly, and the other moves right-handed circularly. Both of them give the maximal speed of light in the really free space about 0.2085 m/s larger than the measured value along the latitudinal direction on the Earth when the net charge Q is zero. It is still very accurate even the net charges is as large as 10^{10} C on the Earth. However, the speed of the left-handed circular light is slightly different from the right-handed circular light, and the difference is

$$\Delta c(R_Q) = \left| |c_+(R_Q)| - |c_-(R_Q)| \right|$$

$$= 2\frac{R_\oplus\left(R_\oplus^2 + a^2 - R_\oplus R_S + R_Q^2\right)^{1/2}\left(R_\oplus^2 + a^2\cos^2\theta\right)}{\left(R_\oplus^2 + a^2\right)^2 + \left(-R_\oplus^2 - a^2 + R_\oplus R_S - R_Q^2\right)a^2\sin^2\theta} c_0. \quad (30)$$

By calculation, the largest difference along the latitudinal direction at Q=0 is 4.76837x10^{-7} *m/s* in the equator of the Earth. When we consider the measurement in the local reference frame, the proper time is used and the speed of light in Eq. (28) becomes

$$c'_\pm = \left(r\sin\theta\frac{d\phi}{d\tau}\right)_\pm = c_0 h_\pm \frac{\rho}{(\Delta - a^2\sin^2\theta)^{1/2}}$$

$$= \pm\frac{r\left(r^2 + a^2 - rR_S + R_Q^2\right)^{1/2}\left(r^2 - rR_S + a^2\cos^2\theta + R_Q^2\right)^{1/2}}{(r^2 + a^2)(r^2 + a^2\cos^2\theta)} c_0$$

$$+ r\left(rR_S - R_Q^2\right)a\sin\theta\frac{\left(r^2 - rR_S + a^2\cos^2\theta + R_Q^2\right)^{1/2}}{(r^2 + a^2\cos^2\theta)}$$

$$\times \frac{1 \mp \left(r^2 + a^2 - rR_S + R_Q^2\right)^{1/2}(a\sin\theta)/(r^2 + a^2)}{(r^2 + a^2)(r^2 + a^2\cos^2\theta) + \left(rR_S - R_Q^2\right)a^2\sin^2\theta} c_0. \quad (31)$$

When we evaluate the difference between the left-handed and right-handed speed of light in the equator, Eq. (30) becomes

$$\Delta c'(R_Q) = \left| |c'_+(R_Q)| - |c'_-(R_Q)| \right|$$

$$= 2\frac{\left(R_\oplus^2 + a^2\cos^2\theta\right)^{1/2}}{\left(R_\oplus^2 + a^2\cos^2\theta - R_\oplus R_S + R_Q^2\right)^{1/2}}$$

$$\times \frac{R_\oplus\left(R_\oplus^2 + a^2 - R_\oplus R_S + R_Q^2\right)^{1/2}\left(R_\oplus^2 + a^2\cos^2\theta\right)}{\left(R_\oplus^2 + a^2\right)^2 + \left(-R_\oplus^2 - a^2 + R_\oplus R_S - R_Q^2\right)a^2\sin^2\theta} c_0. \quad (32)$$

This term is close to the value in Eq. (30) except for the large R_Q case. The difference also reveals that the two latitudinal speeds of light on the Earth are not equal and it has been explicitly pointed out in the Kerr black hole [3]. In such case, two solutions for photons initially going tangent to a circle of constant r in the equatorial plane are

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = 0 \quad (33)$$

and

$$\frac{d\phi}{dt} = -\frac{2g_{t\phi}}{g_{\phi\phi}}, \tag{34}$$

where $g_{t\phi}$ and $g_{\phi\phi}$ are two metric components in the Kerr metric [3]. It is explicit that two solutions are very different, especially one of them reveals the initially zero tangential speed of photon. The tangential speed in Eq. (31) is just one component of the photon's velocity because the photon cannot rest on a circle of radius r. The inhomogeneous speed of light exists and the rotating effect is explicit in the ultra-strong gravity. It is also true on the Earth but not so clear because of the weak gravity.

V. Conclusions

In summary, from the viewpoint of the spacetime established by a charged and rotating source, the speed of light in the really free space is 0.2085 m/s slightly more than it measured in vacuum on the Earth when the totally net charge Q is zero. Besides, the calculations predict that the measured speed of light along the longitudinal direction also have a tiny difference of 5.768x10^{-5} m/s between the equator and two poles. It also points out that the tiny difference on the speed of light exists in the equator when light moves left-handed or right-handed in the latitudinal direction. Light moves left-handed circularly has a speed difference of 4.768x10^{-7} m/s different from the right-handed circularly. Generally speaking, the net charges on the Earth should be less than 10^{10} C so the charging effect is not the main effect on the speed of light. Those results reveal that the speed of light is not homogeneous on the Earth even they are measured in vacuum. It is only homogeneous in the really gravity-free space. In addition, the speed of light measured in the local reference frame by using the proper time is also discussed. The results tell us that the speed of light at two poles is the same as it in the gravity-free space.

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References:

[1]. Robert Guenther, Modern Optics, Wiley, Somerset, 1990.
[2]. Hans C. Ohanian and Remo Ruffini, Gravitation and Spacetime, W. W. Norton & Company, New York, 1994.
[3]. Bernard F. Schutz, A First Course In General Relativity, Cambridge University Press, Cambridge, 1985.
[4]. F. De Felice and C. J. S. Clarke, Relativity On Curved Manifolds, Cambridge University Press, Cambridge, 1990.
[5]. Richard A. Mould, Basic Relativity, Springer, New York, 2002.
[6]. Hans Stephani, Relativity-An Introduction to Special and General Relativity, Cambridge University Press, Cambridge, 2004.

[7]. L. D. Landau and E. M. Lifshitz, The Classical Theory of Fields, Pergamon Press Ltd., Oxford, 1975.
[8]. Charles W. Misner, Kip S. Thorne and John Archibald Wheeler, Gravitation, W. H. Freeman and Company Publishers, San Francisco, 1970.
[9]. Roy P. Kerr, Gravitational field of a spinning mass as an example of algebraically special metrics, Physical Review Letters, **11** (1963), 237-238.
[10]. R. D. Blandford and R. L. Znajek, Electromagnetic extraction of energy from Kerr black holes, Monthly Notices of The Royal Astronomical Society, **179** (1977), 433-456.
[11]. John Stewart, Advanced General Relativity, Cambridge University Press, Cambridge, 1996.
[12]. E. T. Newman, R. Couch, K. Chinnapared, A. Elton, A. Prakash, and R Torrence, Metric of a rotating, charged mass, Journal of Mathematical Physics, **6** (1965), 918-919.
[13]. K.S Virbhadra, Energy Associated with a Kerr-Newman Black Hole, Physical Review D, **41** (1990), 1086-1090.
[14]. S. S. Xulu, Møller energy for the Kerr–Newman metric, Modern Physics Letters A, **15** (2000), 1511-1517.
[15]. Tim Adamo and E. T. Newman, Kerr-Newman metric, Schalorpedia, **9** (2014), 31791.
[16]. Graham Woan, The Cambridge Handbook of Physics Formulas, Cambridge University Press, Cambridge, 2000.