On the Relativity and Equivalence Principles in Quantized Gauge Theory Gravity

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Ongoing proliferation of interpretations of the formalism of quantum mechanics over the course of nearly a century originates in historical choices of wavefunction, and in particular with assignment of geometric and topological attributes to unintuitive internal intrinsic properties of point particle quarks and leptons. Significant simplification and intuitive appeal arise if one extends Dirac's two-component spinors to the full eight-component Pauli algebra of 3D space, providing a geometric representation of wavefunctions and their interactions that is comparatively easily visualized. We outline how the resulting quantized impedance network of geometric wavefunction interactions yields a naturally finite, confined, and gauge invariant model of both the unstable particle spectrum and quantum gravity that is consistent with and clarifies interpretation of both special relativity and the equivalence principle.

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INTRODUCTION

Inspiration for this essay was found in a similarly titled short note[1]. It is no coincidence that 52 years earlier the senior author of that note co-authored the first published paper on *geometric* Clifford wavefunctions[2]. Both play an essential role in this essay, which casts special relativity and the equivalence principle into a web of quantized impedance networks of geometric wavefunction interactions. What follows is organized as shown in the outline below. The first five items provide context - the quantized gauge theory gravity model[3, 4] essential for the last three:

1. *geometric wavefunctions* - to begin we introduce geometric quantization of space, the vacuum wavefunction,
2. *geometric wavefunction interactions* - then vacuum wavefunction interactions modeled by the geometric product,
3. *field quantization* - followed by electromagnetic quantization of its fields
4. *quantized interaction impedances* - with amplitude and phase governed by quantized interaction impedances,
5. *black hole impedance and the Hawking photon* - to quantize gauge theory gravity at the Planck length,
6. *equivalence principle* - explaining origins of gravitational and inertial masses
7. *special relativity* - and the role of quantum logic in special relativity interpretation.
8. *conclusion* - just how strong is geometric Clifford algebra?

A quick overview may be had by jumping to section 6 and referring back to earlier sections as needed.

1. GEOMETRIC WAVEFUNCTIONS

The mathematical language of particle physics is Clifford algebra[5–7]. Modern usage employs the abstract and unwieldy *matrix representation* of Pauli and Dirac, an historical accident. The intent of Grassman and Clifford was to discover an intuitive *spatial algebra* of geometric objects - point, line, plane, and volume elements of Euclid - and their interactions, a *geometric representation*. Clifford himself called it Geometric Algebra.

With the death of Clifford at age 33 in 1879, the absence of an advocate to balance the powerful Gibbs contributed to adoption of his less comprehensive vector algebra. "This was effectively the end of the search for a unifying mathematical language and the beginning of a proliferation of novel algebraic systems..."[8]

Clifford algebra resurfaced without geometric meaning in Pauli and Dirac matrices of the 1920s. However the geometric representation remained dormant until taken up and extended by David Hestenes four decades later[9, 10]. Another four passed before he was awarded the 2002 Oersted medal for "Reforming the Mathematical Language of Physics" by the American Physical Society[11].

Just as absence of the geometric interpretation from mainstream physics can be understood as an historical accident, so can absence of that which governs amplitude and phase of energy transmission in quantum mechanics, the absence of impedance quantization as explained in section 4. As shown in figure 1, synthesis of the two is essential for a functional model.

The geometric vacuum wavefunction model of this essay is comprised of orientable fundamental geometric objects of a minimally complete Pauli algebra of physical 3D space - one scalar point, three vector lines, three pseudovector bivector planes, and one pseudoscalar trivector volume element, as shown at top and left of figure 4.

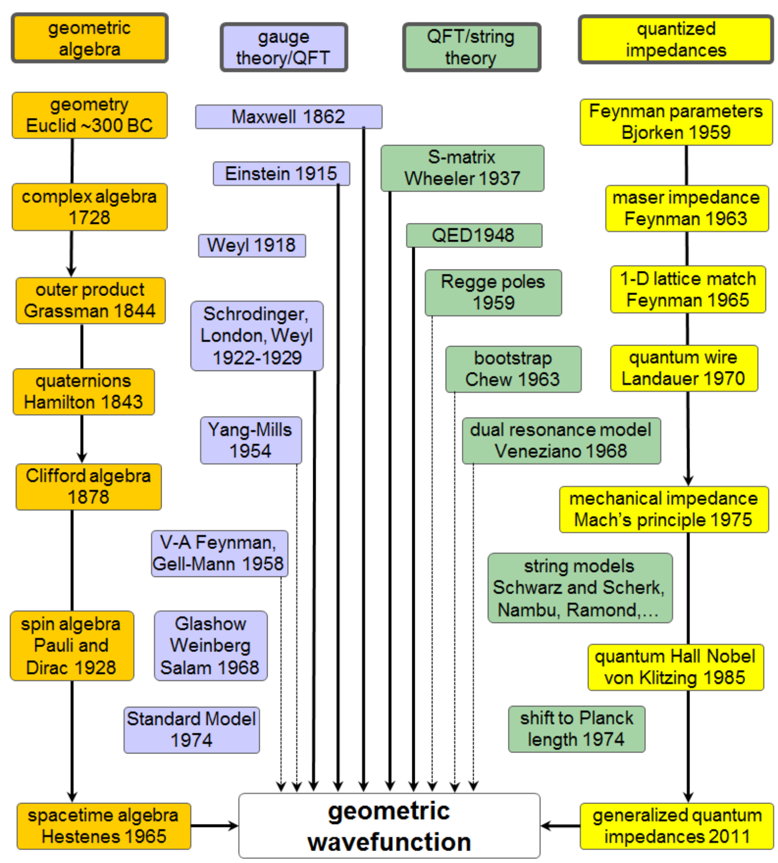

FIG. 1: Four threads of the geometric wavefunction. Connections with early gauge/string theory milestones are reasonably well understood in the model, later not yet so much.

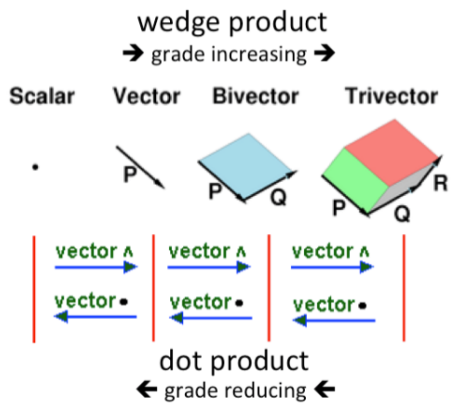

FIG. 2: Geometric algebra components in three dimensions. The two products (dot and wedge or inner and outer) that comprise the geometric product raise and lower the grade. For example, the product of two grade 1 vectors is $ab = a \cdot b + a \wedge b$, the inner product $a \cdot b$ being a grade 0 scalar and the outer a grade 2 bivector[12]

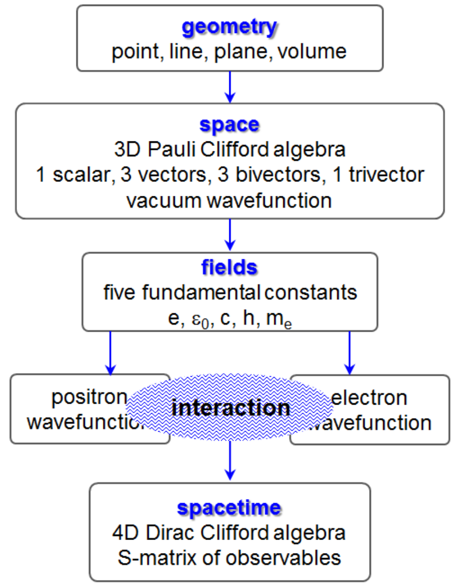

FIG. 3: 'Theoretical minimum' of the GWI model

2. GEOMETRIC WAVEFUNCTION INTERACTIONS

In the geometric wavefunction interaction (GWI) model, interactions are modeled by geometric products. For example, the product of two vectors ab mixes spatial dimension, or *grade*. Mixing of grades makes geometric algebra unique in the ability to handle geometric concepts in any dimension, provides the non-linear process necessary for wavefunction creation and collapse, for shifting linear electromagnetic field energy in the frequency domain.

Figure 3 shows what might be regarded as a theoretical minimum for understanding the GWI model. In this section we address the geometry sans fields, the vacuum wavefunction of the previous section, and the resulting geometric S-matrix of figure 4. What governs interactions in the geometric model sans fields is the algebra. Section 3 introduces quantized electromagnetic fields. What governs (amplitude and phase of) interactions in the electromagnetic geometric model are quantized interaction impedances of section 4.

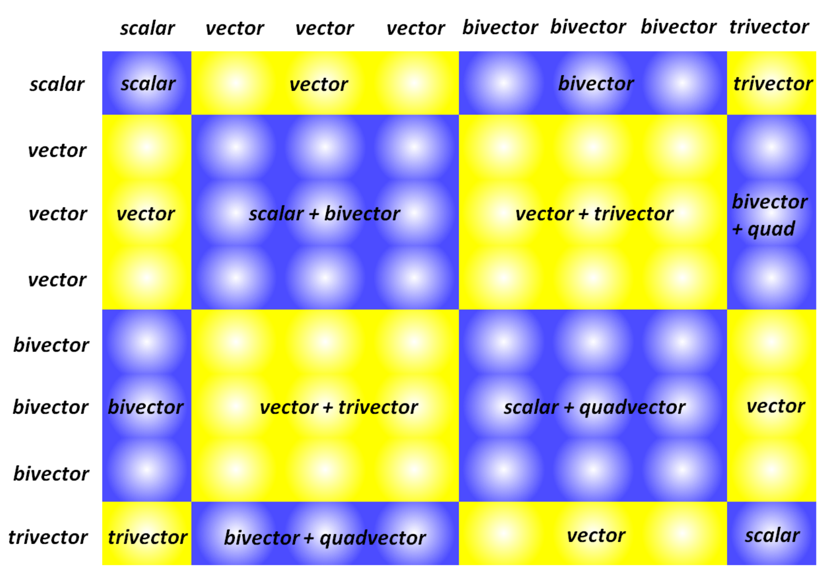

FIG. 4: Dirac S-matrix of *geometric* 'virtual observables' in flat Minkowski spacetime generated by geometric product of 3D Pauli vacuum wavefunctions as shown at top and left of the matrix.

At top and left of figure 4 are vacuum wavefunctions of 3D space. Their geometric product generates the 'virtual scattering matrix', a 4D Dirac algebra in flat Minkowski spacetime. When perturbed, within the constraint of Maxwell's equations any of the 64 modes shown there can couple to any other mode. With number of combinations factorial in number of modes, including orientational degrees of freedom opens very large numbers of possibilities for particle wavefunction composition, more than there are protons in the observable universe. A close look at the combinatorics could offer guidance in the choice of gauge group[13].

Odd grade transition modes (vector and pseudovector trivector) are highlighted in yellow, emerging even eigenmodes (scalar, bivector, and pseudoscalar quadvector) in blue. If the only wavefunction scalar is electric charge, then charge is present in initial and final states, absent in transition modes.

3. FIELD QUANTIZATION

Of itself geometry is not adequate. To manifest physically requires assignment of topologically appropriate fields to geometric objects of the wavefunction. Simplest possibility is to restrict attention to electromagnetic fields only. We know they are quantized - electric charge quantum, magnetic flux quantum, Bohr magneton,... all fundamental constants.

Geometric quantization is present in the wavefunction - geometric objects are individual discrete units. There are no ‘continuum’ geometric objects in wavefunctions. One might argue that this is the origin of electromagnetic quantization, that geometric quantization mandates field quantization. To do so requires five fundamental constants input by hand - electric charge quantum, electric permittivity of free space, speed of light, Planck’s constant, and electron mass (to set the scale of space).

There are no free parameters.

Figure 5 shows the S-matrix that results from assigning fields to the vacuum wavefunction.

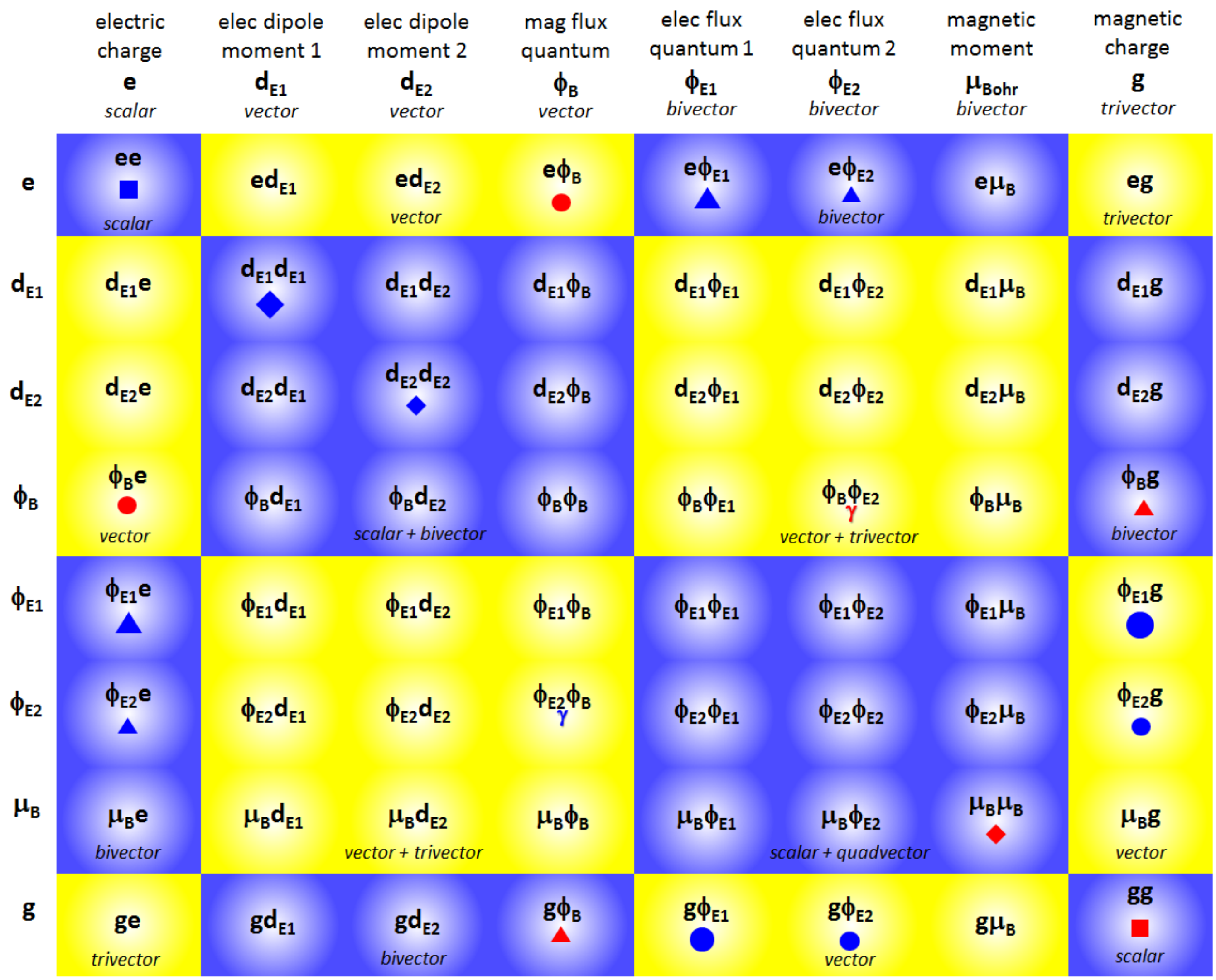

FIG. 5: **S-matrix:** As in the Dirac equation, eight-component wavefunctions at top and left can be associated with electron and positron. Their interaction generates a 4D Dirac algebra of flat Minkowski spacetime, arranged in odd transition (yellow) and even eigenmodes (blue) by geometric grade/dimension. Modes impedances indicated by symbols (triangle, square,...) are plotted in figure 8, opening new windows on the unstable particle spectrum[13].

4. QUANTIZED INTERACTION IMPEDANCES

Photon-electron interaction is the keystone of QED. What governs energy flow in photon-electron interactions is quantum impedance matching in the near field.

Why is this not common knowledge in physics? Missing from formal education of physicists are both photon near-field impedance[14] shown in figure 7 and the corresponding electron dipole impedance[15]. Neither can found in grad school physics texts, the curriculum, or the journals. The same is true of the single free electron quantized impedance network shown in figure 8. What governs energy transmission in photon-electron interactions is absent from mainstream physics.

The oversight can be attributed to **three primary causes**. The first is historical, the second follows from theorists' habit of setting fundamental constants to dimensionless unity, and the third from topological and electromagnetic paradoxes in our systems of units.[16]

Foundations of QED (red in fig.6) were set long before Nobel prize discovery of the scale invariant quantum Hall impedance in 1980 [17]. Prior to that impedance quantization was more implied than explicit in the literature [18–25]. The concept of impedance quantization did not exist, much less *exact* quantization.

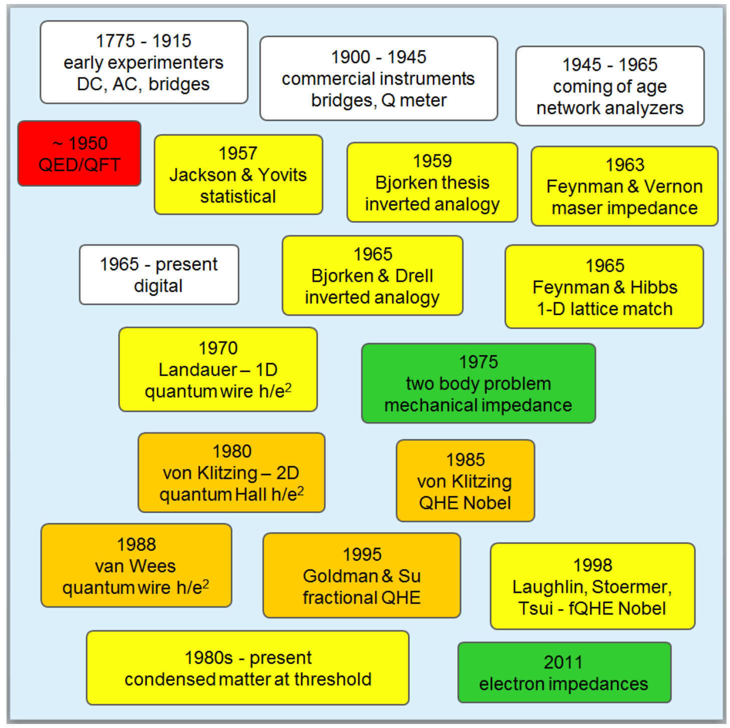

FIG. 6: Development of theory and technology of classical impedances highlighted in white, quantum impedances in yellow (theory) and gold (experiment), generalization of quantum impedances in green, and QED scaffolding in red.

The **second** origin of overlooked quantization is setting fundamental constants to dimensionless unity. Doing so with free space impedance made quantization just a little too easy to overlook. And to no useful purpose. What matters are not absolute values but relative, whether impedances are *matched*.

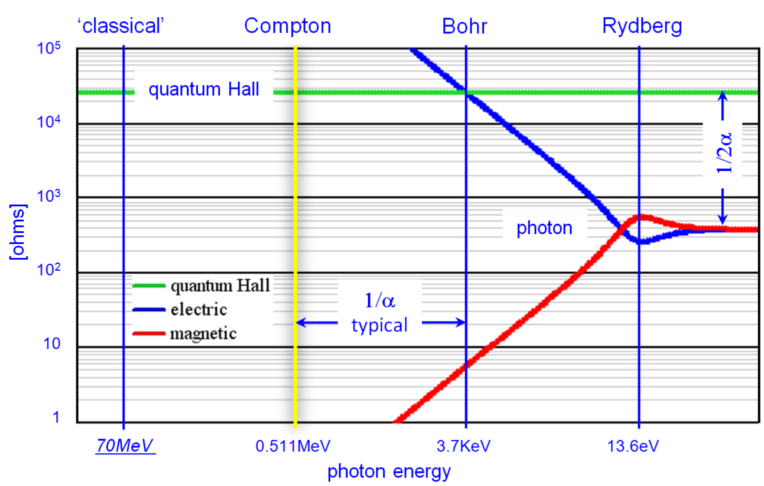

FIG. 7: Photon impedance match to the Hydrogen atom.

The **third** confusion is seen in an approach [20] summarized [21] as "...an analogy between Feynman diagrams and electrical circuits, with Feynman parameters playing the role of resistance, external momenta as current sources, and coordinate differences as voltage drops. Some of that found its way into section 18.4 of..." the canonical text [22]. As presented there, Feynman parameter units are [sec/kg], units not of resistance, but rather *conductance* [26].

It is not difficult to understand how we lost our way. The units of mechanical impedance are [kg/sec]. One would think more [kg/sec] would mean more mass flow. However, the physical reality is more [kg/sec] means more impedance and *less* mass flow. This is one of many interwoven mechanical, electromagnetic, and topological paradoxes [27] found in SI units, which ironically were developed with the intent that they "...would facilitate relating the standard units of mechanics to electromagnetism." [28].

With the confusion that resulted from misinterpreting conductance as impedance and lacking the concept of quantized impedance, the anticipated intuitive advantage [22] of the circuit analogy was lost. Possibility of the jump from well-considered analogy to a quantized photon-electron impedance model was not realized at that time.

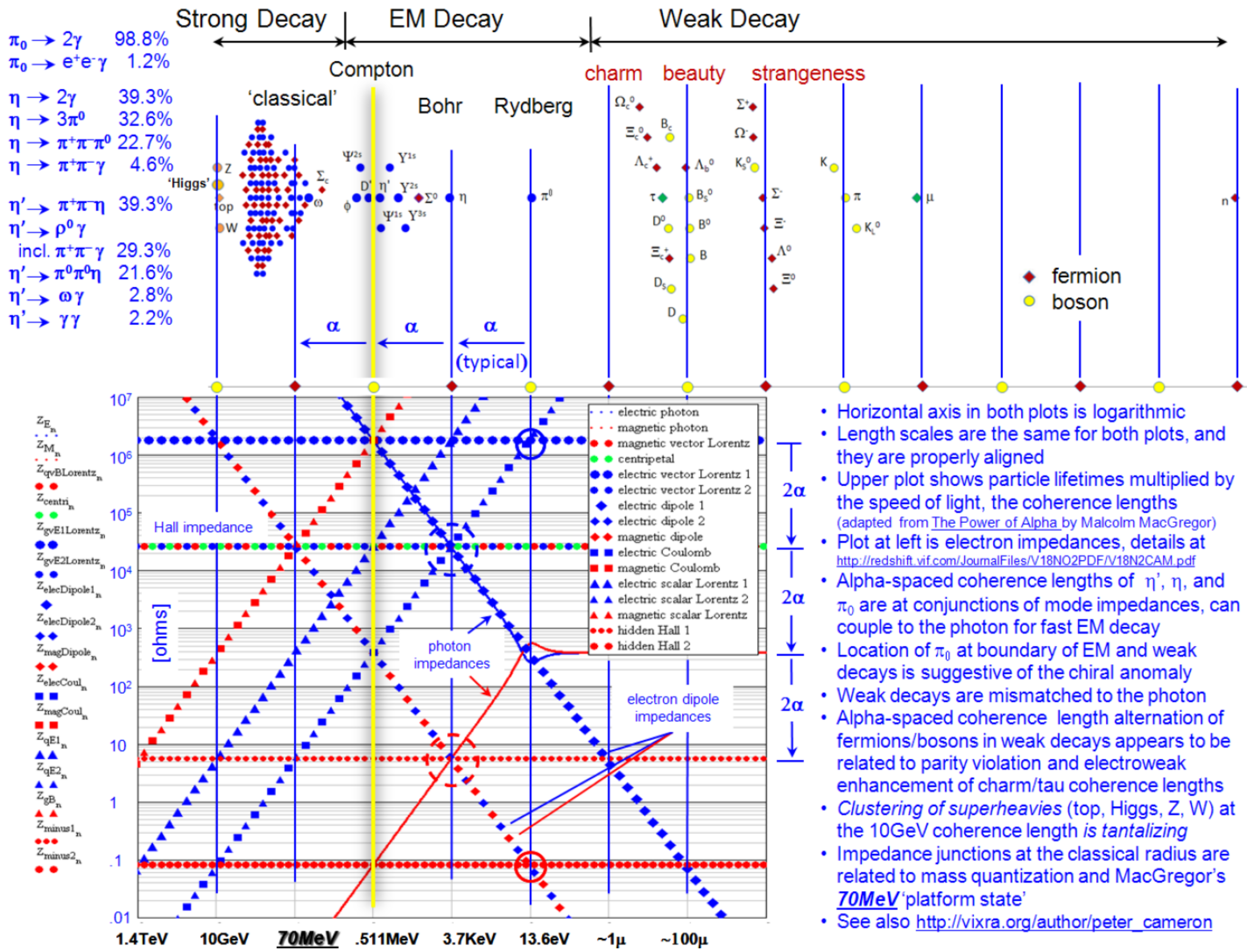

FIG. 8: Correlation of lifetimes/coherence lengths of unstable particle spectrum[29–31] with impedance nodes of the network generated by excitation of the lightest rest mass manifestation of the vacuum wavefunction.

As indicated by the corresponding symbols (triangle, square,...), the quantized impedance network of figure 8 is comprised of a subset of S-matrix modes of figure 5. Correlation follows from the fact that impedances must be matched for energy transmission between modes as required by particle decay.

The single most important consequence of impedance quantization to be understood is the distinction between scale-invariant and scale-dependent impedances. The one is associated with rotations, the other with translations.

Keeping in mind that interaction impedance matching governs amplitude and phase of energy transmission, invariant impedances communicate only phase, not a single measurement observable. The resulting motion is perpendicular to the applied force. No work is done, no energy/information communicated. Phase shifts associated with invariant impedances are the channel of non-local entanglement.

Invariant impedances include quantum Hall of the vector Lorentz force, centrifugal, Coriolis, chiral, and three body. Corresponding potentials are inverse square. Phase shifts of invariant impedances cannot be shielded. Quantum example is Aharonov-Bohm effect of vector Lorentz force. Of particular interest in the present context are the roles of invariant three-body and centrifugal impedances in respectively special relativity and the equivalence principle.

While figure 8 obviously addresses relativistic phenomena, and figure 9 tens-of-orders-of-magnitude moreso, there is no mention of special relativity in the material presented so far. How is this possible?

Similarly, the scale invariant centrifugal impedance, a property of inertial mass, communicates no energy, only phase. By the equivalence principle one would expect that gravitational mass would likewise be comprised of phase shifts only. However it is commonly understood that gravity can do work.

Both conundrums are unraveled in what follows.

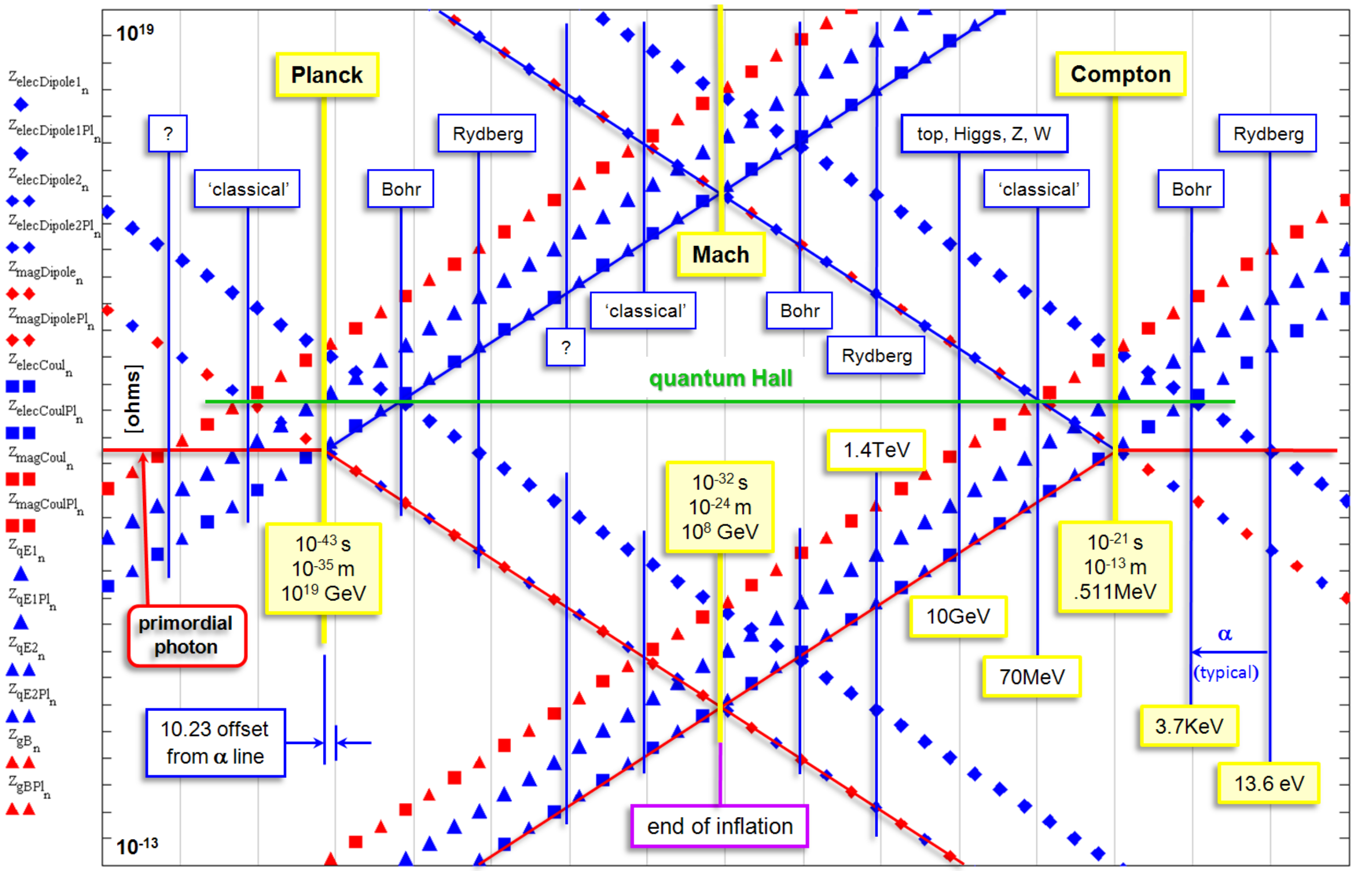

FIG. 9: A subset of the interaction impedance network of geometric wavefunctions defined at the Compton and Planck lengths, showing a .511 Mev photon entering from right and the 'primordial photon' from left. [32–35]

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5. BLACK HOLE IMPEDANCE AND THE HAWKING PHOTON

Just as the energy of a photon whose wavelength is the electron Compton wavelength equals the electron rest mass, the energy of a photon whose wavelength is the Planck particle Compton wavelength is the rest mass of the Planck particle and its associated event horizon. This is the 'electromagnetic black hole'.

Calculating the interaction impedance mismatch between Compton and Planck wavefunctions gives an identity between electromagnetism and gravity[32, 33]. The gravitational force between these two particles is equal to the impedance mismatched electromagnetic force they share. The gravitational constant G, by far the most imprecise of the fundamental constants, cancels out in the calculation. The GWI model delivers exact results at the Planck particle event horizon (and beyond to the singularity, completely decoupled by the infinite mismatch to the dimensionless point). Relativistic curvature corrections appear unneeded (although the unexpected 10.23 offset from the electromagnetic coherence node shown in figure 9 remains to be explained). The model is flat space.

This result suggests that both gravity and rest mass are of electromagnetic origin. While strong classical arguments have been advanced against electromagnetic theories of gravity[36], preliminary examination suggests that such arguments fail when the full consequences of quantum phase coherence are taken into consideration.

Impedance of event horizon at the Planck length is scale invariant Chern-Simons, numerically equal to quantum Hall. Event horizon almost instantaneously radiates its 10^{19} GeV Hawking photon. Impedance mismatch to Compton-scale wavefunctions reflects back all except that part we call gravity. Wavelength of the photon that escapes, the Hawking photon, is of the order of a thousand times the radius of the observable universe. We are in the extreme near field of every particle in the observable universe.

Near field of the Planck particle Hawking photon is the 'graviton' in the GWI model.

6. EQUIVALENCE PRINCIPLE

Previous sections addressed:

- geometric vacuum wavefunctions and their interactions.
- assignment of topologically appropriate quantized electromagnetic fields to the eight fundamental geometric objects of the 3D Pauli vacuum wavefunction via five fundamental constants input by hand.
- introduction of quantized electromagnetic impedances that govern amplitude and phase of energy transmission in wavefunction interactions.
- correlation of quantized impedance network nodes with the unstable particle spectrum.
- impedance matching to event horizon at Planck length to quantize the flat Minkowski spacetime gauge theory gravity equivalent of curved spacetime general relativity.

This completes our necessarily brief attempt to establish context for what follows.

The note that inspired this essay claims "...every gauge theory, aiming at describing gravity, must be in agreement with the basic relativity and equivalence principles of Einstein theory. Moreover, the gauge gravitation theory is to be constructed from these principles."[1]

It was the equivalence principle that led Einstein from special to general relativity. It asserts that **in background independent models**[37] one cannot distinguish between inertial and gravitational forces. In quantized gauge theory gravity one must also give consideration to the basic principle of "...every gauge theory aiming at describing..." quantum gravity - to gauge invariance, to the quantum phase coherence that defines the *observable* boundary of a quantum system.

Quantum phase coherence of gauge fields is maintained via covariant derviatives. Equivalently, the phase shifts generated by quantum impedances create probability distributions, the interference effects we measure. Either way it is the same gauge theory, seen from two complementary perspectives.

Connection between the impedance model and geometric algebra goes deep, to that **coordinate-free background independence** essential for quantum gravity[37]. Geometric Clifford algebra uses a background-free representation. Motion is described with respect to a coordinate frame defined on the object in question rather than an external coordinate system. Similarly, mechanical impedances of massive particles are calculated from Mach's principle applied to the two body problem[38]. Motion is described with respect to a coordinate frame on one of the bodies. The two body problem is inherently background independent. There is no independent observer to whom rotations can be referenced, only spin, a topological property. This shared property of geometric algebra and the impedance model, this background independence, permits scale invariant impedances to be associated with rotation gauge fields of classical gauge theory gravity[39–41], and scale dependent with translation[3].

We have a model in which **inertial mass** of for instance the 'electron' has its origin in electromagnetic field energy of the coupled modes that comprise **self-interacting** wavefunctions[15, 42, 43, 46]. However to the hypothetical 'observer' the wavefunction is static. In the Dirac equation the two-spinor four-component wavefunction is static. It is only with the time derivative, or equivalently the geometric product of two 3D Pauli spatial wavefunctions, that 4D Dirac spacetime emerges. In the 'electron' wavefunction, opposing 'electron' and 'positron' spinor phase evolutions (Feynman's remark that the positron is an electron going backwards in time comes to mind) couple the interacting fields. Relative orientation of phase evolution determines whether interaction with a second wavefunction causes simple collapse (matter-matter) or annihilation into two photons (matter-antimatter).

We have a model in which **gravitational mass** has its origin in **other-interacting** coupling of a given massive particle Compton wavelength to the grossly impedance-mismatched event horizon at the Planck particle Compton wavelength[15, 33, 38], hiding the singularity at the core of every massive particle. Pertinent to note here that unlike other quantum lengths defined by fundamental constants (Rydberg, Bohr, 70MeV, Higgs,...), electric charge is absent at the $\lambda = h/mc$ Compton wavelength. One must work with mechanical impedances, then convert to electrical via the electromechanical oscillator, a topological symmetry breaking transformation from point charge to line charge density.

The original paper[38] that set in motion the present effort was based upon the equivalence principle, the single-body inertial mass of centrifugal force balancing the two-body force of interacting gravitational masses, the moon hanging motionless in the sky, the s-wave electron hanging motionless around the proton,...

Limiting to the equivalence principle by slightly paraphrasing, we claim that the quantized gauge theory presented here is "...in agreement with the basic equivalence principle of Einstein theory, and moreover constructed from that principle."

7. SPECIAL RELATIVITY AND QUANTUM LOGIC

GWI model is naturally finite, confined, and gauge invariant, a consequence of impedance quantization. Finiteness follows from infinite impedance mismatch to both singularity and boundary at infinity. Confinement is flip side of finiteness, energy being reflected from mismatches as it attempts to flow away from the impedance node at the Compton wavelength. And phase shifts due to impedances play the role of covariant derivatives in the GWI model, are the origin of gauge invariance.

GWI model is naturally finite, confined, and gauge invariant at all scales, permits precise calculation from event horizon at the Planck length to event horizon at the edge of the observable universe. It is 'effective' at all energies. Yet there appears no trace of the Lorentz transformation. How is this possible? What quantum logic can support such a model?

Quantum interpretations seek to explain the formalism of quantum mechanics in the language of the philosopher[44–46]. The measurement problem, the problem of the observer, thrives in the world of quantum interpretations, the ongoing proliferation of interpretations assured by less-than-optimal historical choices of wavefunctions at the foundation. This changes with the geometric wavefunction, which brings the internal 'intrinsic' properties of point particle leptons and quarks into real space, provides a powerful intuitive representation. It offers the possibility to address the problem of the observer with a rigorous real space quantum logic, one that naturally extends to special relativity.

It's all about the wavefunction[47].

Wavefunctions are not observable. Observables are wavefunction changes, the energy difference between eigenstates. This is root of the measurement problem. Wavefunctions are not observable.

Foundation of the GWI model is background independence[37]. There is no background in the theoretical minimum of figure 10. Just the interaction of two wavefunctions, the logically rigorous two body problem. There is no third body in the simplest possible logic of quantum mechanics.

In that simplest possible model there are just the two wavefunctions. If there exists an observer, then the logic is simple - the observer must be either or both of the two interacting wavefunctions. However wavefunctions are not observable. Ergo observers are not observable. The observer exists, but can't tell us anything, can't communicate any information, is not observable.

There is no quantum logic of wavefunctions and their interactions that contains an observer. No logic of a multiparticle system whose boundary is defined by phase coherence contains an observer.

The 'observer' exists in the mind of the philosopher/physicist, in a model that incorporates uncountable experiments, both mental and physical, extending back to the origins of consciousness. Observer is an emergent concept. Applying that conceptual model of the observer to a single quantum measurement is not possible without paradox. The sparsest possible quantum logic is essential for coherent quantum interpretation, rooted in a solid understanding of the wavefunction.

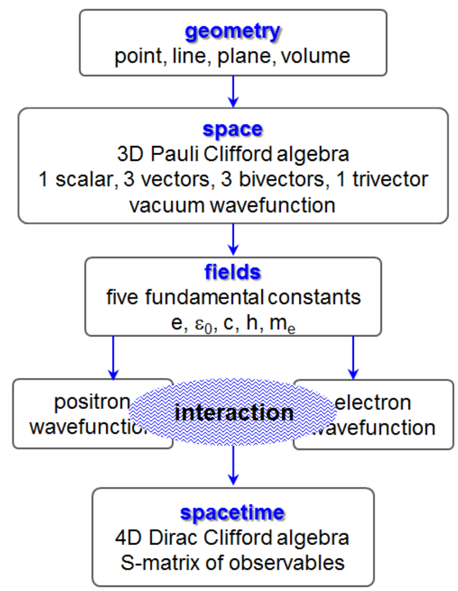

FIG. 10: 'Theoretical minimum' for understanding basics of GWI model

The Lorentz transformation is the Pythagorean theorem, the relation between the sides of a right triangle, the three body problem. It introduces an observer. In the quantum world the observer must interact, however the three body impedance is scale invariant, the potential inverse square. It is the impedance of interactions mediated by the rotation gauge field. Scale invariant impedances communicate only quantum phase, not a single measurement observable. This would seem to explain gauge invariance of special relativity.

There appears no trace of the Lorentz transformation in the GWI model. Yet model calculations yield precise results at relativistic and ultrarelativistic energies. How is this possible?

The answer is quite simple. The GWI model is quite simple, just the two body problem. Special relativity is three body. GWI model contains fundamental quantum mechanics. Special relativity is emergent.

Given that, we claim the GWI model presented here is "...in agreement with the basic relativity and equivalence principles of Einstein theory... and constructed from these principles."[1] Though it might be more accurate to say special relativity is constructed from principles of the GWI model.

CONCLUSION

The possibility that gauge theory gravity and generalized quantum impedances might be linked in such a way as to lead to a viable theory of quantum gravity appears to merit investigation.

ACKNOWLEDGEMENT

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