Autopilot to maintain movement of a drone in a vertical plane at a constant height in the presence of vision-based navigation

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Abstract

In this report we describe correct operation of autopilot for supply correct drone flight. There exists noticeable delay in getting information about position and orientation of a drone to autopilot in the presence of vision-based navigation. In spite of this fact, we demonstrate that it is possible to provide stable flight at a constant height in a vertical plane. We describe how to form relevant controlling signal for autopilot in the case of the navigation information delay and provide control parameters for particular case of flight.

1. **Introduction**

This paper is engineering application of stability theory for differential equations with delays described in [1-5]. In this paper we describe correct operation of autopilot for supply desirable drone flight (movement of a drone in a vertical plane at a constant height). For the finding drone flight parameters was used vision-based navigation [6-15]. For realization vision-based navigation was developed the computer program “Video-navigation of UAV over relief” [6]. This program was tested in Zhejiang Province in east China near the capital Hangzhou using Google Earth data [12]. There always exists noticeable delay in getting information about position and orientation of a drone to autopilot for vision-based navigation because of computer processing image’s big data. In spite of this fact, we demonstrate that it is possible to provide stable flight at a constant height in a vertical plane. We want to describe how to form relevant controlling signal for autopilot in the case of the navigation information delay. For this purpose we use theory of stability for differential equations with delays described in [1-5]. We plan to use the autopilot described in the paper for controlling flight parameters found from vision-based navigation.

Although there is a resistance of engineers to the use of theoretical results on stability of differential equations with delays, the theory of these equations develops intensively. Every year hundreds of papers on stability analysis of delay equations are published.

Let us consider two examples.

Example 1: These two very similar equations arise directly from the Newton second low: $y''(t) = 0$ and $x''(t) = \varepsilon$. Assume that ε is very small and consider the same initial conditions at the point 0: $y(0)=0, y'(0)=0$ and $x(0)=0, x'(0)=0$. Their solutions are $y(t)\equiv 0$ and $x(t)=\frac{\varepsilon}{2}t^2$ respectively. It is clear that $\lim_{t\to\infty} x(t)-y(t)=\infty$, and there is no stability with respect to right-hand side. Conclusion: we need a feedback control to stabilize the equation $x''(t)=f(t)$ with respect to a right-hand side. In this way we come to stability analysis of the delay equation $x''(t)=-\sum_{i=1}^{m} a_i x(t-\tau_i(t))$. $\tau_i(t)>0$ appeared as a result of information, operation or transport delays existing in all real technological processes.

Example 2. There exists a delusion that instead of stability analysis of the delay equation one can use elements of the modern technology, for example, GPS gives us the values of $x(t_1), x(t_2), x(t_3), \ldots$ with very small time intervals $\Delta t_i = t_{i+1}-t_i$. It can be demonstrated, for example, on the delay equation $x''(t)=-ax([t])$, where $[t]$ is the integer part of t and $a>0$. At the moment t we know almost exactly $x([t])$, but this does not help in stabilization.

Actually, it is known from the paper [16] that all solutions of the equation $x''(t)=-ax(t-\tau(t))$ are bounded if and only if $\int_0^{\infty}\tau(t)dt<\infty$. For our equation $x''(t)=-ax([t])$, we have $\tau(t)=t-[t]$ and $\int_0^{\infty}\tau(t)dt=\infty$. This means that there exist unbounded solutions of the equation $x''(t)=-ax([t])$, and this equation is unstable. The direct use of GPS without theoretical basis could not achieve stabilization even the signals from GPS come with very small time intervals.

Results on exponential stability, i.e. all solutions of the homogeneous equation $x''(t)=-\sum_{i=1}^{m} a_i x(t-\tau_i(t))$ tend to zero like $\exp(-\alpha t)$ with positive α, were obtained for the case $m\geq 2$ under corresponding conditions on the coefficients and delays in the form of inequalities in the paper [17].

Stability analysis presents one of the necessary parts in the almost all papers on robotics. Their authors avoid to consider the delay in their models although they accept fact of arising transport, information or executive delay in robotics models. They use the technique of Lyapunov's functions which has the long history (starting with works of N.Krasovskii in 1950s) but is not convenient in many cases for stabilization by delay feedback control.

The current basic engineering method for analyzing a delayed system is replacing the system with delays to the system without delay and using the classical theory of stability (characteristic equations in linear case, and method of Lyapunov's functions – in nonlinear). It is usually achieved in the frame of the following two ideas or their combinations ([18,19]):

A) to extrapolate a motion forward during the delay time,
B) to take into account the estimate error of a current state and to use all possible values of the process for its future analysis.

The use of method B) results in an obvious decrease in the accuracy of control and its effectiveness.

The use of method A) is possible when the system is sufficiently inert and does not have a strong control effect during the delay time. Even in this case, we need to use a complex algorithm. This results in an increase in the time and cost of creating a control system, the cost of computing power for extrapolation. The simplification of the model leads to a decrease in the accuracy and effectiveness of control. Also, when we change and upgrade the system, this big work needs to be carried out again.

If there is a control effect on the behavior of the system during the delay time, the method makes even more expensive and complex - it requires complex iterative schemes, the iterations do not always converge and require a long calculation time, which may insert an additional time delay. This can lead to a complete loss of controllability of the system. In order not to be unfounded from the mathematical point of view, we added an explanation of the shortcomings of approach A) and the justification for the necessity of using new results on stability theory.

It looks that the use of Azbelev's theory of stability of functional differential equations can open new perspectives in the control in robotics. See the book [21] .
In the book [2], based on Azbelev's theory, we developed the stability analysis and methods of estimates of solutions to systems of delay differential equations.
The use of mathematical theory of stability and control of systems with delays allows:

1) It reduces the time and costs for control development.
2) It is easy to modernize the control of the system if the system has been changed.
3) Methods are universal for a wide class of systems.
4) Due to the high accuracy of mathematical methods, the system will have efficient and precise control.
5) There are no additional delays or control failures for complex cases where the controlled system is not inertial, and the control effect is significant during the delay time.

2. **Stability of systems with time delays**

Throughout the paper e denotes the Euler number. L_∞ is the space of essentially bounded measurable functions: $[0,+\infty) \to R$.
Consider the non-homogenous system of differential equations

$$x_i'(t) - \sum_{j=1}^{n} \sum_{k=1}^{m} a_{ij}^k(t)\, x_j(t - \theta_{ij}^k(t)) = f(t), t \in 0, +\infty) \tag{2.1}$$

$$x(\xi) = 0, \xi < 0, i = 1, \ldots, n,$$

Where $A_k(t) = \{a_{ij}^k(t)\}_{i,j=1,\ldots,n}$ are $n \times n$ matrices with entries $a_{ij}^k(t) \in L_\infty$, $\theta_{ij}^k(t) \in L_\infty$ for $k = 1, \ldots, m$, $f(t) = col\{f_1(t), \ldots, f_n(t)\}$, $f_i \in L_\infty$, for $i = 1, \ldots, n$. The components $x_i\colon [0, +\infty) \to \mathbb{R}$ of the vector $x = col\{x_1, \ldots, x_n\}$ are assumed to be absolutely continuous and their derivatives $x_i' \in L_\infty$. A vector-function x is a solution of (2.1) if it satisfies system (2.1) for almost all $t \in [0, +\infty)$.

Denote

$$\theta_{ii}^+(t) = max_{m \geq k \geq 1}\, \theta_{ii}^k(t)$$

$$\theta_{ii}^{-}(t) = min_{m\geq k\geq 1}\, \theta_{ii}^{k}(t)$$

$$\Delta_i = esssup_{t\geq 0}\{\theta_{ii}^{+}(t) - \theta_{ii}^{-}(t)\}.$$

It was shown in Theorem 3.2 in [1] that:

If the following conditions are fulfilled:

(1.1) There exist positive numbers $z_1, \dots, z_n$ such that $\sum_{k=1}^{m} a_{ij}^{k}(t)\, z_i - \sum_{j=1,j\neq i}^{n}\sum_{k=1}^{m}\left|a_{ij}^{k}(t)\right| z_j \geq 1,\, t \in [0,+\infty),\, i = 1,\dots,n$

(1.2) For every $i = 1, \dots, n$ at least one of the conditions (1.2a) or (1.2b) be fulfilled:

(1.2a) there exists m_i such that $a_{ii}^{k}(t) \geq 0,\ a_{ii}^{j}(t) \leq 0, \theta_{ii}^{k}(t) \geq \theta_{ii}^{j}(t)$ for $k = 1,\dots,m_i,\, j = m_{i+1},\dots,m,\, \sum_{k=1}^{m_i} a_{ii}^{k}(t) \geq \frac{1}{e}\sum_{j=m_i+1}^{m}\left|a_{ii}^{j}(t)\right|$ for $t \in [0,+\infty)$ $\int_{t-\theta_{ii}^{+}(t)}^{t}\left\{\sum_{k=1}^{m_i} a_{ii}^{k}(s) - \frac{1}{e}\sum_{j=m_i+1}^{m}\left|a_{ii}^{j}(s)\right|\right\} ds \leq \frac{1}{e},\, t \in [0,+\infty)$ and

(1.2b) there exists m_i such that $a_{ii}^{k}(t) \geq 0,\ a_{ii}^{j}(t) \leq 0, \theta_{ii}^{k}(t) \leq \theta_{ii}^{j}(t)$ for $k = 1,\dots,m_i,\, j = m_{i+1},\dots,m,\, \sum_{k=1}^{m_i} a_{ii}^{k}(t) \geq \sum_{j=m_i+1}^{m}\left|a_{ii}^{j}(t)\right|$ for $t \in [0,+\infty)$ $\int_{t-\theta_{ii}^{+}(t)}^{t}\left\{\sum_{k=1}^{m_i} a_{ii}^{k}(s) - \sum_{j=m_i+1}^{m}\left|a_{ii}^{j}(s)\right|\right\} ds \leq \frac{1}{e},\, t \in [0,+\infty)$

and

$\int_{s}^{s+\Delta_i}\sum_{k=1}^{m_i} a_{ii}^{k}\,(\xi)d\xi \leq \frac{1}{e}\ \forall s \geq 0$, where $\Delta_i = esssup_{t\geq 0}\{\theta_{ii}^{+}(t) - \theta_{ii}^{-}(t)\}$

Then system (2.1) is exponentially stable.

3. **Parameters of drone's motion**

3.1 Nonlinear equations

Let us define the following variables and parameters used in equations of motion for a drone (see Fig. 1) [22]:

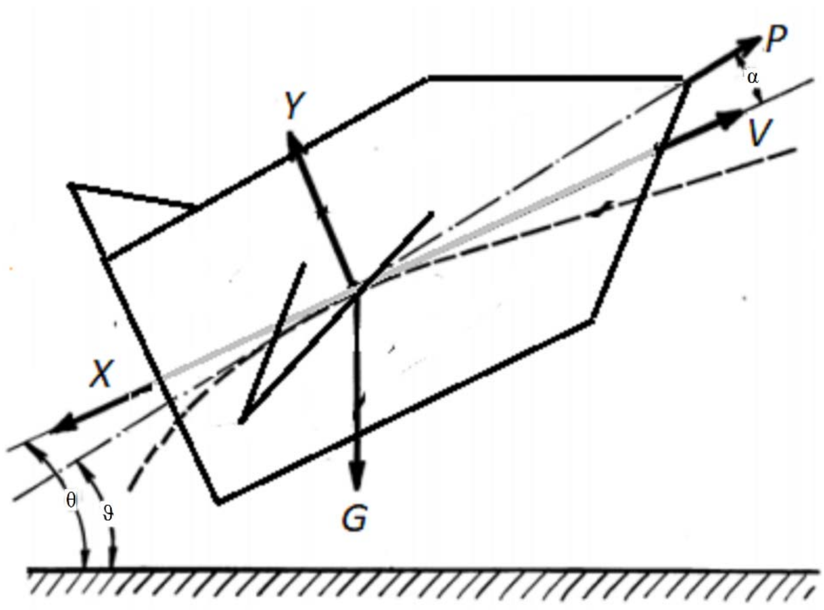

Fig.1 Parameters of drone's longitudinal motion

1) for forces and moments of forces:

P − Tractive force directed along longitudinal drone axis

Y− Carrying force orthogonal to flight velocity

X− Resistance force opposite to V

G − Gravitation force

M_z − Total moment of aerodynamical forces with respect of transversal axis

f_1, f_2, f_3 − Random forces and random moments of forces

2) for variables describing motion:

V− Flight velocity tangent to trajectory (with respect of air)

H – Height above mean sea level of a drone flight

L – Drone path in longitudinal direction

ϑ − Pitch angle, i.e. angle between longitudinal drone axis and horizontal plane

θ − Tilting of velocity about horizontal plane

α − Angle of attack, i.e. angle between longitudinal axis of a drone and projection of drone velocity on the symmetry plane of the drone

3) Drones parameters:

m = G/g − drone mass

J_z − Inertial moment of drone with respect of axis z p - air density

4) Controlling signals:

δ_p– Position of drone central control knob

δ_B– Deviation of drone control elevator

5) External environment parameters:

$U_x$and U_y– wind velocities along axes$x$and$y$, correspondently

It's shown in [22] that forward movement and rotation are described by the system of equations:

$$\begin{cases} m\frac{dV}{dt} = P\big(\delta_p(t), M(V,H)\big)\,cos(\,\alpha) - X(\alpha,V,H) - G\,sin(\,\theta) + f_1(t) \\ mV\frac{d\theta}{dt} = P\big(\delta_p(t), M(V,H)\big)\,sin(\,\alpha) - Y(\alpha,V,H) - G\,cos(\,\theta) - f_2(t) \\ J_z\frac{d^2\vartheta}{dt^2} = M_z\big(\alpha, M(V,H), \dot{\alpha}, \dot{\vartheta}, \delta_B(t)\big) + f_3(t) \\ \frac{dH}{dt} = V\,sin(\,\theta) + U_y(t) \\ \frac{dL}{dt} = V\,cos(\,\theta) + U_x(t) \\ \vartheta = \theta + \alpha \end{cases} \tag{3.1}$$

$P = P(\delta_p, V)$,$X = c_x(\alpha, M)S\frac{\rho(H)V^2}{2}$,$Y = c_y(\alpha, M(V,H))S\frac{\rho(H)V^2}{2}$, $\vartheta = \theta + \alpha$,

$M_z = m_z(\alpha, M(V,H), \dot{\alpha}, \dot{\vartheta}, \delta_B)b_a S\frac{\rho(H)V^2}{2}$, $M(H) \stackrel{\text{def}}{=} \frac{V}{a(h)}$

Here

c_x And c_y- coefficients of resistance and carrying forces, correspondently

m_z - Coefficient of moment

S – Area of winds

b_a - Length of wind chord

$\rho(H)$ - Air density at a flight height

$M = V/a$ - Mach number

a - Sound velocity

$m_z(\alpha, \alpha, \dot{\vartheta}, V, \delta_B, \rho) = m_{1z}(\alpha, M(V,H), \delta_B) + m_{2z}\big(M(V,H), \dot{\alpha}, \dot{\vartheta}\big)$

Aerodynamical damping moment:

$$m_{2z}\big(M(V,H),\dot{\alpha},\dot{\vartheta}\big) = k\left(\frac{L_1}{M(V,H)}\dot{\vartheta} + k'\dot{\alpha}\right)$$

L_1- Distance from tail unit to center of mass

k, k'- Constants

Dependence of parameters on a flight height above mean sea level is defined by the following equations:

$\rho(H) = \rho(0)\left(\frac{T_H(H)}{T(0)}\right)^{\frac{1}{\gamma-1}}$;$a(H) = \sqrt{\gamma R T_H(H)}$; $T_H(H) = T(0) - \beta H$, where

$T_H(H)$ - Temperature at a flight height

$T(0),\ \rho(0)$ - Temperature and air density at mean sea level

γ - Adiabatic constant

R - Gas constant

β - Temperature gradient over height

3.2 Steady state solution

For the constant wind, zero-controlling external small random forces and moments we can find a steady state solution for a drone flight:

$V_0, \theta_0, \alpha_0, \vartheta_0, H_0, (U_x)_0, (U_y)_0$

$U_y(t) = (U_y)_0 + \Delta U_y(t)$

$U_x(t) = (U_x)_0 + \Delta U_x(t)$,

Where $\Delta U_x(t), \Delta U_y(t)$ - small wind fluctuations.

The steady state solution can be obtained from (1) by equating all external small random forces and moments, controlling parameters, wind velocity fluctuations, all derivatives to zero:

$$\begin{cases} G\sin(\theta_0) = P_0\cos(\alpha_0) - (c_x)_0 S\dfrac{\rho_0 V_0^2}{2} \overset{def}{=} -\dfrac{1}{2}(c'_x)_0 S\rho_0 V_0^2 \\ G\cos(\theta_0) = P_0\sin(\alpha_0) + (c_y)_0 S\dfrac{\rho_0 V_0^2}{2} \overset{def}{=} \dfrac{1}{2}(c'_y)_0 S\rho_0 V_0^2 \\ (m_z)_0 = 0 \\ V_0\sin(\theta_0) = -(U_y)_0 \\ L(t) = (V\cos(\theta_0) + (U_x)_0)t + L(0) \\ \theta_0 = \vartheta_0 - \alpha_0 \\ \delta_p(t) = 0;\ \delta_B(t) = 0;\ f_1(t) = 0;\ f_2(t) = 0;\ f_3(t) = 0;\ \Delta U_x(t) = 0;\ \Delta U_y(t) = 0 \end{cases}$$

Here we use the following steady state parameters:

$$(T_H)_0 = T_0 - \beta H_0;\ \ \rho_0 = \rho(0)\left(\frac{T_H(H_0)}{T_0}\right)^{\frac{1}{\gamma-1}};\ \ a_0 = \sqrt{\gamma R(T_H)_0};\ \ M_0 = \frac{v_0}{a_0}$$

$$P_0 = P(0, M_0);\ \ (c_x)_0 = c_x(a_0, M_0);\ \ (c_y)_0 = c_y(a_0, M_0);\ \ (m_z)_0 = m_z(a_0, M_0, 0,0,0)$$

$\left(\frac{\partial c'_x}{\partial M}\right)_0, \left(\frac{\partial c'_y}{\partial M}\right)_0, \left(\frac{\partial c_x}{\partial \alpha}\right)_0, \left(\frac{\partial c_y}{\partial \alpha}\right)_0, \left(\frac{\partial P}{\partial \delta_P}\right)_0, (c'_y)_0, (c'_x)_0, \left(\frac{\partial m_z}{\partial \dot{\alpha}}\right)_0, \left(\frac{\partial m_z}{\partial \alpha}\right)_0, \left(\frac{\partial m_z}{\partial \dot{\vartheta}}\right)_0, \left(\frac{\partial m_z}{\partial M}\right)_0, \left(\frac{\partial m_z}{\partial \delta_B}\right)_0$ - values of the functions and its derivatives for the steady values $V_0, \theta_0, \alpha_0, \vartheta_0, H_0, (U_x)_0, (U_y)_0, \delta_B(t) = 0, \delta_P(t) = 0$

3.3 Linear equations

Since the system (1) is nonlinear, it's too hard to use those equations to analyze stability. We need to linearize those equations on the premise that the parameters ϑ_0, θ_0, V_0, α_0, H_0 corresponding with steady flight get small increments $\Delta\vartheta$,$\Delta\theta$,ΔV,$\Delta\alpha$,ΔHcaused by perturbations action on a flight.

Let us define the following deviations from the steady state:

$$\begin{cases} V = V_0 + \Delta V(t) \\ \theta = \theta_0 + \Delta\theta \\ \vartheta = \vartheta_0 + \Delta\vartheta(t) \\ \alpha = \alpha_0 + \Delta\alpha(t) \\ H = H_0 + \Delta H(t) \end{cases}$$

$$v(t) = \frac{\Delta V}{V_0};\ \ h(t) = \frac{\Delta H}{V_0\tau_a};\ \ \alpha(t) = \Delta\alpha;\ \ \theta(t) = \Delta\theta;\ \ \vartheta(t) = \Delta\vartheta;\ \ v_y(t) = \frac{\Delta U_y}{V_0};\ \ v_x(t) = \frac{\Delta U_x}{V_0};$$

The correspondent parameters are the following:

$$\tau_a = \frac{m}{\rho_0 V_0 S};\ \mu = \frac{b_a m}{2r_z^2\rho_0 S};\ r_z^2 = \frac{J_z}{m}$$

r_z- inertial radius

$$\bar{t} = \frac{t}{\tau_a}; p = \frac{d}{d\bar{t}}$$

$$c'_x(M) \stackrel{\text{def}}{=} c_x(\alpha_0, M) - 2\frac{P(0,M)\cos(\alpha_0)}{\rho_0 S V_0^2};\ c'_y(M) \stackrel{\text{def}}{=} c_y(\alpha_0, M) + 2\frac{P(0,M)\cos(\alpha_0)}{\rho_0 S V_0^2};$$

We can made linearization of (1) in the neighborhood of the found steady state solution:

$$\begin{cases} (p + n_{11})v + n_{12}\alpha + n_{13}\vartheta + n_{14}h = n_p\delta_p + f_1(t) \\ -n_{21}v + (p + n_{22})\alpha - (p + n_{23})\vartheta + n_{24}h = f_2(t) \\ n_{31}v + (n_0 p + n_{32})\alpha + (p^2 + n_{33}p)\vartheta + n_{34}h = -n_B\delta_B(t) + f_3(t) \\ -n_{41}v + n_{42}\alpha - n_{42}\vartheta + ph = v_y(t) \end{cases}$$

$$n_{11} = \frac{M_0}{2}\left(\frac{\partial c'_x}{\partial M}\right)_0 + (c_x)_0;\ n_{12} = \frac{1}{2}\left(\left(\frac{\partial c_x}{\partial \alpha}\right)_0 - (c_y)_0\right);$$

$$n_{13} = \frac{1}{2}(c'_y)_0;\ n_{14} = \frac{\beta V_0 \tau_0}{2(T_H)_0}\left[\frac{M_0}{2}\left(\frac{\partial c'_x}{\partial M}\right) - \frac{(c_x)_0}{\gamma - 1}\right];$$

$$n_{21} = -\left(\frac{M_0}{2}\left(\frac{\partial c'_y}{\partial M}\right)_0 + (c_y)_0\right);\ n_{22} = \frac{1}{2}\left(\left(\frac{\partial c_y}{\partial \alpha}\right)_0 + (c_x)_0\right);$$

$$n_{23} = \frac{1}{2}(c'_x)_0;\ n_{24} = \frac{\beta V_0 \tau_0}{2(T_H)_0}\left[\frac{M_0}{2}\left(\frac{\partial c'_y}{\partial M}\right)_0 - \frac{(c_y)_0}{\gamma - 1}\right];$$

$$n_{31} = -\mu M_0\left(\frac{\partial m_z}{\partial M}\right)_0;\ n_{32} = -\mu\left(\frac{\partial m_z}{\partial \alpha}\right)_0;$$

$$n_{33} = -\frac{\mu}{\tau_\alpha}\left(\frac{\partial m_z}{\partial \dot{\vartheta}}\right)_0;\ n_{34} = -\mu\frac{\beta\tau_\alpha V_0}{2(T_H)_0}M_0\left(\frac{\partial m_z}{\partial M}\right)_0;$$

$$n_0 = -\frac{\mu}{\tau_\alpha}\left(\frac{\partial m_z}{\partial \dot{\alpha}}\right)_0;$$

$$n_{41} = \sin(\theta_0);\ n_{42} = \cos(\theta_0);$$

$$n_p = \frac{\left(\frac{\partial P}{\partial \delta_P}\right)_0 \cos(\alpha_0)}{\rho_0 s {V_0}^2};\ n_B = -\mu\left(\frac{\partial m_z}{\partial \delta_B}\right)_0; p = \frac{d}{d\bar{t}}$$

The typical real values of the coefficients n_{ij} can be found from the Table 1.

Table 1

Coefficients of equatuions for longitudinal motion

	Drone					
	lightweight	middle		heavy		
	H = 11 km M = 0,9 τ_a = 3,8 s	H = 0, landing	H = 4 km M = 0,65 τ_a = 2,1 s	H = 8 km M = 0,8 τ_a = 2,5 s	H = 12 km M = 0,9 τ_a = 4 s	H = 0, landing
n_{11}	0,024	0,12	0,019	0,026	0,048	0,12
n_{12}	-0,11	-0,28	0,019	-0,025	-0,079	-0,12
n_{13}	0,2	0,4	0,3	0,1	0,17	0,3
n_{14}	$-4{,}3\cdot10^{-4}$	—	$-4{,}4\cdot10^{-4}$	$-4\cdot10^{-4}$	$-4{,}2\cdot10^{-4}$	—
n_{21}	-0,4	-0,8	-0,6	-0,36	-0,68	-0,65
n_{22}	2,4	2,4	2,66	3	2,4	2,35
n_{23}	0	0,02	0	0	0	0,015
n_{24}	$-1{,}22\cdot10^{-2}$	—	$-1{,}28\cdot10^{-2}$	$-1{,}1\cdot10^{-2}$	$-1{,}2\cdot10^{-2}$	—
n_{31}	0	0	0	0	-1,2	0
n_0	0,4	0,59	0,59	1,17	0,68	0,9
n_{32}	38	6,6	10,63	42	36	8
n_{33}	2,45	1,67	1,69	2,5	2,42	2,35
n_{34}	-0,053	—	-0,055	-0,05	-0,05	—
$n_в$	49	15,2	24,5	28	46	8,4
n_p	0,022	0,019	0,021	0,02	0,02	0,019

3.4 Adjusting the system to a proper form

We have to adjust our system to a proper form to apply the theory of stability.

$$\begin{cases} \upsilon'(t) = -n_{11}\upsilon(t) - n_{12}\alpha(t) - n_{13}\vartheta(t) - n_{14}h(t) = n_p\delta_p(t-\tau) \\ \alpha'(t) = \vartheta'(t) + n_{21}\upsilon(t) - n_{22}\alpha(t) + n_{23}\vartheta(t) - n_{24}h(t) \\ \vartheta''(t) = -n_0\alpha'(t) - n_{33}\vartheta'(t) - n_{31}\upsilon(t) - n_{32}\alpha(t) - n_{34}h(t) - n_B\delta_B(t-\tau) \\ h'(t) = n_{41}\upsilon(t) - n_{42}\alpha(t) + n_{42}\vartheta(t) \end{cases}$$

Let's start with linear substitution. It brings the system to a form where all diagonal coefficients are non-zero.

The first substitution is following:

$h(t) = \lambda(t) - Wv(t) \Rightarrow h'(t) = \frac{d\lambda(t)}{dt} - W\frac{dv(t)}{dt}$;

Now we have:

$$\begin{cases} v'(t) = -n_{11}v(t) - n_{12}\alpha(t) - n_{13}\vartheta(t) - n_{14}(\lambda(t) - Wv(t)) = n_p\delta_p(t-\tau) \\ \alpha'(t) = \vartheta'(t) + n_{21}v(t) - n_{22}\alpha(t) + n_{23}\vartheta(t) - n_{24}(\lambda(t) - Wv(t)) \\ \vartheta''(t) = -n_0\alpha'(t) - n_{33}\vartheta'(t) - n_{31}v(t) - n_{32}\alpha(t) - n_{34}\big(\lambda(t) - Wv(t)\big) - n_B\delta_B(t-\tau) \\ \lambda'(t) = W\left(-n_{11}v(t) - n_{12}\alpha(t) - n_{13}\vartheta(t) - n_{14}\big(\lambda(t) - Wv(t)\big) + n_p\delta_p(t-\tau)\right) + \\ +n_{41}v(t) - n_{42}\alpha(t) + n_{42}\vartheta(t) \end{cases}$$

The second linear substitution is used here to decrease the order of the system.

$\vartheta'(t) = \varphi(t) + b_0\vartheta(t)$;

$$\begin{cases} v'(t) = -n_{11}v(t) - n_{12}\alpha(t) - n_{13}\vartheta(t) - n_{14}(\lambda(t) - Wv(t)) + n_p\delta_p(t-\tau) \\ \alpha'(t) = \varphi(t) + b_0\vartheta(t) + n_{21}v(t) - n_{22}\alpha(t) + n_{23}\vartheta(t) - n_{24}(\lambda(t) - Wv(t)) \\ \vartheta'(t) = \varphi(t) + b_0\vartheta(t) \\ \varphi'(t) = -n_B\delta_B(t-\tau) + \big((-n_{23} - b_0)n_0 - b_0(n_{33} + b_0)\big)\vartheta(t) + \\ +\big((-Wn_{24} - n_{21})n_0 + Wn_{34} - n_{31}\big)v(t) + \\ +(-n_{33} - b_0 - n_0)\varphi(t) + (n_0n_{24} - n_{34})\lambda(t) - \alpha(t)(-n_0n_{22} + n_{32}) \\ \lambda'(t) = W\delta_p(t-\tau)n_p + (W^2n_{14} - Wn_{11} + n_{41})v(t) + (-Wn_{13} + n_{42})\vartheta(t) + \\ +(-Wn_{12} - n_{42})\alpha(t) - W\lambda(t)n_{14} \end{cases}$$

δ_P and δ_B are control parameters. We want to express them as linear combinations of our original variables:

$\delta_P(t-\tau) = p_1v(t-\tau) + p_2\alpha(t-\tau) + p_3\vartheta(t-\tau) + p_4h(t-\tau)$;

$\delta_B(t-\tau) = b_1v(t-\tau) + b_2\alpha(t-\tau) + b_3\vartheta(t-\tau) + b_4h(t-\tau)$

Taking this into account the system will be the following:

$$\begin{cases} v'(t) = -n_{11}v(t) - n_{12}\alpha(t) - n_{13}\vartheta(t) - n_{14}(\lambda(t) - Wv(t)) + \\ +n_p\left(p_1v(t-\tau) + p_2\alpha(t-\tau) + p_3\vartheta(t-\tau) + p_4\big((\lambda(t-\tau) - Wv(t-\tau))\big)\right) \\ \alpha'(t) = \varphi(t) + b_0\vartheta(t) + n_{21}v(t) - n_{22}\alpha(t) + n_{23}\vartheta(t) - n_{24}(\lambda(t) - Wv(t)) \\ \vartheta'(t) = \varphi(t) + b_0\vartheta(t) \\ \varphi'(t) = -b_0(\varphi(t) + b_0\vartheta(t)) - \\ - n_0\left(\varphi(t) + b_0\vartheta(t) + n_{21}v(t) - n_{22}\alpha(t) + n_{23}\vartheta(t) - n_{24}\big(\lambda(t) - Wv(t)\big)\right) \\ -n_{33}(\varphi(t) + b_0\vartheta(t)) - n_{31}v(t) - n_{32}\alpha(t) - n_{34}(\lambda(t) - Wv(t)) - n_B(b_1v(t-\tau)) + \\ +b_2\alpha(t-\tau) + b_3\vartheta(t-\tau) + b_4(\lambda(t-\tau) - Wv(t)) \\ \lambda'(t) = W\big(p_1v(t-\tau) + p_2\alpha(t-\tau) + p_3\vartheta(t-\tau) + p_4(\lambda(t-\tau) - Wv(t-\tau))\big)n_p + \\ +(W^2n_{14} - Wn_{11} + n_{41})v(t) + (-Wn_{13} + n_{42})\vartheta(t) + \\ +(-Wn_{12} - n_{42})\alpha(t) - W\lambda(t)n_{14} \end{cases} \tag{3.1}$$

1.1 Autopilot

For the case when steady state parameters cannot provide stability of the desirable steady state trajectory themselves, we need to use autopilots (Fig.2.). An autopilot states the controlling parameters δ_p, δ_B to be functions of the output controlled parameters$(\upsilon(t);\ h(t);\ \alpha(t);\ \theta(t);\ \vartheta(t))$, which are deviations from the desirable steady state trajectory. The values of the output parameters can be obtained by autopilot from navigation measurements, for example, from vision-based navigation, inertial navigation, satellite navigation and so on. On the basis these navigation measurements, the autopilot forms controlling signals to decrease undesirable deviation. Unfortunately, there always exists some in getting information about the output controlled parameters to autopilot for any navigation measurements. So we have a problem, because of the lack of some necessary information for controlling. In this paper we demonstrate that we are possible even for such conditions with the time delay to get controlling signal providing a stable flight.

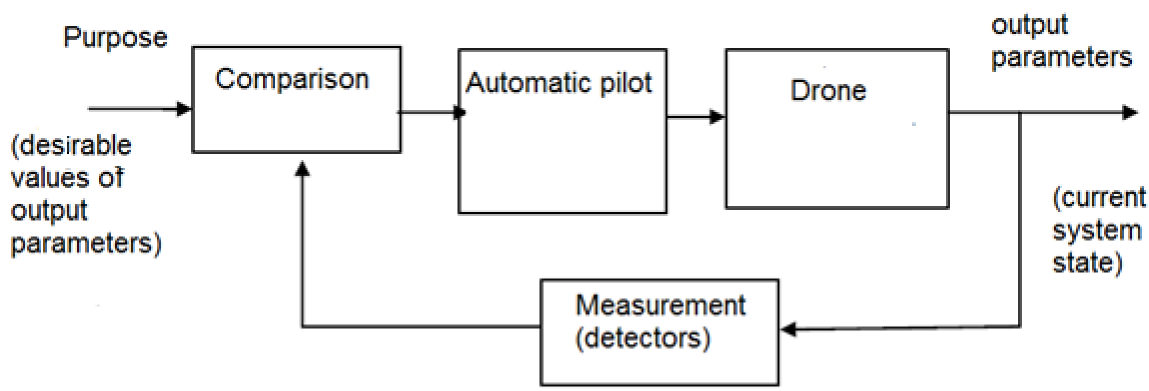

Fig.2 Automatic control

4 Applying the theory to a particular case

Apply the condition (1.1) of Theorem 3.2 in [1], described in section 2 of this paper to the system (3.1)

Consider the system of inequalities:

$$\begin{cases} \left(n_p p_4 W + n_{14} W - n_p p_1 + n_{11}\right) z_1 - \left|-n_p p_2 + n_{12}\right| z_2 - \left|-n_p p_3 + n_{13}\right| z_3 - \left|-n_p p_4 + n_{14}\right| z_5 \geq 1 & (4.1) \\ n_{22} z_2 - |n_{24} W + n_{21}| z_1 - |n_{23} + b_0| z_3 - z_4 - |n_{24}| z_5 \geq 1 & (4.2) \\ -b_0 z_3 - z_4 \geq 1 & (4.3) \\ (b_0 + n_0 + n_{33}) z_4 - |-n_0 n_{21} - n_0 n_{24} W - n_{21} + n_{34} W - n_B b_1 + n_B b_4 W| z_1 - |n_0 n_{22} - n_{32} - n_B b_2| z_2 - & \\ -|n_0 b_0 + n_{33} b_0 + b_0^2 + n_0 n_{23} + n_B b_3| z_3 - |n_0 n_{24} - n_{34} - n_B b_4| z_5 \geq 1 & (4.4) \\ (-n_p p_4 W + n_{14} W) z_5 - \left|-W^2 n_p p_4 + W^2 n_{14} + W n_p p_1 - W n_{11} + n_{41}\right| z_1 - \left|-n_p p_2 + n_{12} + \frac{n_{42}}{W}\right| z_2 - & \\ - \left|-n_p p_3 + n_{13} - \frac{n_{42}}{W}\right| z_3 \geq 1 & (4.5) \end{cases}$$

Take other coefficients from <u>the first column</u> of Table 1 we get following:
$n_{11} = 0.024, n_{12} = -0.11, n_{13} = 0.2, n_{14} = -0.00043, n_{21} = -0.4, n_{22} = 2.4, n_{23} = 0, n_{24} = -0.0122, n_{31} = 0, n_{32} = 38, n_{33} = 2.45, n_{34} = -0.053, n_0 = 0.4, n_B = 49, n_P = 0.022, n_{41} = 0, n_{42} = 1;$

We choose $z_1 = 1 \cdot |z|, z_2 = 1 \cdot |z|, z_3 = 10^{-10} \cdot |z|, z_4 = 10^{-10} \cdot |z|, z_5 = 1 \cdot |z|,$
where $|z| \gg 1, |z| \cdot 10^{-10} \gg 1$
From (4.3)

$$b_1 = -\frac{n_{21}(n_0 + 1)}{n_B}$$

$$b_2 = \frac{n_0 n_{22} - n_{32}}{n_B}$$

$$b_3 = -\frac{{b_0}^2 + n_0 b_0 + n_{33} b_0 + n_0 n_{32}}{n_B}$$

$$b_4 = \frac{n_0 n_{24} - n_{34}}{n_B}$$

$$b_0 \geq -n_0 - n_{33} + 1 / (|z| \cdot 10^{-10})$$

From (4.3)

$$b_0 \geq -(1 + 1 / (|z| \cdot 10^{-10}))$$

We choose $b_0 = -1.001$
Then we can calculate parameters b_1, b_2, b_3, b_4:
$b_1 =$ 0.01142857143, $b_2 =$ -0.7559183673, $b_3 =$ 0.03777242857, $b_4 =$ 0.0009820408163

From (4.2)

$$n_{22} - |n_{24}W + n_{21}| - (|n_{23} + b_0| + 1) \cdot 10^{-10} - |n_{24}| \geq \frac{1}{|z|}$$

$$\frac{n_{22}}{|n_{24}|} - \frac{(|n_{23} + b_0| + 1)}{|n_{24}|} \cdot 10^{-10} - 1 - \frac{1}{|z||n_{24}|} - \frac{n_{21}}{n_{24}} \geq W$$

$$\geq -\frac{n_{22}}{|n_{24}|} + \frac{(|n_{23} + b_0| + 1)}{|n_{24}|} \cdot 10^{-10} + 1 + \frac{1}{|z||n_{24}|} - \frac{n_{21}}{n_{24}}$$

$$162.95 - \frac{(|n_{23}+b_0|+1)}{|n_{24}|} \cdot 10^{-10} - \frac{1}{|z||n_{24}|} \geq W \geq -228.51 + \frac{(|n_{23}+b_0|+1)}{|n_{24}|} \cdot 10^{-10} + \frac{1}{|z||n_{24}|}$$

We choose $W = -63$
We choose

$$\left|-n_p p_2 + n_{12} + \frac{n_{42}}{W}\right| = |-n_p p_2 + n_{12}| = \left|\frac{n_{42}}{2W}\right|$$

$$\left|-n_p p_3 + n_{13} - \frac{n_{42}}{W}\right| = |-n_p p_3 + n_{13}| = \left|\frac{n_{42}}{2W}\right|$$

so

$$p_2 = -\frac{\left|\frac{n_{42}}{2W}\right| + |n_{12}|}{n_p}, \quad p_3 = -\frac{\left|\frac{n_{42}}{2W}\right| + |n_{13}|}{n_p};$$

Then we can calculate parameters p_2, p_3:
$p_2 =$ -5.360750359, $p_3 =$ 9.451659450

We introduce new variable $p_5 = p_1 - Wp_4$.

Substituting $p_1 = p_5 + Wp_4$ we get conditions for parameters p_4 and p_5 from (4.1) and (4.5):

$$\begin{cases} \left(n_{14}W - \left|\frac{n_{42}}{2W}\right|(1+10^{-10}) + n_{11}\right) - n_P p_5 - \left|-n_p p_4 + n_{14}\right| > 0 \\ (n_{14}W - \left|\frac{n_{42}}{2W}\right|(1+10^{-10})) - n_p p_4 W - \left|W^2 n_{14} + W n_p p_5 - W n_{11} + n_{41}\right| > 0 \end{cases}$$

$$\begin{cases} -0.022p_5 + 0.04315349210 - |0.022p_4 + 0.00043| > 0 \\ 0.022p_4 - 0.007506507971 - |0.00309 + 0.022p_5| > 0 \end{cases} \quad (4.6)$$

We find the parameters and choose the particular ones that satisfy the requirements:

$p_1 = -35, p_4 = 0.5512345678$

Now it's possible to find the requirements for delay:

Apply the conditions (1.2) of Theorem 3.2 in [1], described in section 2 of this paper to the system (3.1)

From eq. for $v'(t)$

$\left(n_{14}W + n_{11} - n_p p_1\right)\tau \leq \frac{1}{e}$

$0.821\tau \leq 0.368$

$\tau \leq 0.448$

$\left(n_{14}W + n_{11} - n_p p_1 - \left|n_p p_4 W\right|\right)\tau \leq \frac{1}{e};$

$0.057\tau \leq 0.368$

$\tau \leq 6.445$

From eq. for $\lambda'(t)$

$\left(n_{14}W - n_p p_4 W\right)\tau \leq \frac{1}{e}$

$0.791\tau \leq 0.368$

$\tau \leq 0.465$

Finally,

$\tau \leq 0.448$

Delay in seconds

$\tau_s \leq 0.448\tau_a = 0.448 * 3.8\ sec = 1.703\ sec$

5 Computer simulations

We made numerical simulation of the system (4.1)-(4.5) for different values of time delay (fig. 3-5). Nonzero initial conditions was used. We can see that the system (4.1)-(4.5) is stable even for values of time delay larger than theoretical one $\tau = 0.448$

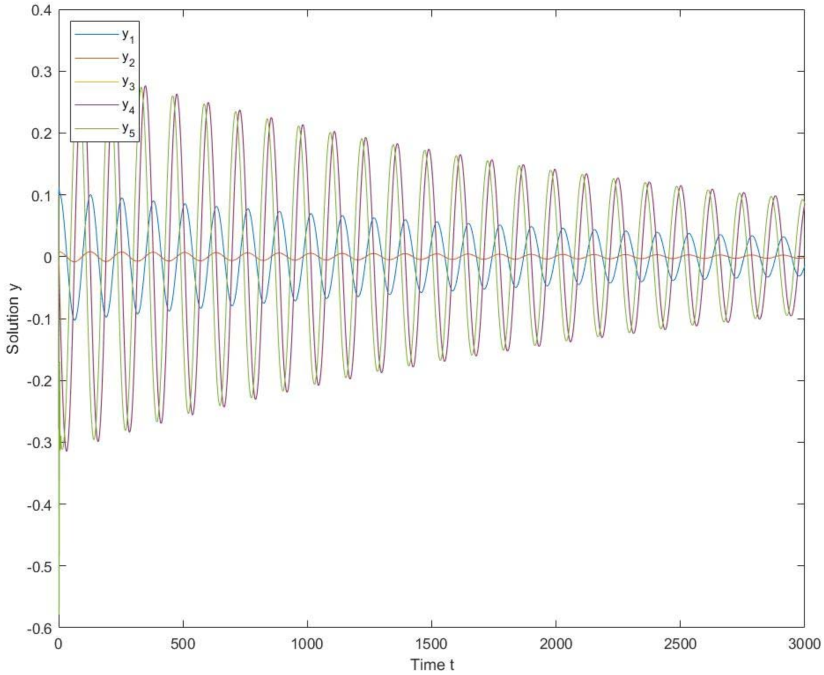

Fig. 3 Numerical simulation of the system (4.1)-(4.5) for time delay $\tau = 1$

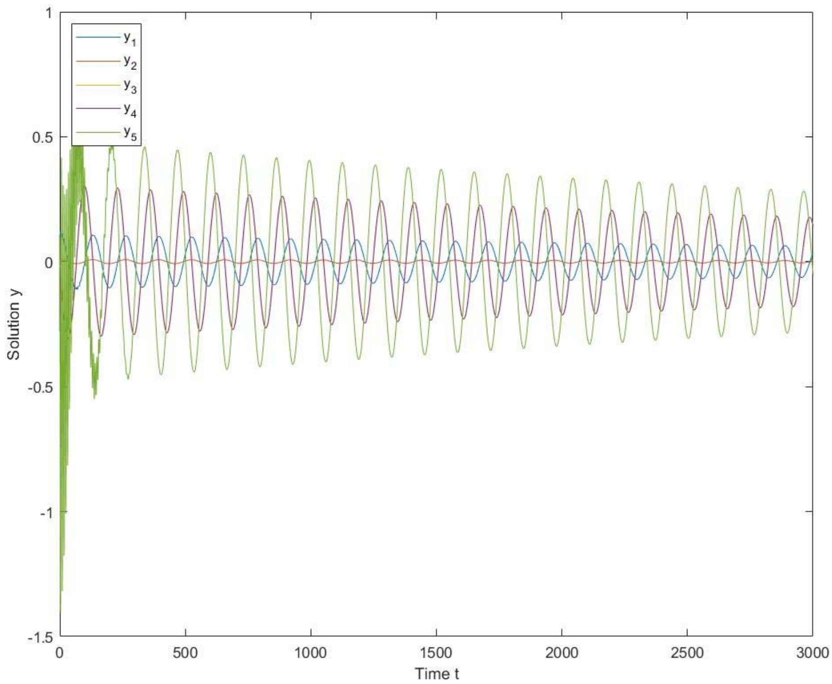

Fig. 4 Numerical simulation of the system (4.1)-(4.5) for time delay $\tau = 1.9$

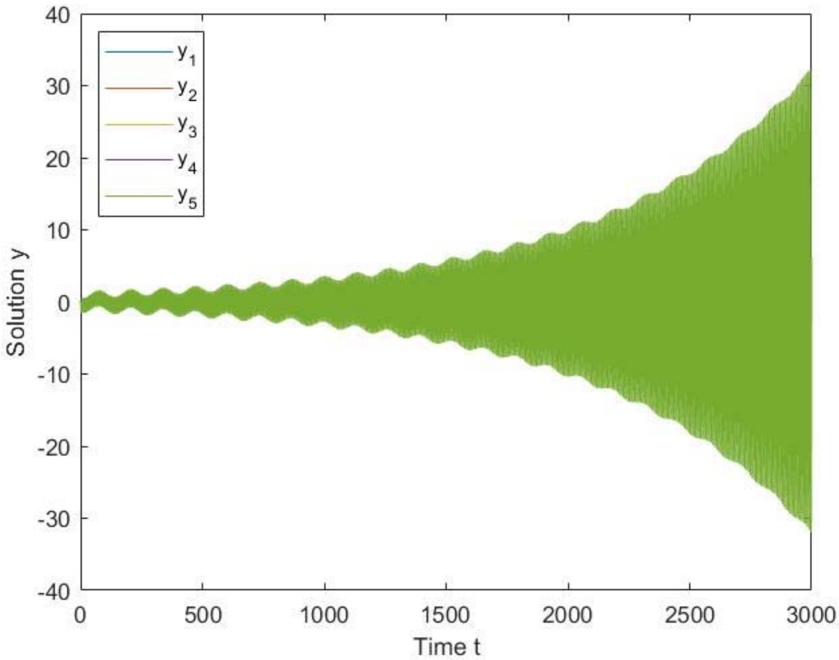

Fig. 5 Numerical simulation of the system (4.1)-(4.5) for time delay
$\tau = 2$

6 Conclusion

As a result, we proved that it's possible to maintain stable movement of a drone even when time delay exists in transfer information about output control parameters from navigation measurement devices to autopilot. We found control parameters for a particular case of flight and estimated max possible delay of the system.

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