Charge Stability Approach to Finite Quantum Field Theory: An Alternative to Renormalization

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This paper analyses charge stability and applies the resulting stability principle to resolve divergence issues in quantum field theory without renormalization. For quantum electrodynamics (QED), stability is enforced by requiring that the positive electromagnetic field energy be balanced by a negative interaction energy between the observed electron charge and a local vacuum potential. Then in addition to the observed core mechanical mass m, an electron system consists of two electromagnetic mass components of equal magnitude M but opposite sign; consequently, the net electromagnetic mass is zero. Two virtual, electromagnetically dressed mass levels $m \pm M$, constructed to form a complete set of mass levels and isolate the electron-vacuum interaction, provide essential S-matrix corrections for radiative processes involving infinite field actions. Total scattering amplitudes for radiative corrections are shown to be convergent in the limit $M \to \infty$ and equal to renormalized amplitudes when Feynman diagrams for all mass levels are included. In each case, the infinity in the core mass amplitude is canceled by the average amplitude for electromagnetically dressed mass levels, which become separated in intermediate states and account for the stabilizing interaction energy between an electron and its surrounding polarized vacuum. In this manner, S-matrix corrections in QED are shown to be finite for any order diagram in perturbation theory, all the while maintaining the mass and charge at their physically observed values. Charge stability corrections, applied to one-loop diagrams of non-Abelian gauge theory, also yield finite results without renormalization. The results demonstrate that quantum field theory is scale invariant.

I. INTRODUCTION

A long-standing enigma in particle physics is how an elementary charged particle such as an electron can be stable in the presence of its own electromagnetic field (see [1, 2] and cited references). Critical accounting for charge stability is essential since radiative corrections in quantum field theory (QFT) involve self-interactions that can change the mass and charge of an electron. This analysis identifies and accounts for the hidden interaction that energetically stabilizes a charged particle such that its mass and charge assume their physically observed values - it expands the scope of [3] to include non-Abelian gauge theory.

The agreement between renormalized QED theory and experiment confirms the effect of vacuum fluctuations on the dynamics of elementary particles to astounding accuracy. For example, electron anomalous magnetic moment calculations currently agree with experiment to about 1 part in a trillion [4, 5]. This achievement is the result of more than six decades of effort since the relativistically invariant form of the theory took shape in the works of Feynman, Schwinger, and Tomonaga (see Dyson's unified account [6]). The agreement leaves little doubt that QED predictions are correct; however, the renormalization technique [7, 8] used to overcome divergence issues in radiative corrections offers little insight into the underlying physics behind electron stability in the high-energy regime. Recall that divergent integrals occur in scattering amplitudes for self-energy processes and arise in sums over intermediate states of arbitrarily high-energy virtual particles. This stymied progress until theoretical improvements were melded with renormalization to isolate the physically significant parts of radiative corrections by absorbing the infinities into the electron mass and charge. Although the renormalization method used to eliminate ultraviolet divergences results in numerical predictions in remarkable agreement with experiments, redefinition of fundamental physical constants remains an undesirable feature of the current theory.

Our main purpose is to develop an alternative to mass and charge renormalization in QFT. We begin by revisiting the classical self-energy problem where we define an energetically stable electric charge. We then apply the resulting stability principle to derive a S-matrix correction for loop processes. S-matrix corrections for stability are simply constructed using core amplitudes from the literature, involve two additional Feynman diagrams associated with dressed core mass (DCM) states, and account for the action of the vacuum back on

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the charge via an opposing vacuum current. After defining divergent integrals for DCM diagrams, we verify that net S-matrix corrections in QED for vacuum polarization, electron self-energy, and vertex processes are finite to all orders in perturbation theory. Finally, we apply the method to one-loop diagrams in non-Abelian gauge theory.

II. FORMULATION

Regarding an electron as a point particle [9], the classical electrostatic self-energy $e^2/2a \equiv \alpha\Lambda_\circ$ diverges linearly as the shell radius $a \to 0$, or energy cutoff $\Lambda_\circ \to \infty$, where $-e$ is the charge and $\alpha = e^2/4\pi\hbar c$ is the fine-structure constant. However, Weisskopf [10, 11] showed using Dirac's theory [12] that the charge is effectively dispersed over a region the size of the Compton wavelength due to pair creation in the vacuum near an electron, and the self-energy only diverges logarithmically. Feynman's calculation [13] in covariant QED yields an electromagnetic mass-energy

$$m_{em} = \frac{3\alpha m}{2\pi}\left(\ln\frac{\Lambda_\circ}{mc^2} + \frac{1}{4}\right) , \tag{1}$$

where m is the electron mass. In the absence of a compensating negative energy, (1) signals an energetically unstable electron. It is the key ultraviolet divergence problem in QED, whose general resolution will result in finite amplitudes for all radiative corrections. In this section we derive a stability condition and apply it to develop corrections to scattering amplitudes for otherwise divergent processes.

To ensure that the total electron mass is its observed value, renormalization theory posits that a negatively infinite "bare" mass must exist to counterbalance m_{em}. For lack of physical evidence, negative matter is naturally met with some skepticism (see Dirac's discussion [14] of the classical problem, for example). Nevertheless, energies that hold an electron together are expected to be negative, and we can understand their origin by first considering the source for the electrical energy required to assemble a classical charge in the rest frame. Recall that the work done in assembling a charge from infinitesimal parts is equal to the electromagnetic field energy. Since the agents that do the work must draw an equivalent amount of energy from an external energy source (well), the well's energy is depleted and the total energy

$$\mathcal{E} = mc^2 + \mathcal{E}^+_{em} + \mathcal{E}_w \tag{2}$$

of the system including matter, electromagnetic field $\mathcal{E}^+_{em}$, and energy well $\mathcal{E}_w$ is constant. For an elementary particle, could the depleted energy well be the surrounding vacuum?

From another point of view, consider an electron and its neighboring vacuum treated as two distinct systems that can act on one another. Suppose the electron acts on the vacuum to polarize it creating a potential well, then there must be an opposing reaction of vacuum back on the electron. The resulting vacuum potential Φ_{vac} confines the observed core charge akin to a spherical capacitor as shown in Fig. 1, and the interaction energy

$$\mathcal{E}_w \to \mathcal{E}^-_{em} \equiv -e\Phi_{vac} \tag{3}$$

is assumed to just balance $\mathcal{E}^+_{em}$ resulting in a stability condition

$$\begin{aligned} \mathcal{E}^+_{em} + \mathcal{E}^-_{em} &= 0 \\ m^+_{em} + m^-_{em} &= 0 , \end{aligned} \tag{4}$$

where the mass-energy equivalence $\mathcal{E}^\pm_{em} = m^\pm_{em}c^2$ has been used to obtain an equivalent expression in terms of electromagnetic masses. Therefore, the net mass-energy of a free electron is attributed entirely to the observed core mechanical mass m. In contrast to Poincaré's theory [15] wherein internal non-electromagnetic stresses hold an electron together, external vacuum electrical forces are assumed to provide charge stabilization and energy balance via a steady state polarization field surrounding the electron. Corresponding to

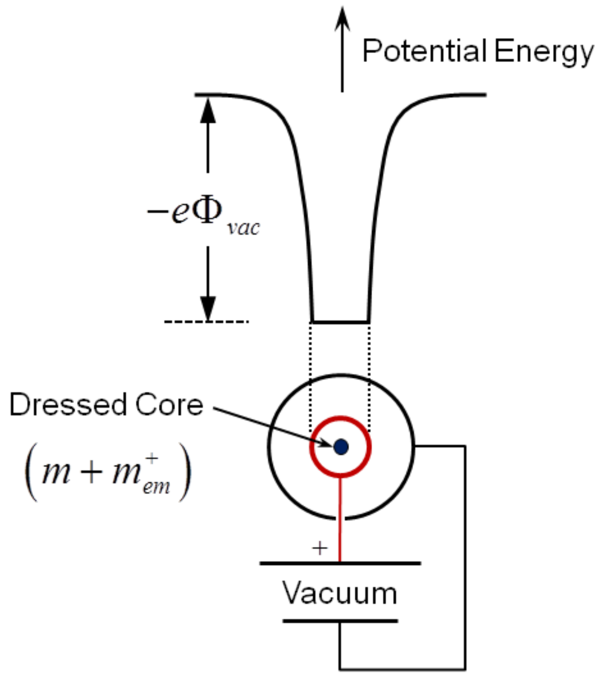

FIG. 1: Effective vacuum potential confines core electron charge similarly to spherical capacitor. Since the stability principle requires $\mathcal{E}_{em}^{+} - e\Phi_{vac} = 0$, the total energy of core electron in the well and dressed in its electromagnetic field is just its observed mass-energy.

a divergent self-action process, we require a mechanism whereby the core charge interacts locally with the polarized vacuum according to (3).

The energy of the core charge in the potential well of Fig. 1 is

$$\mathcal{E}_{core}^{-} = mc^2 + \mathcal{E}_{em}^{-} \equiv m_b c^2 \, , \tag{5}$$

where m_b may be identified with the bare mass, and

$$m_b + m_{em}^{+} = m \tag{6}$$

captures the mass renormalization condition which is equivalent to (2) with (3) and (4). However, notice that the bare mass corresponds to a core electron dressed in negative electromagnetic energy; hence, its characterization as a "mechanical mass" is a misnomer (see [16] for example). Only the core mass is observable, and only it is expected to appear in the Lagrangian if one takes (4) seriously. In renormalization theory, however, one starts with a bare electron, self-interaction dresses it with positive electromagnetic energy, and (6) is subsequently applied to redefine the mass. On the other hand, suppose we start with the observed electron charge; then taking into account (2), (3), and (4), m_{em}^{+} and m_{em}^{-} are always present, and the total mass reduces to the observed core mechanical mass. Starting with this premise, we can formulate a finite theory of radiative corrections that accounts for all possible electromagnetically dressed intermediate states, and no asymmetry necessitating a redefinition of mass and charge is introduced. For the ensuing development, relativistic notation defined in [17] is employed, and natural units are assumed; that is, $\hbar = c = 1$.

Equations (2) and (4) suggest that a stable electron consists of three rest mass components: a core mass m and two electromagnetic masses $m_{em}^{\pm}$ that are assumed large in magnitude but finite until the final step of the development. We can think of $m_{em}^{\pm}$ as components of an electromagnetic vacuum (zero net energy) which are tightly bound to the core mass and inseparable from the core and each other, at least for finite field actions. Considering all non-vanishing masses constructed from the set $\{m, \, m_{em}^{+}, \, m_{em}^{-}\}$, we are led to define a complete set of mass levels $m + \lambda M$, where $\lambda = \{0, \pm 1\}$ and $M \equiv |m_{em}^{\pm}|$. In the following, an electromagnetically dressed core mass (DCM) refers to a composite particle with mass levels $m \pm M$. Associated DCM 4-momenta are $p_{dcm} = p \pm P_M$, where $\{p,\, P_M\}$ correspond to $\{m,\, M\}$, respectively.

To transition this charge stability model into quantum theory, first consider a free particle state $|p,\, m\rangle$ with momentum satisfying $p^2 \equiv p_\mu p^\mu = m^2$, where $p^\mu = (p^\circ,\, \vec{p})$ and $p_\mu = g_{\mu\nu} p^\nu$ are contravariant and covariant momentum 4-vectors, respectively. Metric tensor $g_{\mu\nu}$ has non-zero components

$$g_{00} = -g_{11} = -g_{22} = -g_{33} = 1\,.$$

Spin is omitted in $|p,\, m\rangle$ since it is inessential to the subsequent development, and the rest mass is included because it is the fundamental particle characteristic which varies in stability corrections to the S-matrix [see (16)]. We employ the relativistic normalization

$$\langle p',\, m\, |p,\, m\rangle = 2E\left(\vec{p},\, m\right)(2\pi)^3\, \delta\left(\vec{p} - \vec{p}\,'\right)\ ,$$

where $E\left(\vec{p},\, m\right) = \sqrt{\vec{p}^{\,2} + m^2}$. Now construct the superposition

$$|\chi\rangle = \frac{1}{\sqrt{2}} \sum_{\lambda = \pm 1} \left|\Upsilon_\lambda^{dcm}\left(p\right)\right\rangle \tag{7}$$

of DCM states

$$\left|\Upsilon_\lambda^{dcm}\left(p\right)\right\rangle = |p + \lambda P_M\,,\, m + \lambda M\rangle\ , \tag{8}$$

where the core 4-momentum is dispersed per an uncertainty $\Delta p \equiv \lambda P_M$. DCM states are normalized according to

$$\begin{aligned}\left\langle \Upsilon_{\lambda'}^{dcm}\left(p'\right)\right|\, \Upsilon_\lambda^{dcm}\left(p\right)\rangle &= 2E\left(\vec{p} + \lambda \vec{P}_M\,,\, m + \lambda M\right)(2\pi)^3\, \delta\left(\vec{p} - \vec{p}\,' + \left(\lambda - \lambda'\right)\vec{P}_M\right)\ ,\\ &\simeq 2E\left(\vec{P}_M\,,\, M\right)(2\pi)^3\, \delta\left(\vec{p} - \vec{p}\,'\right)\delta_{\lambda\lambda'}\end{aligned}$$

where the latter form follows upon assuming $M \gg m$ and requiring the vector components satisfy

$$\left|P_M^i\right| \gg \left|p^i - p'^i\right|\ ,\ i = 1, 2, 3$$

thereby excluding a zero in the delta-function argument at infinity for $\lambda' \neq \lambda$. While $\left|p^i - p'^i\right|$ is arbitrarily large in an integral over p'^i in the delta function, it is assumed small compared to $\left|P_M^i\right|$. The expected momentum and mass are given by

$$\frac{\langle \chi\, |\{p_{op}, m_{op}\}|\, \chi\rangle}{\langle \chi\, |\chi\rangle} = \{p, m\}\ ,$$

where $\{\, p_{op},\, m_{op}\}$ are corresponding operators. Therefore, the composite state (7) is energetically equivalent to the core mass state $|p,\, m\rangle$ as required by (2) and (4). A core electron dressed with positive or negative energy as in (8) is a transient state that is sharply localized within a spacial interaction region $r \simeq \hbar/Mc$ in accordance with Heisenberg's uncertainty principle [18] $\Delta p^\mu \Delta x^\mu \geq \hbar/2$ (no implied sum over μ). Scattering amplitudes for low-energy processes are assumed unaffected because the energies are insufficient to induce a separation of tightly bundled states (8) in (7). For infinite field actions, however, DCM states may become separated in intermediate states with infinitesimally small lifetimes; in this case, we shall need to account for both core and DCM scattering amplitudes. To account for all possible intermediate states in QED and satisfy (4), both mass levels $m \pm M$ are required; this generalizes the classical model depicted in Fig. 1 which assumed that only a positive energy electron interacts with the vacuum potential well.

Since the interaction region reduces to a point as $M \to \infty$ for DCM states, self-interaction effects vanish, and an electromagnetically dressed electron interacts only with the polarized vacuum. The vacuum potential is generated by a net positive current in close proximity to the core electron charge since $\Phi_{vac} > 0$. Therefore, suppose a dressed electron is located at space-time position x_1 such that it is constrained to interact only with an opposing vacuum current as indicated in Fig. 2. The current density at a neighboring point $x_2 \neq x_1$ is distinct from that of the dressed core and reversed in sign; that is,

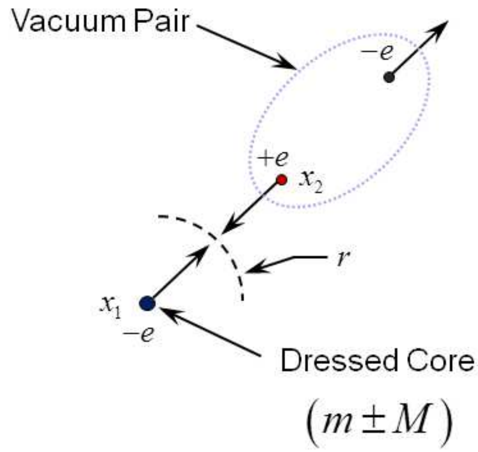

FIG. 2: Dressed core electron interacts with opposing vacuum current resulting in an exchange of the core and vacuum electrons and a sign reversal of the DCM scattering amplitude relative to the core.

$$\mathrm{sgn}\left[j_\mu\left(x_2\right)\right] = -\mathrm{sgn}\left[j_\mu\left(x_1\right)\right] \ . \tag{9}$$

With core current defined by the normal product [19, 20]

$$j_\mu\left(x_1\right) = -\frac{e}{2}\left[\bar{\psi}\gamma_\mu\psi - \bar{\psi}_c\gamma_\mu\psi_c\right]_{x_1} = -eN\left[\bar{\psi}\gamma_\mu\psi\right]_{x_1} \ ,$$

where γ^μ are Dirac matrices, the vacuum current operator at x_2 may be generated by interchanging the field operator ψ with its charge conjugate ψ_c to satisfy (9) and model an exchange of dressed core and vacuum electrons via the e^+e^- annihilation process suggested in Fig. 2 , then

$$j_\mu\left(x_2\right) = eN\left[\bar{\psi}\gamma_\mu\psi\right]_{x_2} \ .$$

Similarly to (9), the Hamiltonian density at nearby points must satisfy

$$\mathrm{sgn}\left[\mathcal{H}_{int}(x_2)\right] = -\mathrm{sgn}\left[\mathcal{H}_{\mathrm{int}}(x_1)\right] \ , \tag{10}$$

where $\mathcal{H}_{\mathrm{int}}(x) = j_\mu\left(x\right)A^\mu\left(x\right)$ in the interaction representation [21], and $A^\mu\left(x\right)$ is the radiation field. From (10) we anticipate a sign reversal in the DCM scattering amplitude relative to that for the core mass since second-order S-matrix [22] corrections involve a product $\mathcal{H}_{int}\left(x_1\right)\mathcal{H}_{int}\left(x_2\right)$.

For radiative corrections containing primitive divergences, evaluation of S-matrix charge stability corrections associated with DCM states entails a core mass replacement

$$m \to m + \lambda M \tag{11}$$

in fermion lines internal to loops as indicated in Fig. 3; that is, in each fermion propagator [23]

$$iS_F\left(p, m\right) = \frac{i}{\not{p} - m + i\varepsilon} \ ,$$

where $\not{p} = \gamma_\mu p^\mu$. Resulting loop-operator amplitudes are averaged over mass levels; that is, $\lambda = \pm 1$. For an external line entering a loop, the momentum is similarly modified

$$p \to p + \lambda P_M \ , \tag{12}$$

since the propagator is required to have a pole at $m+\lambda M$. To regulate singularities for soft photon emissions, a fictitious photon mass $\mu \neq 0$ is introduced [13] to form a modified photon propagator

$$iD_F^{\alpha\beta}(k) = \frac{-ig^{\alpha\beta}}{k^2 - \mu^2 + i\varepsilon} \, ,$$

wherein the Feynman gauge is assumed. Since infrared divergent amplitudes involve a ratio $|m/\mu|$, a replacement

$$\mu \to \eta\mu \, , \tag{13}$$

where $\eta = M/m$, is also required to ensure reduction to known results.

In summary, the total loop-operator associated with a self-energy or vertex part is defined by

$$\Omega = \Omega_{core} + \Omega_{dcm} \, , \tag{14}$$

where Ω_{core} accounts for self-interaction effects involving the core mass and Ω_{dcm} enforces stability via interaction of DCM states with an opposing current of the polarized vacuum. Ω_{dcm} is evaluated by substituting (11), (12), and (13) into known Ω_{core}. In addition to mass m, Ω_{core} depends on external momenta $\{k, p, q\}$ for Feynman diagrams in Fig. 3. For notational simplicity, any dependence on an external momentum parameter is suppressed during construction of Ω_{dcm} because $\{p, q\}$ are implicitly dependent on the core mass. Since Ω_{core} and Ω_{dcm} are both divergent for loop corrections, their improper integrals must be temporarily regulated using an energy cutoff $\Lambda_\circ$ or by dimensional regularization. For reasons clarified below, we assume an energy cutoff, then the net amplitude (14) is convergent and reduces to expected results if we define

$$\Omega = \lim_{\Lambda_\circ \to \infty} \left[\Omega_{core}\,(m, \Lambda_\circ) + \Omega_{dcm}(m, \Lambda_\circ)\right] \, , \tag{15}$$

where

$$\Omega_{dcm}\,(m, \Lambda_\circ) = -\frac{1}{2} \lim_{\eta \to \infty} \sum_{\lambda = \pm 1} \Omega_{core}\,(m + \lambda M,\, \Lambda)\big|_{M = \eta m,\, \Lambda = \eta \Lambda_\circ} \, . \tag{16}$$

The overall minus sign in (16) ensures that the core charge associated with a DCM state interacts with an opposing vacuum current as required by (9). The self-energy is defined by

$$M \equiv \eta m \, , \tag{17}$$

where $m > 0$ is the unit of mass measure; the corresponding rule for the cutoff is

$$\Lambda \equiv \eta \Lambda_\circ \, . \tag{18}$$

Scaling rules (17) and (18) are required for consistent definition of the integrals – they ensure that $\Lambda \gg M$ for arbitrarily large M, synchronize cutoff to $\Lambda_\circ$, and yield a well defined limit as $\eta \to \infty$ in (16). As verified in Sec. IV, the operator Ω_{dcm} is independent of $\{P_M,\, M\}$ for $M \gg m$. In contrast to the regulator technique of Pauli and Villars [24], the above method employs physically meaningful electromagnetically dressed mass levels (albeit virtual only), and we assume that the same principle applies to all self-energy processes in QFT without introduction of auxiliary constraints.

For a Yang-Mills charge coupling g, the self-energy is likewise assumed to be an arbitrarily large value M, which must be offset by an interaction energy $-M$ with the vacuum, so that the charge is stable, and the measured mass is the core mass m. Again, for quantum mechanics, there exists two dressed mass states $m \pm M$ in addition to the core mass. Since the physical argument is not dependent on whether the theory is Abelian (as in QED) or non-Abelian, the rules (11) and (12) used in the stability correction (16) apply to charge carrying gauge bosons as well as fermions.

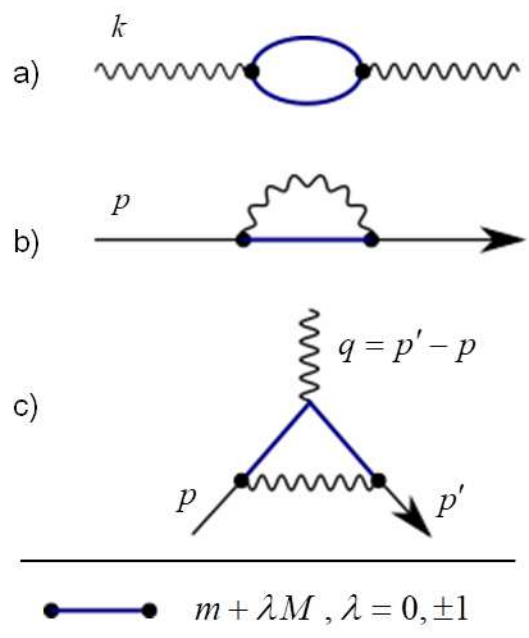

FIG. 3: Baseline radiative corrections: a) photon self-energy, b) fermion self-energy, and c) vertex involve the core mass only in internal fermion lines. Two additional diagrams, obtained by replacing the core mass with electromagnetically dressed mass levels $m \pm M$, are required for each radiative process to account for interaction with an opposing vacuum current and ensure stability.

III. DIVERGENT INTEGRALS

Here we develop integration formulae required for evaluation of stability corrections using cutoff and dimensional regularization. In the p-representation, loop diagrams involve four-dimensional integrals over momentum space, and the real parts of scattering amplitudes contain integrals of the form [25]

$$D(\Delta) = \frac{1}{i\pi^2}\int \frac{d^4p}{(p^2-\Delta)^n} = \frac{(-1)^n}{\pi^2}\int \frac{d^4p_\varepsilon}{(p_\varepsilon^2+\Delta)^n} \ , \tag{19}$$

where Δ depends on the core mass, momentum parameters external to the loop, and integration variables. On the right side of (19), a Wick rotation has been performed via a change of variables $p = (ip_\varepsilon^\circ, \vec{p}_\varepsilon)$, so that the integration can be performed in euclidean space where $p_\varepsilon^2 = p_\varepsilon^\circ p_\varepsilon^\circ + \vec{p}_\varepsilon \cdot \vec{p}_\varepsilon$. Integrals for the divergent case ($n = 2$) must be regulated such that they are consistently defined for core and dressed core masses. For the core mass, D is regularized using a cutoff $\Lambda_\circ$ on $s = |p_\varepsilon|$. In four-dimensional polar coordinates, we have

$$D(\Delta, \Lambda_\circ) = \frac{1}{\pi^2}\int d\Omega \int_0^{\Lambda_\circ} ds \ \frac{s^3}{\left[s^2+\Delta\right]^2} \ . \tag{20}$$

For DCM states, Δ depends on $|m \pm M| \simeq \eta m$ with $\eta \gg 1$, and the domain of integration in (20) must be scaled according to (18); consequently, we need to evaluate

$$D_{dcm} = D\left[\Delta(\eta m), \eta\Lambda_\circ\right] \ .$$

With a change of variables $s = \eta t$ and taking the limit $\eta \to \infty$, we obtain

$$D_{dcm} = D(\Delta_\circ, \Lambda_\circ) \ , \tag{21}$$

where

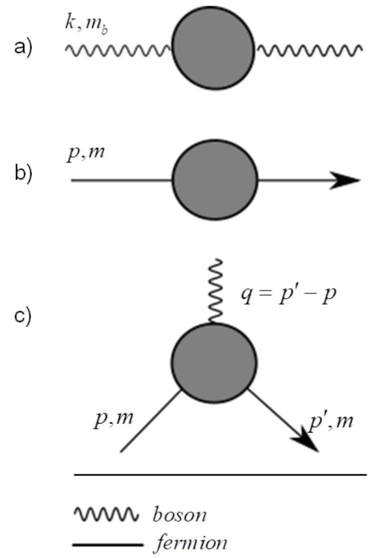

FIG. 4: Generic self-energy and vertex diagrams: a) boson SE, b) fermion SE, and c) vertex.

$$\Delta_\circ = \lim_{\eta \to \infty} \eta^{-2} \Delta(\eta m) \,. \tag{22}$$

For example, considering the standard divergent integral [25]

$$D_\circ \equiv D\left(\Delta = m^2, \Lambda_\circ\right) = \ln \frac{\Lambda_\circ^2}{m^2} - 1 + O\left(\frac{m^2}{\Lambda_\circ^2}\right) \,, \tag{23}$$

we see that $D_\circ$ is invariant under scaling rules (17) and (18); that is,

$$D_\circ = D\left(M^2, \Lambda\right) \,. \tag{24}$$

In contrast to the cutoff method, dimensional regularization evaluates a Feynman diagram as an analytic function of space-time dimension d. For $n = 2$ and $d^4p \to d^dp$ in (19), D may be evaluated using [17, 26]

$$\begin{aligned} D\left(\Delta, \sigma\right) &= \pi^{-\sigma} \Gamma\left(\sigma\right) \Delta^{-\sigma} \\ &= \frac{1}{\sigma} - \gamma - \ln \Delta + O\left(\sigma\right) \,, \end{aligned} \tag{25}$$

where $\sigma = 2 - d/2$ and $\gamma = 0.577...$ is Euler's constant. For $\sigma \neq 0$, the limit $\Lambda_\circ \to \infty$ may be taken since σ regulates the integral. The argument Δ in (25) has the form

$$\Delta\left(m\right) = am^2 + b\left(k^2 \vee p^2\right) + cq^2 \,, \tag{26}$$

where $\{k, p, q\}$ are external momentum parameters indicated in Fig. 4, and $\{a, b, c\}$ depend on Feynman parameters. Shaded blobs in Fig. 4 involve one-particle irreducible amputated correlation functions, and the external bosons to blob (a) may be either charged or uncharged, and either massive or massless.

For DCM states, D_{dcm} must yield consistent results for both cutoff and dimensional regularization methods. Considering the requirements used to derive (21) and employing appendix formulae in [26], we conclude

$$D_{dcm} = D\left(\Delta_\circ, \sigma\right) \,. \tag{27}$$

Approximations for dressed parameters

$$(m+\lambda M)^2 \simeq \eta^2 m^2 \tag{28}$$

$$(p+\lambda P_M)^2 \simeq P_M^2 = M^2 + \delta \simeq \eta^2 m^2 \ , \tag{29}$$

ensure that the regulated integral D_{dcm} in (21) or (27) and Ω_{dcm} in (16) are independent of individual mass levels ($\lambda = \pm 1$) for $M \gg m$. In the expansion of P_M^2 about M^2 on the right side of (29), any off-shell term δ is assumed bounded and therefore negligible compared to M^2.

Computing (22) using (26) with (28) and (29), external momentum parameters go on-shell in $\Delta_\circ$; that is,

$$p^2 \to m^2 \ , k^2 \to m_b^2 \ , \ q^2 \to 0 \ ,$$

which we recognize as on-shell renormalization conditions. Dressed momentum transfer q_{dcm} is assumed bounded, so $\lim_{\eta\to\infty} \eta^{-2} q_{dcm}^2 = 0$. If particle masses external and internal to the blob in Fig. 4 (a) are both zero, choose $a = 1$, $c = 0$, and make the replacement

$$m_b \to m_b + \lambda M \left.\right|_{m_b=0, M=\eta\mu_\circ} \tag{30}$$

in $\Delta\,(m = m_b)$, where $M \equiv \eta\,\mu_\circ$ is the gauge boson self-energy, and $\mu_\circ$ represents one unit of mass measure. Thus, $\Delta_\circ$ is non-zero for all $m \geq 0$, and the net S-matrix amplitude computed from (15) is well defined since it involves a factor

$$\frac{\Gamma\,(\sigma)}{\Delta^\sigma} - \frac{\Gamma\,(\sigma)}{\Delta_\circ^\sigma} = -\ln\frac{\Delta}{\Delta_\circ} \ .$$

In addition to a divergent part, Ω_{dcm} in (15) may include a finite part that cancels a like term in Ω_{core}.

IV. QED APPLICATIONS

Let us apply the foregoing theory with integration formulae given above to verify that the net amplitudes for second order radiative corrections in QED are convergent and agree with results obtained via renormalization theory. Cutoff and dimensional regularization approaches will be used to illustrate the method.

A. Vacuum polarization

The photon self-energy associated with Fig. 3 (a) results in a propagator modification [22]

$$iD_F'^{\alpha\beta} = iD_F^{\alpha\beta} + iD_F^{\alpha\mu}\,\left(i\Pi_{\mu\nu}\right)\,iD_F^{\nu\beta} \ ,$$

where

$$\Pi_{\mu\nu} \equiv \Pi_{\mu\nu}^{core} + \Pi_{\mu\nu}^{dcm}$$

is a polarization tensor generalized to include the stability (aka DCM) correction, and whose core mass term

$$\Pi_{\mu\nu}^{core}\,(k,\,m) = \frac{ie^2}{(2\pi)^4}\int d^4p\,\mathrm{Tr}\left[\gamma_\mu S_F(p,m)\gamma_\nu S_F(p-k,m)\right] \tag{31}$$

follows from the Feynman-Dyson rules [6, 13]. In consequence of Lorentz and gauge invariance [8] or by direct calculation, it factors into

$$\Pi_{\mu\nu}^{core}\,(k,\,m) = \Pi_{core}\left(k^2,\,m\right)\left(k_\mu k_\nu - g_{\mu\nu}k^2\right) \ , \tag{32}$$

where $\Pi_{core}\left(k^2, m\right)$ is a real scalar function. As is well known, the contribution from terms $k_\mu k_\nu$ vanishes due to current conservation upon connection to an external fermion line.

Since the scattering amplitude is in general a complex analytic function, it follows from Cauchy's formula that the real and imaginary parts are related by a dispersion relation [27]. The imaginary part is divergence free and may be obtained by replacing Feynman propagators with cut propagators on the mass shell according to Cutkosky's cutting rule [28] or, alternatively, via calculation in the Heisenberg representation as shown in Källén [29]. In particular for vacuum polarization, the real part for the core mass is given by

$$\Pi_{core}\left(k^2,\, m\right) = \frac{1}{\pi}\int_{4m^2}^{4\Lambda_\circ^2} ds\,\frac{g\left(\frac{4m^2}{s}\right)}{s-k^2} \tag{33}$$

with imaginary part

$$g\left(w\right) = \frac{\alpha}{3}\sqrt{1-w}\left(1+w/2\right).$$

Applying (16), using (28), and performing a change of variables $s=\eta^2 t$ in (33), we have

$$\begin{aligned}\Pi_{dcm} &= -\frac{1}{2}\lim_{\eta\to\infty}\left[\Pi_{core}\left(k^2,\, m+\eta m\right)+\Pi_{core}\left(k^2,\, m-\eta m\right)\right] \\ &= -\frac{1}{\pi}\lim_{\eta\to\infty}\int_{4m^2}^{4\Lambda_\circ^2} dt\,\frac{g\left(\frac{4m^2}{t}\right)}{t-\eta^{-2}k^2}\,.\end{aligned} \tag{34}$$

Letting $\eta\to\infty$, we see that (34) is equivalent to the subtracted core amplitude evaluated on the light cone

$$\Pi_{dcm} = -\Pi_{core}(k^2=0,\, m)$$

which is associated with a correction to the bare charge in renormalization theory, but here the correction represents an interaction between the observed core electron charge associated with a transient DCM state and a polarization current that is required for charge stability in the intermediate state. Combining (33) and (34), we obtain a once-subtracted dispersion relation

$$\begin{aligned}\Pi &= \Pi_{core}+\Pi_{dcm} \\ &= \frac{k^2}{\pi}\int_{4m^2}^{\infty} ds\,\frac{g\left(\frac{4m^2}{s}\right)}{s\left(s-k^2\right)}\end{aligned} \tag{35}$$

in agreement with renormalized QED.

B. Fermion self-energy

The fermion self-energy operator for the core mass corresponding to the Feynman diagram in Fig. 3 (b) is given by

$$\Sigma_{core}\left(p,m\right) = \frac{-ie^2}{\left(2\pi\right)^4}\int d^4k\,\gamma_\mu\,S_F\left(p-k,m\right)\,\gamma^\mu\,\frac{1}{k^2-\mu^2}\,. \tag{36}$$

After standard reduction and dimensional regularization, Σ_{core} simplifies to

$$\Sigma_{core}\left(p,m\right) = \frac{\alpha}{2\pi}\left\{S_1+\int_0^1 dx\,\left[2m-\not{p}x+\sigma\left(\not{p}x-m\right)\right]\,D\left(\Delta,\sigma\right)\right\}\,, \tag{37}$$

where $D\left(\Delta,\sigma\right)$ is given by (25) with

$$\Delta = \left(1-x\right)\left(m^2-xp^2\right)+x\mu^2\,.$$

The integral expression in (37) is equivalent to a form given in Peskin & Schroeder [30], while the term

$$S_1 = -\frac{1-\sigma}{4}\not{p}$$

follows from appendix formulae in [25] and represents a surface contribution arising from a term linear in k during reduction of (36).

Evaluation of Σ_{dcm} using (16) reduces to negating (37) and replacing $\Delta \to \Delta_\circ$ according to (27); we obtain

$$\Sigma_{dcm}\left(p,m\right) = -\frac{\alpha}{2\pi}\left\{S_1 + \int_0^1 dx\ \left[2m - \not{p}x + \sigma\left(\not{p}x - m\right)\right]\ D\left(\Delta_\circ,\sigma\right)\right\}\ , \tag{38}$$

where

$$\Delta_\circ = m^2\left(1-x\right)^2 + x\mu^2$$

follows from (22) using (28), (29), and (13). Terms involving $(\lambda P_M, \lambda M)$ have canceled in the average over DCM mass levels yielding a function of the core mass and momentum only. The net correction, including all three mass levels in Fig. 3 (b), is given by (cf. [13])

$$\begin{aligned}\Sigma &= \Sigma_{core} + \Sigma_{dcm} \\ &= \frac{\alpha}{2\pi}\int_0^1 dx\ \left(2m - \not{p}x\right)\ln\frac{m^2\left(1-x\right)^2 + x\mu^2}{\left(m^2 - xp^2\right)\left(1-x\right) + x\mu^2}\ ,\end{aligned} \tag{39}$$

where the limit $\sigma \to 0$ has been taken to recover four-dimensional space-time. With a change of variables $x = 1 - z$, (39) is seen to be identical to the renormalized result given in Bjorken & Drell [31].

The processes in Fig. 3 (b), including iterations, results in a modified propagator [6, 22]

$$\begin{aligned}iS'_F &= iS_F + iS_F\ \left(-i\Sigma\right)\ iS'_F \\ &= \frac{i}{\not{p} - m - \Sigma + i\varepsilon}\ ,\end{aligned} \tag{40}$$

which has the desired pole at $\not{p} = m$ since (39) vanishes on the mass shell

$$\mathit{\Sigma}\left(p^2 = m^2\right) = 0\,. \tag{41}$$

Upon identifying

$$m^+_{em} = \mathit{\Sigma}_{core}\left(\not{p} = m,\ \mu = 0\right) \tag{42}$$

$$m^-_{em} = \mathit{\Sigma}_{dcm}\left(\not{p} = m,\ \mu = 0\right) \tag{43}$$

we see that (41) is equivalent to the stability principle (4). Reverting to cutoff $\Lambda_\circ$ using (19), it follows that (42) reduces to Feynman's result (1); for derivation, see [25]. In the language of renormalization theory, the bare mass in the propagator [17]

$$iS'_F\left(p,m\right) = \frac{i}{\not{p} - m_b - \Sigma_{core} + i\varepsilon}$$

must be renormalized using (6) with (42).

C. Vertex

A second-order correction to a corner involves a replacement

$$ie\gamma^\mu \to ie\left(\gamma^\mu + \mathit{\Lambda}^\mu\right) , \tag{44}$$

where the vertex function $\mathit{\Lambda}^\mu$ for the core mass corresponding to Fig. 3 (c) is approximated by [13]

$$\Lambda^\mu_{core}\left(q,m\right) = \gamma^\mu L + a^{(2)}\frac{i}{2m}\sigma^{\mu\nu}q_\nu + O\left(\frac{q^2}{m^2}\right) . \tag{45}$$

for small q^2. The divergent constant

$$L = \frac{\alpha}{4\pi}\left(D_\circ + \frac{11}{2} - 4\ln\frac{m}{\mu}\right) , \tag{46}$$

and $\sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right]$ are spin matrices. Note that $L = \frac{\alpha}{2\pi}r$, where r is given by Eq. (23) in [13]. The coefficient $a^{(2)} = \frac{\alpha}{2\pi}$ is the second-order contribution to the anomalous magnetic moment first derived by Schwinger [32] and verified experimentally by Foley & Kusch [33].

Inserting (45) into (16), using (13), and accounting for the invariance of $D_\circ$ (24) under scaling rules (17) and (18), the stability correction is $\Lambda^\mu_{dcm} = -\gamma^\mu L$, where finite terms in (45) of order $O\left(\frac{q}{m}\right)$ with replacements $m \to m + \lambda M$ and $q \to q_{dcm} \equiv q + \lambda\left(P'_M - P_M\right)$ vanish in the limit $M \to \infty$ since q_{dcm} is assumed bounded. Therefore, the total vertex function $\Lambda^\mu = \Lambda^\mu_{core} + \Lambda^\mu_{dcm}$ is convergent, and $\mathit{\Lambda}^\mu$ satisfies the usual renormalization condition for a vertex

$$\Lambda^\mu|_{q^2=0,\not p=\not p'=m} = 0 .$$

This completes verification that lowest-order S-matrix corrections are finite without renormalization.

V. GENERALIZATION TO HIGHER ORDERS

The next task is to show that higher-order radiative corrections are convergent. The proof closely follows methods in the references; therefore, we keep our remarks brief highlighting required modifications.

Irreducible (skeleton) diagrams include second-order self-energy (SE) and vertex (V) parts discussed in Sec. IV plus infinitely many higher-order primitively divergent V-parts. Using Dyson's expansion method [22], second-order SE- and V-part operators for the core mass are

$$\Sigma_{core} = mA - \left(\not p - m\right)B + \Sigma , \tag{47}$$

$$\mathit{\Pi}_{core} = C + \mathit{\Pi} , \tag{48}$$

$$\mathit{\Lambda}^\mu_{core} = \gamma^\mu L + \mathit{\Lambda}^\mu , \tag{49}$$

where $\{A,\, B\,, C\,,\, L\}$ are logarithmically divergent coefficients depending on $D_\circ$ – specifically, $A = \frac{3\alpha}{4\pi}\left(D_\circ + \frac{3}{2}\right)$ using (1) and $B = L$ from (46). Insignificant finite terms can depend on the regularization method used; for example, compare $C = -\Pi_{dcm} = \frac{\alpha}{3\pi}\left(D_\circ + \ln 4 - \frac{2}{3}\right)$ from (34) with [25]. Higher-order primitively divergent V-parts are also of the form (49) since $K = 0$ in the divergence condition

$$K = 4 - \frac{3}{2}f_e - b_e \geq 0 ,$$

wherein $f_e\left(b_e\right)$ are the number of external fermion (boson) lines; in this case, $L\left(D_\circ\right)$ is a power series in α.

To determine the interaction of an electromagnetically dressed core with the polarized vacuum, we apply (16) with (24) to obtain

$$\Sigma_{dcm} = -\left[mA - \left(\not p - m\right)B\right] , \tag{50}$$

$$\mathit{\Pi}_{dcm} = -C , \tag{51}$$

$$\mathit{\Lambda}^\mu_{dcm} = -\gamma^\mu L , \tag{52}$$

where the vanishing of the finite parts $\{\Sigma,\, \Pi,\, \Lambda^{\mu}\}$ as $M \to \infty$ is both a physical requirement and a consequence of their convergent integrals. In this way, (14) yields convergent results

$$\Sigma = \Sigma_{core} + \Sigma_{dcm} \tag{53}$$

$$\Pi = \Pi_{core} + \Pi_{dcm} \tag{54}$$

$$\Lambda^{\mu} = \Lambda^{\mu}_{core} + \Lambda^{\mu}_{dcm} \tag{55}$$

for all irreducible diagrams; therefore, SE-part insertions

$$iS_F \to iS_F + iS_F\,(-i\Sigma)\, iS_F \tag{56}$$

$$iD_F^{\alpha\beta} \to iD_F^{\alpha\beta} + iD_F^{\alpha\mu}\left(-ig_{\mu\nu}k^2\Pi\right) iD_F^{\mu\beta} \tag{57}$$

into lines, and V-part insertions

$$\gamma^{\mu} \to \gamma^{\mu} + \Lambda^{\mu} \tag{58}$$

into corners of a skeleton diagram yield no additional divergences.

For reducible vertex diagrams, the V-part resolves into a skeleton along with SE- and V-part insertions. With replacements (56), (57), and (58) in the skeleton, the vertex operator again reduces to the form (49), where $L \to L_s$ is the skeleton divergence. In general, L_s depends on multiple functions $D_\circ$ corresponding to all possible charged fermion masses arising from photon self-energy insertions which may in turn contain SE- and V-parts. Since each $D_\circ$ is invariant under (17) and (18) and Λ^{μ} vanishes as $M \to \infty$, (16) yields $\Lambda^{\mu}_{dcm} = -\gamma^{\mu} L_s$ similarly to (52); therefore, the complete reducible V-part given by (55) is convergent.

For reducible self-energy diagrams, a skeleton with SE insertions is handled in the same way as reducible vertex diagrams. However, vertex insertions into fermion and photon SE skeletons involve overlapping divergences that require further analysis [34, 35]. Integration of Ward's identities yields expressions of the same form as (47) and (48); in this case, the coefficients $\{A,\, B\,, C\}$ are all power series in α depending on $D_\circ$, and vertex insertions in SE-parts are convergent upon including stability corrections (50) and (51). We conclude that infinite field actions excite mass levels $m \pm M$ uniformly in all connected fermion lines internal to overlapping loops; for a specific example, apply (15) to calculate the real part of the fourth-order vacuum polarization kernel [36] using the dispersion method given in Sec. IV. Therefore, a diagram with overlapping divergences is not a special case for implementation of stability corrections.

The complete propagators, replacing fermion and photon lines in a skeleton diagram, are given by

$$iS_F'(p) = \frac{i}{\not p - m - \Sigma^* + i\varepsilon}\,,$$

$$iD_F'^{\alpha\beta}(k) = \frac{-ig^{\alpha\beta}}{k^2\left[1 + \Pi^*\right] + i\varepsilon}\,,$$

where $\{\Sigma^*,\, \Pi^*\}$ are given by sums over all proper SE-parts. Similarly, the most general vertex replacing a corner in a skeleton diagram is given by a sum over all proper V-parts. Since both core and DCM contributions are included for each sub-diagram, the complete propagators and vertices are well defined (convergent).

VI. NON-ABELIAN APPLICATION

Finally, we apply the stability method to compute radiative corrections in non-Abelian gauge theory [37]. In the following examples, we focus on key one-loop diagrams discussed in [30, 38].

For gauge bosons in lieu of photons in Fig. 3, the core amplitude differs from QED only by a group factor; therefore, finite S-matrix amplitudes, including stability corrections in (15), are given by

$$\Pi^{ab}(GB) = tr\left(t^{a}t^{b}\right)\Pi(QED) \tag{59}$$

$$\Sigma(GB) = t^{a}t^{a}\,\Sigma(QED) \tag{60}$$

$$\Lambda^{a}_{\mu}(GB) = t^{b}t^{a}t^{b}\Lambda_{\mu}(QED)\,, \tag{61}$$

where $\{t^{a},\, t^{b}\}$ are hermitian generator matrices of some representation R of a semisimple Lie group G. The t-matrices occur in a fermion/gauge-boson vertex $ig\gamma^{\mu}t^{a}$, and they satisfy commutation relations

$$\left[t^{a},\, t^{b}\right] = if^{abc}t^{c}\,,$$

where f^{abc} are structure constants of the group. The structure constants, which arise in three- and four-gauge-boson vertices, satisfy

$$f^{acd} f^{bcd} = C_2(G)\, \delta^{ab} \,,$$

where $C_2(G)$ is the Casimir operator for the adjoint representation of group G. In (59),

$$tr\left(t^a t^b\right) = C(R)\, \delta^{ab} \,,$$

where $C(R)$ is a constant. For the special unitary group $SU(N)$ of degree N, $C_2(G) = N$, and $C(R) = \frac{1}{2}$ for matrices of the fundamental representation.

In addition to the fermion loop diagram in Fig. 3 (a), massless vector bosons in Fig. 5 give [30]

$$\Pi^{ab}_{\mu\nu}\left(k^2\right) = C_2(G)\, \delta^{ab} \left(g_{\mu\nu} k^2 - k_\mu k_\nu\right) \Pi_{core}\left(k^2\right) \tag{62}$$

$$\Pi_{core}\left(k^2\right) = \frac{ig^2}{(4\pi)^{d/2}} \int_0^1 dx\, \frac{\Gamma(\sigma)}{\Delta^\sigma} \left[(-1+\sigma)(1-2x)^2 + 2\right] \,, \tag{63}$$

where x is a Feynman parameter, and $\Delta = -k^2 x(1-x)$. For massive bosons, the propagators in the loops would be modified

$$\frac{1}{p^2 - m_b^2} \frac{1}{(p+k)^2 - m_b^2} = \int_0^1 \frac{dx}{\left[P^2 - \Delta(m_b)\right]^2} \,,$$

where the usual change of variables $P = p + xk$ has been made for loop integration parameter p, and

$$\Delta(m_b) = m_b^2 - k^2 x(1-x) \,. \tag{64}$$

To evaluate the stability contribution, we make the replacement (30) in (64); therefore, from (16) and (27), Π_{dcm} is simply obtained by negating (63) and replacing

$$\frac{1}{\Delta^\sigma} \rightarrow \frac{1}{\Delta_\circ^\sigma} \,,$$

where $\Delta_\circ = \mu_\circ^2$ using (22). From (14), the net amplitude

$$\Pi\left(k^2\right)(GB) = -\frac{ig^2}{(4\pi)^2} \int_0^1 dx\, \ln\left(\left|\frac{\Delta}{\mu_\circ^2}\right|\right) \left[-(1-2x)^2 + 2\right] \tag{65}$$

is finite. Note that all three diagrams must be combined to eliminate quadratic divergences before the stability correction is computed. More generally, individual diagrams may need to be combined to eliminate quadratic divergences that occur when applying (16); for example, computation of individual vacuum polarization amplitudes $\Pi^{\mu\nu}_{LL}, \Pi^{\mu\nu}_{LR}, \Pi^{\mu\nu}_{RL}, \Pi^{\mu\nu}_{RR}$ for left- and right-handed currents in weak-interaction gauge theory yields core amplitudes of order m^2 which at first appears problematic. For the stability correction, numerator factors proportional to λM vanish immediately for each current upon summing over λ; however, $O\left(M^2\right)$ terms vanish only upon taking the sum of DCM corrections for {LL, LR, RL, RR} currents. For equal masses in each loop segment, the sum reduces to (34), and the total amplitude, equivalent to (35), is finite. For an exhaustive list of Feynman diagrams for electroweak theory, see [39].

The total amplitude for the three-gauge-boson/fermion vertex shown in Fig. 6 is given by

$$ig\Lambda^\mu(GB) = \frac{ig^3 f^{abc} t^b t^c}{(2\pi)^4} 2 \int dxdydz\, \delta(x+y+z-1)\left(I^\mu_{core} + I^\mu_{dcm}\right) \tag{66}$$

$$I^\mu_{core} = I^\mu_1 + I^\mu_2 \tag{67}$$

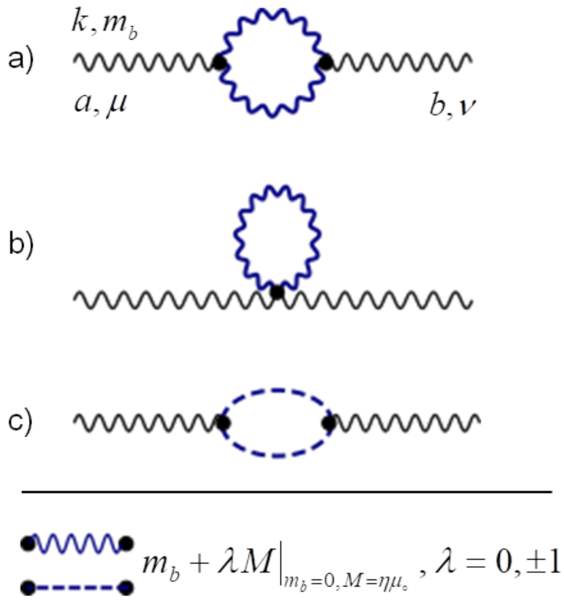

FIG. 5: Order g^2 gauge boson self-energy corrections: a) gauge boson loop, b) four-gauge-boson vertex, and c) ghost loop.

$$
\begin{aligned}
I_1^{\mu} &= \int^{\Lambda_\circ} \frac{d^d l}{\left(l^2-\Delta\right)^3} \gamma_\nu \not{l} \gamma_\rho \left(-g^{\mu\nu} l^\rho + 2 g^{\nu\rho} l^\mu - g^{\rho\mu} l^\nu\right) \,, \\
&= -3 i \pi^2 \frac{\Gamma(\sigma)}{\Delta^\sigma} \gamma^\mu
\end{aligned}
\tag{68}
$$

$$
\begin{aligned}
I_2^{\mu} &= \int \frac{d^4 l}{\left(l^2-\Delta\right)^3} N_f(m,\, q) \,, \\
&= -\frac{i\pi^2}{2\Delta} N_f(m,\, q)
\end{aligned}
\tag{69}
$$

where $l = k - p' x - p y$,

$$
\Delta = -xyq^2 + m^2 z^2 + \mu^2 (1-z) \,, \tag{70}
$$

μ is a small boson mass, and

$$
N_f(m,\, q) = m^2 \gamma^\mu f_1 + q^2 \gamma^\mu f_2 - i \sigma^{\mu\nu} q_\nu m f_3 \,, \tag{71}
$$

wherein $\left\{f_1 = 2x - 7z^2,\ f_2 = 2(xy - x - y),\ f_3 = 2z(1-z)\right\}$ depend on Feynman parameters (x, y, z). The cutoff is retained in the divergent part I_1^μ as a reminder that for computation of the stability correction, $\Lambda_\circ \to \eta \Lambda_\circ$ followed by a change of variables $l \to \eta l$. Now apply (16) and (27) to obtain

$$
I_{dcm}^{\mu} = 3 i \pi^2 \frac{\Gamma(\sigma)}{\Delta_\circ^\sigma} \gamma^\mu + \frac{i\pi^2}{2\Delta_\circ} N_f(m,\, q=0) \,, \tag{72}
$$

where from (22), $\Delta_\circ = \Delta\left(q^2 = 0\right)$. Finally the stabilized integral is given by

$$
\begin{aligned}
I^\mu &= I_{core}^\mu + I_{dcm}^\mu \\
&= \ln\left(\frac{\Delta}{\Delta_\circ}\right) + finite \,,
\end{aligned}
\tag{73}
$$

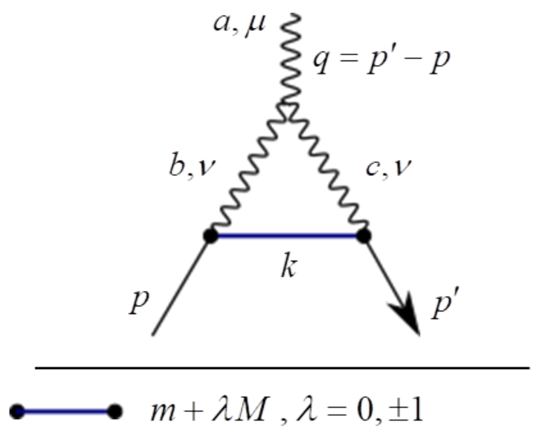

FIG. 6: Three-gauge-boson vertex.

and the complete amplitude (66) is finite without renormalization.

Since renormalization theory accounts for the core S-matrix amplitude only, the seven diagrams considered in Figs. 3, 5, and 6 yield a running coupling constant characterized by the Callan-Symanzik [40, 41] β function [42, 43]

$$\beta\left(g\right)=\frac{\partial g}{\partial\ln\mu}=-\frac{g^{3}}{16\pi^{2}}\left[\frac{11}{3}C_{2}\left(G\right)-\frac{4}{3}n_{f}C\left(r\right)\right]\ ,$$

wherein μ is the energy scale, and n_f is the number of fermion species contributing in Fig. 3 (a). In contrast, the coupling constant is static in the proposed stability formulation of scattering amplitudes; thus, $\beta = 0$, and QFT is scale invariant.

VII. CONCLUDING REMARKS

For QED, we developed a model for a stable elementary charge wherein a hidden interaction between the electromagnetically dressed charge and an opposing polarization current offsets the positive electromagnetic (EM) field energy. Concise rules for constructing S-matrix corrections for the electromagnetically dressed core were developed and applied to resolve divergence issues to all orders, and we maintained the fermion mass and charge as observed fundamental constants throughout. Our findings provide compelling evidence for negative and positive EM mass components in virtual intermediate states of infinitesimally short duration. Since there is no renormalization in this approach, the EM coupling is a constant independent of the energy scale; therefore, QED is scale-invariant. Likewise, for non-Abelian gauge theory, we found that one-loop diagrams are finite without renormalization if stability corrections are included. Model predictions agree precisely with renormalized QFT and therefore are consistent with current experiments also. These predictions are in sharp contrast to renormalization group arguments [44, 45] which contend that the coupling constant, e in QED or g in non-Abelian gauge theory, scales with energy. The stability approach for computing finite amplitudes in QFT is simpler compared to renormalization, and it more accurately characterizes the physics involved in radiative processes since it includes the reaction (16) of the vacuum back on a core charge.

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