Modified general relativity and the Klein-Gordon equation in curved spacetime

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Abstract

From the existence of a line element field $(A^\beta, -A^\beta)$ on a four-dimensional time oriented Lorentzian manifold with metric, the Klein-Gordon equation in curved spacetime, $\nabla_\mu \nabla^\mu \Psi = k^2 \Psi$, can be constructed from one of the pair of regular vectors in the line element field, its covariant derivative and associated spinor-tensor; and scalar product for spins 1,1/2 and 0, respectively. The left side of the asymmetric wave equation can then be symmetrized. The symmetric part, $\tilde{\Psi}_{\alpha\beta}$, is the Lie derivative of the metric, which links the Klein-Gordon equation to modified general relativity for spins 1,1/2 and 0. Modified general relativity is intrinsically hidden in the Klein-Gordon equation for spins 2 and 3/2. Massless gravitons do not exist as force mediators of gravity in a four-dimensional time oriented Lorentzian spacetime. The diffeomorphism group Diff(M) is not restricted to the Lorentz group. $\tilde{\Psi}_{\alpha\beta}$ can instantaneously transmit information to, and quantum properties from, its antisymmetric partner $K_{\alpha\beta}$ along A^β. This establishes the concept of entanglement.

Keywords
quantum mechanics; Klein-Gordon; general relativity; gravitational energy-momentum; quantum field theory; quantum gravity

1. Introduction

It has been nearly a century since Schrödinger [1] wrote down his equation describing non-relativistic quantum mechanics. In the same year of 1926, Klein [2], Gordon [3] and Fock [4] developed the relativistic quantum mechanical wave equation; mainly referred to as the Klein-Gordon (KG) equation. Over a decade before that, Einstein [5] formulated general relativity (GR) in 1915. And yet today, there is still not a full understanding of the relationship between the two fundamental theories of physics: quantum theory and general relativity. The quantization of gravity has been the major approach to unite the two theories, with string theory and loop quantum gravity the two mainstream proposals. However, those and other theories of quantum gravity have well documented successes and failures [6, 7]. The opposite approach of gravitizing quantum mechanics attempts to bring quantum theory in line with the principles of general relativity as discussed in [8] (and other references therein). Rather than trying to force quantum theory on general relativity, or vice versa, this article investigates if a connection between quantum theory and general relativity exists naturally.

The Klein-Gordon equation for a free field with a particular spin in Minkowski spacetime is fundamental to the formulation of quantum field theory (QFT). In curved spacetime, the covariant KG equation $\nabla_\mu \nabla^\mu \Psi = k^2 \Psi$ is assumed to be the rudimentary equation for the development of quantum theory. This is an asymmetric wave equation with the field Ψ considered to represent spins 0,1,2,1/2 and 3/2.

From the existence in curved spacetime of a smooth, real, non-vanishing vector field A^β, the KG equation can be constructed from it, its covariant derivative $\Psi_{\alpha\beta} = \nabla_\alpha A_\beta$ and related spinor-tensor;

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and its scalar product $\varphi = A_\beta A^\beta$ for spins $1, 1/2$ and 0, respectively. In that sense, A^β plays the role of a fundamental quantum vector field. The left side of the KG wave equation can then be symmetrized. As GR involves symmetric tensors, the only possibility to associate GR directly with quantum theory in curved spacetime is through the symmetric part of the KG equation. In that regard, it is noteworthy that the Lie derivative of the metric with respect to A^β *is* the symmetric part of the KG equation, $\tilde{\Psi}_{\alpha\beta}$, for spins $1, 1/2$ and 0. This immediately exhibits a geometrical property of quantum theory which has been neglected.

Einstein developed GR in a four-dimensional Riemannian spacetime. Recently, a modified Einstein equation of general relativity was developed in the article: Modified general relativity [9]. Modified general relativity (MGR) is the natural extension of GR to a four-dimensional time oriented Lorentzian spacetime. On a Lorentzian manifold satisfying certain conditions, a smooth regular line element field $(A^\beta, -A^\beta)$ exists from which a symmetric tensor is introduced in MGR: $\Phi_{\alpha\beta}$. It describes the energy-momentum of the gravitational field itself and completes GR. $\Phi_{\alpha\beta}$ is constructed from the Lie derivative of both the metric and the unit vectors collinear with one of the pair of regular vectors in the line element field. Hence, the Lie derivative of the metric is involved with both $\tilde{\Psi}_{\alpha\beta}$ and $\Phi_{\alpha\beta}$ for spins $1, 1/2$ and 0; and it will be established that GR resides intrinsically in the symmetric part of the KG equation for spins 2 and 3/2. Thus, MGR is linked to quantum theory naturally as discussed in detail in section 2.

In section 3, some interesting results appear from the study of the spin-2 KG equation. Gravity, described by the metric, is a long-range *effective* force. If gravitons are the exchange particles of gravity, they must be massless. In a spacetime described by a four-dimensional time oriented Lorentzian manifold with a torsionless and metric compatible connection, it is shown that massless gravitons governed by the spin-2 KG equation cannot be described with the metric; and massless spin-2 "particles" do not couple to a non-zero energy-momentum tensor. Massless gravitons therefore do not act as force mediators of gravity. This result should not be viewed controversially. It is well known that GR, as a classical field theory, does not require particle exchange to describe the effective force of gravity; that is nicely done by the curvature of spacetime. Furthermore, there is nothing in the formalism of QFT that requires GR to be quantized. That was noted by Feynman who said: [10] "It is still possible that quantum theory does not absolutely guarantee that gravity *has* to be quantized". Gravity, unlike the other three known forces in nature, does not require the exchange of particles to describe its long-range force behavior. This explains why gravity is so much weaker than the long-range electromagnetic force which involves the photon as the exchange particle.

The wave-particle duality and quantum entanglement are discussed in section 4. The KG equations for spin-1 bosons and spin-1/2 fermions must contain the symmetric tensor $\tilde{\Psi}_{\alpha\beta}$ in addition to the traditional antisymmetric tensor $K_{\alpha\beta}$. These particles move as a wave at all spacetime coordinates and are guided by local changes in the gravitational field. This property is similar in concept to the pilot wave of the de Broglie-Bohm theory [11, 12]. But it does not involve the mysterious guiding equation which describes the pilot wave. The Lie derivative of the metric is constructed from the diffeomorphism group Diff(M) which is not restricted to the Lorentz group. It is possible to transmit information instantaneously between $\tilde{\Psi}_{\alpha\beta}$ and $K_{\alpha\beta}$ along the quantum vector, which establishes the concept of entanglement.

2. Constructing the Klein-Gordon equation from the quantum vector A^β in curved spacetime

Curved spacetime is described by the four-dimensional time oriented Lorentzian manifold with a +2 signature metric, $(M, g_{\alpha\beta})$. The connection on the manifold is torsionless and metric compatible. The Lorentzian manifold is assumed to be compact with a vanishing Euler-Poincaré characteristic. It admits a smooth regular line element field $(\boldsymbol{A}, -\boldsymbol{A})$.

In curved spacetime, the KG equation is fundamentally assumed to be

$$\nabla_\mu \nabla^\mu \Psi = k^2 \Psi \tag{1}$$

where $k = \frac{m_0 c}{\hbar}$ and m_0 is the rest mass attributed to each particle of a given spin. This is an asymmetric wave equation with the field Ψ considered to represent spins 0,1,2,1/2 and 3/2. More specifically in the language of QFT, Ψ is a function of spacetime and an operator in a linear vector space with particular algebraic properties. This article is concerned with the first property of Ψ in curved spacetime and its intrinsic relationship to MGR.

The real quantum vector field A^β is related to (1) for spins 1,1/2 and 0 as shown in the subsections below. This is accomplished by using the (0,2) tensor

$$\Psi_{\alpha\beta} := \nabla_\alpha A_\beta \tag{2}$$

and symmetrizing it according to

$$\begin{aligned}\Psi_{\alpha\beta} &= \frac{1}{2}(\nabla_\alpha A_\beta + \nabla_\beta A_\alpha) + \frac{1}{2}(\nabla_\alpha A_\beta - \nabla_\beta A_\alpha) \\ &:= \frac{1}{2}\tilde{\Psi}_{\alpha\beta} + \frac{1}{2}K_{\alpha\beta}.\end{aligned} \tag{3}$$

While the composition of the KG equation in terms of A^β for these spins seems straightforward, there are some subtle points to discuss.

Firstly, the symmetric tensor $\tilde{\Psi}_{\alpha\beta}$ is the Lie derivative of the metric along A^β: $\pounds_A g_{\alpha\beta} = \nabla_\alpha A_\beta + \nabla_\beta A_\alpha$. This directly connects MGR

$$\frac{8\pi G}{c^4}\tilde{T}_{\alpha\beta} = G_{\alpha\beta} + \Phi_{\alpha\beta} \tag{4}$$

to quantum theory for spins 1,1/2 and 0 by the relation

$$\Phi_{\alpha\beta} = \frac{1}{2}\tilde{\Psi}_{\alpha\beta} + u^\lambda(u_\alpha \nabla_\beta A_\lambda + u_\beta \nabla_\alpha A_\lambda) \tag{5}$$

where u^β is a timelike unit vector collinear with A^β, $\Phi_{\alpha\beta}$ is the energy-momentum tensor of the gravitational field, $\tilde{T}_{\alpha\beta}$ is the matter energy-momentum tensor and $G_{\alpha\beta}$ is the Einstein tensor.

Secondly, the Lorentz constraint

$$\nabla_\alpha A^\alpha = 0 \tag{6}$$

cannot be invoked for any spin because it would force both $\tilde{\Psi}_{\alpha\beta}$ and $\Phi_{\alpha\beta}$ to vanish. $\tilde{\Psi}_{\alpha\beta}$ cannot vanish because even weak gravitational fields gravitate; the gravitational field along A^β is not constant so $\pounds_A g_{\alpha\beta} \neq 0$. From (5), Φ, the trace of the energy-momentum of the gravitational field with respect to the metric, is given by

$$\Phi = \nabla_\alpha A^\alpha \tag{7}$$

in an affine parameterization where the geodesic term $2u^\lambda u^\alpha \nabla_\alpha X_\lambda$ vanishes.

Thirdly, the additional divergenceless constraint

$$\nabla_\alpha \tilde{\Psi}^{\alpha\beta} = 0 \tag{8}$$

cannot apply to the spin-1 and spin-1/2 fields which is shown in the following discussion.

2.1. Spin-1 Klein-Gordon equation

If Ψ is the real vector field A^β, (1) yields the spin-1 equation

$$\nabla_\alpha \nabla^\alpha A^\beta = k^2 A^\beta \tag{9}$$

or its equivalent from (3)

$$\nabla_\alpha(\tilde{\Psi}^{\alpha\beta} + K^{\alpha\beta}) = 2k^2 A^\beta. \tag{10}$$

If $\tilde{\Psi}^{\alpha\beta}$ is divergenceless, then (10) generates the Lorentz constraint (6)

$$\nabla_\beta \nabla_\alpha K^{\alpha\beta} = -\frac{1}{2}[\nabla_\alpha, \nabla_\beta]K^{\alpha\beta} = -R_{\alpha\beta}K^{\alpha\beta} = 2k^2 \nabla_\beta A^\beta = 0 \tag{11}$$

because $R_{\alpha\beta}$ and $K^{\alpha\beta}$ have opposite symmetries. The Lorentz constraint forces $\tilde{\Psi}_{\alpha\beta}$ to vanish because $g^{\alpha\beta}$ is non-degenerate. Thus, (8) cannot hold for the spin-1 field. The spin-1 KG equation in curved spacetime contains *both* symmetric and antisymmetric components. The relationship of $\tilde{\Psi}_{\alpha\beta}$ to the energy-momentum tensor of the gravitational field by (5), does not permit a pure spin-1 vector field. $\Phi = \nabla_\alpha A^\alpha \neq 0$ is an intrinsic part of the spin-1 field. This is contrary to the traditional belief in curved spacetime [13, 14], that the antisymmetric tensor $K^{\alpha\beta}$ in the Proca equation

$$\nabla_\alpha K^{\alpha\beta} = k^2 A^\beta \tag{12}$$

with the Lorentz constraint, completely describe a neutral spin-1 boson for $k \neq 0$, and the photon for $k = 0$. Furthermore, this form of the Proca equation in curved spacetime forces the spin-1 wave equation to be expressed as $\nabla_\alpha \nabla^\alpha A^\beta = k^2 A^\beta + \nabla_\alpha \nabla^\beta A^\alpha$ which violates the fundamental assumption that the spin-1 KG equation shall be of the form (9). This problem is automatically resolved with the inclusion of the symmetric part of the KG equation according to (10).

Symmetrization of the asymmetric tensor $\Psi_{\alpha\beta}$ into the Lie derivative of the metric and the Faraday tensor for electrodynamics when $k = 0$, is somewhat similar to the approach that Einstein presented [15, 16, 17] to unify gravity and the electromagnetic field. He generalized the Riemannian metric as an asymmetric tensor with the symmetric part representing the gravitational field in a Riemannian spacetime, and the antisymmetric component with six remaining degrees of freedom describing the electromagnetic field. However, one major problem with this theory was that it did not encompass quantum theory; whereas $\Psi_{\alpha\beta}$ comes from quantum theory and while $\tilde{\Psi}_{\alpha\beta}$ is not the metric, it is the flow of the metric along the quantum vector.

2.2. Spin-1/2 Klein-Gordon equation

The spin-1/2 KG equation in curved spacetime expressed in terms of its spinor indices is

$$\nabla_\alpha \nabla^\alpha \Psi^{A\dot{A}} = k^2 \Psi^{A\dot{A}}. \tag{13}$$

It is well known [18, 19] that the two index spinors $\varphi^{A\dot{B}}$ and $\varphi_{A\dot{B}}$ can be expressed in terms of the associated tensors A^β and A_β as

$$\varphi^{A\dot{B}} = \sigma_\beta^{A\dot{B}} A^\beta \tag{14}$$

and

$$\varphi_{A\dot{B}} = \sigma^\beta_{A\dot{B}} A_\beta. \tag{15}$$

The Hermitian connecting quantities $\sigma_\beta^{A\dot{B}}$ transform as a spacetime vector on the index β and as spinors on the index $A = 1, 2$ and conjugate index $\dot{B} = 1, 2$. Covariant derivatives of spinors are introduced in the same formalism as that for tensors by adopting the spinor affinities $\Gamma^A_{\alpha B}$ and defining

$$\nabla_\alpha \Psi_A = \partial_\alpha \Psi_A - \Gamma^B_{\alpha A} \Psi_B, \;\; \nabla_\alpha \Psi^A = \partial_\alpha \Psi^A + \Gamma^A_{\alpha B} \Psi^B \tag{16}$$

for the spinors Ψ_A and Ψ^A respectively. The covariant derivative of a mixed index spinor-tensor is defined as

$$\nabla_\alpha \Psi^{\beta A} = \partial_\alpha \Psi^{\beta A} + \Gamma^\beta_{\alpha\kappa} \Psi^{\kappa A} + \Gamma^A_{\alpha B} \Psi^{\beta B} \tag{17}$$

and the covariant derivative of the connection quantities is postulated to vanish

$$\nabla_\kappa \sigma^\alpha_{A\dot{B}} = 0. \tag{18}$$

Equation (13) is then equivalent to

$$\sigma_\beta^{A\dot{A}} \nabla_\alpha \nabla^\alpha A^\beta = k^2 \Psi^{A\dot{A}} \tag{19}$$

using (14). This can be rewritten in terms of $\Psi^{\alpha\beta} = \nabla^\alpha A^\beta$ and symmetrized as in (3) to give

$$\sigma_\beta^{A\dot{A}} \nabla_\alpha (\tilde{\Psi}^{\alpha\beta} + K^{\alpha\beta}) = 2k^2 \sigma_\beta^{A\dot{A}} A^\beta. \tag{20}$$

Similar to the spin-1 situation, the divergenceless condition (8) cannot be employed because it would require both $\tilde{\Psi}_{\alpha\beta}$ and $\Phi_{\alpha\beta}$ to vanish. The spin-1/2 KG equation contains *both* symmetric and antisymmetric components.

Φ is an intrinsic part of the spin-1/2 field. Φ provides the association with gravity that is traditionally attributed to the Ricci scalar R. An adjustment to the Dirac equations in curved spacetime to remove their dependence on R is required. This is accomplished by requiring the Dirac equations to be solutions of their parent KG equation.

Remark 1. *The Dirac equations in curved spacetime must be modified to be solutions of their parent spin-*1/2 *KG equation.*

The Dirac equations in curved spacetime

$$(\gamma^\nu \nabla_\nu - k)\Psi^A = 0, \;\; (\gamma^\nu \nabla_\nu + k)\Psi^{\dot{A}} = 0 \tag{21}$$

are taken to be factorizations of the spin-1/2 KG equation. The gamma matrices γ^μ in curved spacetime are assumed to satisfy the anticommutation relation

$$\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}. \tag{22}$$

The product of the Dirac factorizations

$$(\gamma^\mu \nabla_\mu + k)(\gamma^\nu \nabla_\nu - k)\mathit{\Psi}^{A\dot{A}} = 0 \tag{23}$$

must yield the spin-1/2 KG equation. Using

$$\nabla_\mu \gamma^\nu = 0 \tag{24}$$

we have

$$\gamma^\mu \gamma^\nu \nabla_\mu \nabla_\nu \mathit{\Psi}^{A\dot{A}} = k^2 \mathit{\Psi}^{A\dot{A}} \tag{25}$$

which is expressed in the literature [20, 21] (with the metric having a +2 signature) as

$$\nabla_\mu \nabla^\mu \mathit{\Psi}^{A\dot{A}} = (k^2 + \frac{1}{4}R)\mathit{\Psi}^{A\dot{A}}. \tag{26}$$

With only the algebra of (22), the spin-1/2 KG equation in curved spacetime is not precisely recoverable due to the additional $\frac{1}{4}R$ term. This term can be eliminated by inserting the scalar $\Omega = \frac{1}{2}\sqrt{R}$ into (21) to obtain the modified Dirac equations

$$(\gamma^\nu \nabla_\nu + \Omega - k)\Psi^A = 0, \;\; (\gamma^\nu \nabla_\nu + \Omega + k)\Psi^{\dot{A}} = 0. \tag{27}$$

By defining the algebras

$$\{\gamma^\mu \nabla_\mu, \Omega\} = 0 \tag{28}$$

and

$$\{\Omega, \Omega\} = \frac{R}{2}, \tag{29}$$

the product of the modified Dirac equations in curved spacetime yields their parent spin-1/2 KG equation (13).

2.3. Spin-0 Klein-Gordon equation

When Ψ is a scalar field φ, the spin-0 Klein-Gordon equation is

$$\nabla_\alpha \nabla^\alpha \varphi = k^2 \varphi. \tag{30}$$

With φ defined in terms of the quantum vector field A^β as

$$\varphi := A^\beta A_\beta, \tag{31}$$

and using (3) and (8), we have

$$\begin{aligned} \nabla_\alpha \nabla^\alpha \varphi &= 2\nabla_\alpha (A_\beta \mathit{\Psi}^{\alpha\beta}) \\ &= A_\beta \nabla_\alpha K^{\alpha\beta} + \frac{1}{2}(\tilde{\mathit{\Psi}}_{\alpha\beta}\tilde{\mathit{\Psi}}^{\alpha\beta} + K_{\alpha\beta}K^{\alpha\beta}) \end{aligned} \tag{32}$$

where $\tilde{\mathit{\Psi}}^{\alpha\beta} K_{\alpha\beta} = 0$ because the tensors in the product have opposite symmetries.

2.4. Spin-2 and spin-3/2 Klein-Gordon equations

The KG equation for the symmetric spin-2 field is

$$\nabla_\mu \nabla^\mu \tilde{\Psi}_{\alpha\beta} = k^2 \tilde{\Psi}_{\alpha\beta}. \tag{33}$$

$\tilde{\Psi}_{\alpha\beta}$ is constructed from the symmetrization of the general (0,2) tensor field $\Psi_{\alpha\beta}$ according to

$$\begin{aligned} \Psi_{\alpha\beta} &= \frac{1}{2}(\Psi_{\alpha\beta} + \Psi_{\beta\alpha}) + \frac{1}{2}(\Psi_{\alpha\beta} - \Psi_{\beta\alpha}) \\ &:= \frac{1}{2}\tilde{\Psi}_{\alpha\beta} + \frac{1}{2}C_{\alpha\beta}. \end{aligned} \tag{34}$$

Gravitons are the particles associated with the metric. The symmetric spin-2 field must therefore involve the metric as a field variable to enable gravitons to be described by the spin-2 KG equation. The spin-2 field must be divergenceless and traceless with respect to the metric; it has 5 degrees of freedom if $k \neq 0$. $\tilde{\Psi}_{\alpha\beta}$ cannot be equivalent to the Lie derivative of the metric because the traceless attribute of the spin-2 field would lead to the trivial solution of $\tilde{\Psi}_{\alpha\beta}$.

An expression for $\tilde{\Psi}_{\alpha\beta}$ can be obtained by generalizing the results from [9] where the divergenceless collection of tensors in the Orthogonal Decomposition Theorem can be defined to consist of the Lovelock tensors, namely the metric and the Einstein tensor in a four-dimensional spacetime; and a collection of non-Lovelock tensors represented by $h_{\alpha\beta}$, which are independent of both $g_{\alpha\beta}$ and $G_{\alpha\beta}$. $h_{\alpha\beta}$ has dimensions of L^{-2} which eliminates the super-energy divergenceless tensors, such as the traces of the Chevreton and Bach tensors which depend on L^{-4}. With $\tilde{\Psi}_{\alpha\beta}$ defined to be a linear combination of symmetric tensors consisting of the matter energy-momentum tensor, the energy-momentum tensor of the gravitational field, the Lovelock tensors and the non-Lovelock tensors with $\Lambda = 0$ as discussed in [9]:

$$\tilde{\Psi}_{\alpha\beta} = \frac{a}{c}\tilde{T}_{\alpha\beta} + b(G_{\alpha\beta} + \mathit{\Phi}_{\alpha\beta} + h_{\alpha\beta}) \tag{35}$$

where a and b are arbitrary constants. By requiring both $\tilde{\Psi}_{\alpha\beta}$ and $h_{\alpha\beta}$ to be traceless, equation (4) is recovered for a non-degenerate inverse metric by setting $a = -\frac{1}{2}$ and $b = \frac{c^3}{16\pi G}$. It follows that

$$\tilde{\Psi}_{\alpha\beta} = \frac{c^3}{16\pi G} h_{\alpha\beta} \tag{36}$$

where $h_{\alpha\beta}$ is a collection of tensors that are divergenceless, traceless and not a concomitant of the metric and its first two derivatives. Thus, MGR is hidden in the spin-2 field $\tilde{\Psi}_{\alpha\beta}$.

Spin-3/2. A spin-3/2 field can be described by the vector-spinor wave equation

$$\nabla_\mu \nabla^\mu \Psi_{\alpha A\dot{A}} = k^2 \Psi_{\alpha A\dot{A}}. \tag{37}$$

Using (15), this equation can be symmetrized and written as

$$\sigma^\beta_{A\dot{A}}(\nabla_\mu \nabla^\mu - k^2)\tilde{\Psi}_{\alpha\beta} = 0. \tag{38}$$

MGR is hidden in the spin-3/2 field.

Some consequences of these results are now discussed.

3. Massless spin-2 "particles"

Gravitons are taken to be massless particles because of the $\frac{1}{r^2}$ long-range effective force behaviour of gravity. They have spin-2 so that they can couple to the energy-momentum tensor. However, in a four-dimensional spacetime with a metric compatible connection, when the mass vanishes,

$$\nabla_\mu \nabla^\mu h_{\alpha\beta} = 0 \tag{39}$$

is the equation describing a massless spin-2 "particle". Because $h_{\alpha\beta}$ is independent of the metric, this equation cannot describe a massless spin-2 graviton.

Furthermore, massless spin-2 "particles" cannot couple to a non-zero energy-momentum tensor as force mediators for gravity. If we calculate the interaction of the total matter energy-momentum tensor with $h_{\alpha\beta}$, we obtain

$$\begin{aligned} S_h^{int} &= -\frac{1}{2c}\int \tilde{T}^{\alpha\beta} h_{\alpha\beta}\sqrt{-g}d^4x \\ &= -\frac{c^3}{16\pi G}\int G^{\alpha\beta} h_{\alpha\beta}\sqrt{-g}d^4x \end{aligned} \tag{40}$$

since $\int \Phi^{\alpha\beta} h_{\alpha\beta}\sqrt{-g}d^4x = 0$ because $h_{\alpha\beta}$ is divergenceless and $\nabla_\mu(u^\lambda u^\alpha) = 0$. The variation of the functional S_h^{int} with respect to $h_{\alpha\beta}$ must vanish. This requires $G^{\alpha\beta}$ to vanish because by definition, $h_{\alpha\beta}$ is independent of both the metric and the Einstein tensor. $\Phi^{\alpha\beta}$ is orthogonal to and independent of $h_{\alpha\beta}$. Thus, $\int \Phi^{\alpha\beta}\delta h_{\alpha\beta}\sqrt{-g}d^4x = 0$ and $\Phi^{\alpha\beta}$ vanishes. It follows that $\tilde{T}^{\alpha\beta}$ vanishes and there is no coupling to the matter energy-momentum tensor. Massless spin-2 "particles" do not couple to any types of matter but can occupy the vacuum in accordance with $R^{\alpha\beta} = 0$.

The hierarchy problem

The hierarchy problem of particle physics can be stated as the question: why is the force of gravity so much weaker than the other three known forces in nature? In the case of electrodynamics, if both gravity and electrodynamics have long-range massless force mediators, why is the electromagnetic force 10^{40} times stronger than that of gravity? The electroweak force is 10^{24} times stronger than gravity. And as the name suggests, the strong nuclear force presents the largest disparity to gravity at nuclear dimensions.

At the basis of this problem is the notion that gravity *must* be quantized. However, the symmetric spin-2 KG equation in a 4-dimensional Lorentzian spacetime with a torsionless and metric compatible connection, excludes massless gravitons as force mediators of gravity. This starkly contrasts with the spin-1 KG equation for a massless photon which mediates the electromagnetic field; similarly for the electroweak force and the massive spin-1 W and Z bosons, and the spin-1 massless gluons mediating the strong nuclear force. Gravity has no massless particles that act as force mediators. Thus, the hierarchy problem is explained without the need of extra spatial dimensions inherent in string theory; or any other theory that involves massless gravitons in a Lorentzian spacetime with a metric compatible connection. Of course, this result depends on the assumption that the spin-2 KG equation completely describes spin-2 particles in curved spacetime.

4. Wave-particle duality and quantum entanglement

It has been established that neutral spin-1 bosons and spin-1/2 fermions are described by (10) and (20), respectively. These equations and the spin-0 bosons (30) must have antisymmetric and symmetric components. The antisymmetric tensor $K_{\alpha\beta}$ satisfies

$$\nabla_\mu K_{\alpha\beta} + \nabla_\beta K_{\mu\alpha} + \nabla_\alpha K_{\beta\mu} = 0 \tag{41}$$

from which the wave equation

$$\nabla_\mu \nabla^\mu K_{\alpha\beta} = -2k^2 K_{\alpha\beta} - 2K^\mu{}_{[\alpha} R_{\beta]\mu} - 2K^{\mu\sigma} R_{\mu\alpha\sigma\beta} - 2\nabla_{[\alpha}\nabla^\mu \tilde{\Psi}_{\beta]\mu} \tag{42}$$

is obtained using (10). This establishes the wave nature of these particles. Each particle, or quantum corpuscle in the case of a light quantum which has no rest mass, moves as a wave at all spacetime coordinates in the microworld.

The symmetric tensor $\tilde{\Psi}_{\alpha\beta}$ is the Lie derivative of the metric. Given a diffeomorphism $\phi : M \longrightarrow M$, $\tilde{\Psi}_{\alpha\beta}$ is constructed from the pullback ϕ_{t*} of the metric under the diffeomorphism group, Diff(M). The Lorentz group is a subgroup of Diff(M) so the pullback of the metric is not restricted to the Lorentz group.

The metric at a point on the regular vector A^β, far from a given point p on that vector, can be pulled back instantaneously to the neighbourhood of p; or pushed forward from p with $(\phi_t^{-1})^*$. The metric describes the gravitational field and the geometry of spacetime. Hence, the rate of change of the gravitational field along A^β is not limited to the speed of light; and could be instantaneous with the caveat that there may be a presently unknown upper bound to the speed that spacetime itself can transfer information. This does not conflict with the Lorentzian behaviour of gravitational waves, which carry energy; information is not energy and has no mass equivalent. $\tilde{\Psi}_{\alpha\beta}$ can instantaneously transmit information to, and quantum properties from, its antisymmetric partner $K_{\alpha\beta}$ along A^β. This establishes the global nature of quantum theory and the concept of entanglement.

The last term in (42) is constructed from the flow of the gravitational field along the quantum vector. This term is sensitive to changes in the local gravitational field. In that sense, in the microworld, it can guide the wave in response to local changes due to gravity. In the classic dual slit experiments, this term would sense the presence of a detector near a particular slit. It would guide the particle to the slit which has no impediments, as nature seems to prefer paths of least resistance. No interference pattern would be observed. The gravitationally guided wave is similar in concept to the pilot wave of the de Broglie-Bohm theory [11, 12]. But it does not suffer from the mysterious quantum potential [22] integral to pilot wave theory.

From (4) and (5)

$$\frac{1}{2}\tilde{\Psi}_{\alpha\beta} = \frac{8\pi G}{c^4}\tilde{T}_{\alpha\beta} - G_{\alpha\beta} - u^\lambda(u_\alpha \nabla_\beta A_\lambda + u_\beta \nabla_\alpha A_\lambda). \tag{43}$$

Equations (42) and (43) are deterministic. In the microworld, particles in a gravitational field move as a wave according to these equations. But the Lie derivative of the metric is not constrained by any scale of measurement. Quantum theory and MGR are therefore linked together in the micro and macro worlds, although one theory or the other may be more dominant at a particular scale of measurement. In the macroworld, our de Broglie wave lengths are so small they are essentially undetectable.

When we undertake a measurement on a particle in the microworld by bombarding it with light quanta or other particles, chaos or destruction is inherent in the process of the measurement. Despite the fact that the measurement may constrain the wave behaviour of the particle, it does not permanently destroy the wave property of the particle when detected as a particle; and it does not suddenly become a wave when detected as such, because in both cases, the particle behaves as a wave at all spacetime coordinates according to (42). The experiment can be designed to detect a wave or a particle, but the reality of the microworld is not determined by the experiment; reality exists in the quantum-world before and after a measurement is performed on an entity in it. This is contrary to the Copenhagen interpretation [23](and references therein) of quantum theory where the reality of the microworld is declared after a measurement is performed on a particle in it. The statistical rules of quantum theory are then imposed to interpret the results of the measurement.

The complementarity principle [24] provides entities in the microworld with both wave and particle characteristics. But any given experiment is designed to detect a wave *or* a particle, but not both due to the Heisenberg uncertainty principle. The wave-particle duality does not challenge the reality of the microworld. Rather, the duality of the microworld melds into the macroworld by allowing the experimenter to determine the wave or particle characteristics as designed in the experiment. That a particle always has wave characteristics is supported by a recent experiment which has observed the simultaneous behaviour of light acting as both a wave and a stream of particles [25].

5. Conclusion

The results in this article are obtained directly from the rudimentary Klein-Gordon equation of relativistic quantum mechanics in curved spacetime, and modified general relativity. Gravity is not quantized and quantum theory is not geometrized. Rather, the Lie derivative of the metric provides a natural connection between quantum theory and modified general relativity.

The KG equation for spins $1, 1/2$ and 0 is constructed from a smooth, real, regular quantum vector field A^β. Its symmetric part, $\tilde{\Psi}_{\alpha\beta}$, is the Lie derivative of the metric along the quantum vector. Because $\tilde{\Psi}_{\alpha\beta}$ cannot vanish, it provides the missing link to gravity for these spins.

A spin-2 decomposition for $\tilde{\Psi}_{\alpha\beta}$ is obtained in terms of a collection of tensor fields independent of the Lovelock tensors. It follows that MGR is intrinsically contained in $\tilde{\Psi}_{\alpha\beta}$ for spins 2 and 3/2. The metric does not appear as a field variable in the spin-2 KG equation. Massless spin-2 "particles" do not couple to a non-zero energy-momentum tensor as force mediators for gravity, but can occupy the vacuum. Thus, massless gravitons in a time oriented four-dimensional spacetime do not exist. Unlike the other three known fundamental forces in nature, no particle exchange is required to explain the force of gravity; that is nicely done by the curvature of spacetime. That massless gravitons do not exist explains the hierarchy problem of particle physics.

$\tilde{\Psi}_{\alpha\beta}$ links the deterministic microworld to the macroworld. Spin-1 bosons and spin-1/2 fermions move as a wave at all spacetime coordinates in the microworld, which exists before and after a measurement.

The diffeomorphism group Diff(M) is not restricted to the Lorentz group. $\tilde{\Psi}_{\alpha\beta}$ can instantaneously transmit information to, and quantum properties from, its antisymmetric partner $K_{\alpha\beta}$ along A^{β}. This establishes the nonlocal behaviour of quantum theory and the concept of entanglement.

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