The Complex Form of Relativistic Maxwell's Equations

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We outline the relativistic formalism which gives a more comprehensive explanation of the complexification scheme. Such issues as considering the Higgs Boson as a soliton depends on Lorentz invariance and relativistic causality constraints. We relate the complexification of Maxwell's equations to models of nonlocal micro and macro phenomena. In this chapter we relate the electromagnetic fields, $F^{\mu\nu}$ and A_μ, the potentials to the gravitational field, $G_{\mu\nu}$. We examine, for example, the manner in which advanced potentials may explain the remote connectedness which is indicated by the Clauser test of Bell's theorem. Similar arguments apply to Young's double slit experiment. The collective coherent phenomena of superconductivity is also explainable by considering the relativistic field theoretic approach in which wave equations are solved in the complex Minkowski space.

1. Relativistic Conditions for Maxwell's Equations in Complex Geometries and Invariance of the Line Element

This section introduces the relativistic form of Maxwell's equations. The fields $\underline{E}$ and $\underline{B}$ are defined in terms of ($\underline{A}, \phi$), the four vector potential; and the relativistic form of $\underline{E}$ and $\underline{B}$ is presented in terms of the tensor field, $F_{\mu\nu}$ (where indices μ and ν run 1 to 4). We then complexity $F_{\mu\nu}$ and determine the expression for the four vector potential $A_\mu = \left(\underline{A}_j, \phi\right)$ in terms of $F_{\mu\nu}$. (index j runs 1 to 3). Discussion of line element invariance is given in terms of the fields $F_{\mu\nu}$.

In Chap. 6 we describe the complex form of A_μ fields and through the formalism in this section we can relate this to the complex forms of $\underline{E}$ and $\underline{B}$. We utilize Weyl's action principle to demonstrate the validity of the use of the complex form of $F_{\mu\nu}$ [1]. Weyl relates the gravitational potential, $G_{\mu\nu}$, to the electromagnetic 'geometrizing' potential A_μ, or geometrical vector, using the principle of stationary action for all variations $\delta G_{\mu\nu}$ and δA_μ [2]. The quantity A_μ, or vector potential, which we identify with A^μ by symmetry relations on the complex conjugate of A, is related to $F_{\mu\nu}$, the electromagnetic force field, E by a set of gauge invariant relations. The electromagnetic force $F_{\mu\nu}$ is independent of the gauge system. The curl of A_μ has the important property

$$F_{\mu\nu} = \frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu} \tag{1}$$

where $F_{\mu\nu}$ is antisymmetric or $F_{\mu\nu} = -F_{\nu\mu}$, and changing A_μ to $A_\mu^{'} = A_\mu + \partial\phi / \partial x_\mu$ is a typical gauge transformation where the intrinsic state of the world remains unchanged.

Defining the 4-vector potential as A_μ, which is written in terms of the 3-vector $\underline{A}_j$ and ϕ, where ϕ is the fourth or temporal component of the field. The indices μ, ν run 1 to 4 and j runs 1 to 3.

Then Maxwell's equations in compact notation in their usual tensor form in terms of $F_{\mu\nu}$, (for c = 1) are

$$F_{\mu\nu} = \begin{pmatrix} 0 & -B_z & B_y & E_x \\ B_z & 0 & -B_x & B_y \\ -B_y & B_x & 0 & E_z \\ -E_x & -E_y & -E_z & 0 \end{pmatrix} \tag{2}$$

then the equations $\nabla \times \underline{E} = -(1/c)(\partial \underline{B} / \partial t)$ and $\nabla \cdot \underline{B} = 0$ can be written

$$\text{as} \frac{\partial F_{\mu\nu}}{\partial x} + \frac{\partial F_{\mu\nu}}{\partial y} + \frac{\partial F_{\mu\nu}}{\partial z} = 0 \tag{3}$$

or $\nabla \times \underline{F}_{\mu\nu} = 0$ for $x^1 = x, \; x^2 = y, \; x^3 = z,$ and $x^4 = t$.

To complexity the elements of $F_{\mu\nu}$ we can choose the conditions, for

$$\begin{gathered} (F_{41}, F_{42}, F_{43}) = i\underline{E} \text{ and } (F_{23}, F_{32}, F_{12}) = \underline{B}, \\ \text{or} \\ (E_x, E_y, E_z) = i\underline{E} \text{ and } (B_x, B_y, B_z) = \underline{B}. \end{gathered} \tag{4}$$

The complex conjugate of the electric and magnetic fields are written in terms of the complex conjugate of $\underline{F}$ or $F_{\mu\nu}^* = -F^{\mu\nu}$. Tin this regard there is a useful theorem that states [3] $\nabla_{123} \times \underline{F} = \nabla^4 \cdot F^*$ or $(\nabla_{xyz} \times F = \nabla^t \cdot F^*)$. For $(F^{23*}, F^{31*}, F^{12*}) = i\underline{E}$ and $(F^{41*}, F^{42*}, F^{43*}) = -\underline{B}$ we then will obtain $\partial F^{\mu\nu*} / \partial x^\nu = 0$ or $\nabla \cdot \underline{F}^* = 0$ which gives the same symmetry between real and imaginary components as ours and Inomata's formalism [4].

The expressions for the other two Maxwell equations $\nabla \cdot \underline{E} = 4\pi\rho$ and $\nabla \times \underline{B} = \frac{1}{c}\frac{\partial \underline{E}}{\partial t} + \underline{J}_e$ can be obtained by introducing the concept of the vector potential in the Lorentz theory as first noticed by Minkowski [5]; we have the 4-vector forms $(\phi_1, \phi_2, \phi_3) = \underline{A}$ and $\phi_4 = i\phi$, so that $\underline{B} = \nabla \times \underline{A}$ and

$\underline{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \underline{A}}{\partial t}$. This yields $F_{\mu\nu} = \frac{\partial A_\nu}{\partial x^\mu} - \frac{\partial A_\mu}{\partial x^\nu}$ or $\underline{F} = \nabla \times \underline{A}$ for the vector and scalar potentials $\underset{\sim}{A} = (A_1, A_2, A_3, \phi)$. If A is a solution to $\underline{F} = \nabla \times \underline{A}$ then $\phi_\mu^{'} + \frac{\partial \phi}{\partial x^\mu}$ also is also a solution by gauge invariance conditions and $\nabla \cdot \underline{A} + \frac{1}{c}\frac{\partial \phi_4}{\partial t} = 0$. We term the fourth component of A as ϕ or ϕ_4 interchangeably. Then from Lorentz theory we have the 4D form as $\frac{\partial A^\mu}{\partial x^\mu} = 0$ or $\nabla \cdot \underline{A} = 0$. We now write the equations for $\nabla \cdot \underline{E} = 4\pi\rho$ and $\nabla \cdot \underline{B} = \frac{1}{c}\frac{\partial \underline{E}}{\partial t} + \underline{J}_e$ as

$$\frac{\partial F^{\mu\nu}}{\partial x^\nu} = s^\mu \text{ or } \nabla \cdot \underline{F} = s. \tag{5}$$

The most general covariant transformation group of electromagnetic field equations, which are more general than the Lorentz group, is formed by affine transformations which transform the equation of the light cone, $s^2 = 0$ into itself. The properties of the spacetime manifold are defined in terms of the constraints of the line element, which relate to the gravitational potential, $G_{\mu\nu}$. We also form an analogy of the metric space invariant to the electromagnetic source vector, s_μ [6]. The Lorentz group contains the Lorentz transformations as well as inversion with respect to a 4D sphere, or hyperboloid in real coordinates. Frank [7] discusses the Weyl theory and gives a proof that the Lorentz group together with the group of ordinary affine transformations, is the only group, in which Maxwell's equations are covariant [7]. Recall that an affine transformation acts as $x^\mu = \alpha_\nu^\mu x^\nu$ with an inverse $x^\nu = \alpha_\mu^{-\iota} x^{'\mu}$. The affine group contains all linear transformations and the group of affine transformations transforms $s^2 = 0$ on the light cone into itself.

In the Weyl geometry, where we have from before, $\underline{F} = \nabla \times \phi$ and

$$\nabla \cdot \underline{F} = \frac{1}{\sqrt{g}}\frac{\partial \sqrt{g} F^\mu}{\partial x^\mu} \tag{6a}$$

and

$$\nabla^\mu \cdot F = \frac{1}{\sqrt{g}}\frac{\partial \sqrt{g} F^\nu}{\partial x^\nu} \tag{6b}$$

with the signature (+,+,+,-) and where $\sqrt{g}$ is the square root of the metric tensor representation of $g_{\mu\nu}$, which is proportional to x,y,z. Then using the theorem in Pauli [8],

$$\nabla_\mu \cdot \nabla \times \underline{F} = \nabla_\mu \nabla \cdot \underline{F} - \square F_\mu \tag{7}$$

and from before, $\nabla \cdot \underline{F} = s$ and since $\nabla \cdot \phi = 0$ and then $\partial \phi^\mu / \partial x^\mu = 0$ and we have from

$$\nabla_\mu \cdot \nabla \times \underline{A} = \nabla_\mu \nabla \cdot \underline{A} - \Box \phi_\mu = s_\mu \tag{8}$$

or

$$\Box A_\mu = -s_\mu \tag{9}$$

for our potential equation, where $\Box$ is the D'Alembertian 4-space operator, and

$$\Box = \partial^\mu \partial_\mu = \eta^{\mu\nu} \partial_\nu \partial_\mu = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = \nabla^2_{R^3} - \frac{\partial^2}{\partial t^2} \tag{10}$$

where $\eta^{\mu\nu}$ is a metrical like transform.

The important aspect of this consideration [9] is our ability to relate the electromagnetic potential to a corresponding spacetime metric interval s or s^2. Hence, we can construct the invariant relations for our fields in terms of our Lorentz invariance 4-space conditions where the $\sqrt{g}$ relates to s and $g_{\mu\nu}$ to s^2. We relate the introduction of a complex spacetime to the complex expansion of the electric and magnetic fields in this section and demonstrate their self-consistency. We examine this in more detail at the end of this section where we consider a generalized affine connection. We relate the electromagnetic potential, A_μ and ϕ_4 to $g_{\mu\nu}$ as $\sqrt{g}$ and also to the square root of the invariant, or s.

The key to the relationship of complex $F_{\mu\nu}$ and complex spacetime is the analogy between ϕ and $g_{\mu\nu}$. We can relate the electromagnetic scalar potential into the interval of time as in Eq. (9), $\Box A_\mu = \phi = -s_\mu$ and we make the analogy of the electromagnetic potential, A_μ to the gravitational potential, $G_{\mu\nu}$ which is related to the invariance conditions on s^2. Both potentials are related to space-like or time-like interval separation. Note that in the $\Box A_\mu = -s$ equation we have a $\sqrt{g}$ factor in order to form the invariant. In the equation for s^2, the invariant is found directly as $s^2 = g_{\mu\nu} x^\mu x^\nu$. We address the set of invariant relations for the case of complex $\underline{E}$ and $\underline{B}$ fields at the end of this section. We relate this then to the deSitter algebras and the complex Minkowski metric.

We associate the E_x component of $F_{\mu\nu}$ or $F_{41} = E_x$ with ϕ_4' as follows:

$$F_{41} = E_x = \phi_4' \frac{e}{r^2} \tag{11}$$

in which $4\pi e$ or e is associated with electric charge on the electron. This approximation is made in the absence of a gravitational field. Maxwell's equations are intended to apply to the case in which no field of force is acting on the system or in the special system of Galilean coordin-ates $A^\mu = \left(A_x, A_y, A_z, \phi\right)$, where $A^j = \left(A_x, A_y, A_z\right)$ is the vector potential and ϕ is the scalar potential and A^μ is the covariant form. For the contravariant form, we have $A_\mu = \left(-A_x - A_y - A_z, \phi\right)$, and in empty space we have

$$\Box A_\mu = 0\,. \tag{12}$$

In non-empty space then

$$\Box A^{\mu} = J^{\mu} \tag{13}$$

or we can write this as

$$\nabla^2 \underline{A}^{\mu} - \mu\varepsilon \frac{\partial^2 \underline{A}^{\mu}}{\partial t^2} = -\underline{J}^{\mu} \tag{14}$$

which is true only approximately for the assumption of flat space for Galilean coordinate transformations. This is the condition which demands that we consider the weak Weyl limit of the gravitational field.

The invariant integral, $\mathcal{J}$ for $F^{\mu\nu}$ is given by

$$\mathcal{J} = \frac{1}{4}\int F^{\mu\nu} F_{\mu\nu}\sqrt{g}\, d\tau \tag{15}$$

where $d\tau$ stands for *dx,dy,dz*.

The quantity, $\mathcal{L}$ is called the action integral of the electromagnetic field. Weyl [10] demonstrated that the action integral is a Lagrangian function, or

$$\mathcal{L} = \int dt \iiint \frac{1}{2}\left(B_x^2 + B_y^2 + B_z^2 - E_x^2 - E_y^2 - E_z^2\right) dxdydz \tag{16}$$

which is of the form $\mathcal{L} = (T - V)dt$. Note the definition for the kinetic energy, T and the potential energy, V for the Hamiltonian is $H = T + V$ the Lagrangian, $\mathcal{L} = T - V$. By describing an electron in a field by Weyl's formalism one has a more general, but more complicated, formalism than the usual Einstein-Galilean formalism [11]. We write a generalized Lagrangian, $\mathcal{L}$ in terms of complex quantities. For example, we form a modulus of the complex vector $\underline{B}$ as $|B|^2 = \underline{B}\underline{B}^* = \underline{B}_{\mathrm{Re}}^2 + \underline{B}_{\mathrm{Im}}^2$. This is the Lagrangian form for the real components of $\underline{E}$ and $\underline{B}$ in 4-space with $E = E_{\mathrm{Re}} + iE_{\mathrm{Im}}$ and $B = B_{\mathrm{Re}} + iB_{\mathrm{Im}}$ for the complex forms of $\underline{E}$ and $\underline{B}$. The complex Lagrangian in complex 8-space becomes

$$\mathcal{L} = \iint dt_{\mathrm{Re}} dt_{\mathrm{Im}} \iiint_{\mathrm{Re}} \iiint_{\mathrm{Im}} \frac{1}{2}\left(B_{\mathrm{Re}}^2 - E_{\mathrm{Re}}^2 + B_{\mathrm{Im}}^2 - E_{\mathrm{Im}}^2\right)\ dx_{\mathrm{Re}} dy_{\mathrm{Re}} dz_{\mathrm{Re}} dx_{\mathrm{Im}} dy_{\mathrm{Im}} dz_{\mathrm{Im}} \tag{17}$$

This is an 8D integral, six over space, two over time (not represented here) where all quantities of the integrand are real because they are squared quantities. We also write an expression for a generalized Poynting vector and energy relationship. There are two equations which define a vector quantity, A_{μ} in electromagnetic theory which corresponds to the gravitational potential, $G_{\mu\nu}$ (which relates to the metric, $g_{\mu\nu}$). We have

$$\frac{\partial}{\partial g_{\mu\nu}}\left(\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right)=\frac{1}{2}\mathrm{E}^{\mu\nu} \tag{18}$$

and

$$\frac{\partial}{\partial A_{\mu}}\left(\frac{1}{4}F^{\mu\nu}F_{\mu\nu}\right)=-\underline{J}^{\mu} \tag{19}$$

where $\mathrm{E}^{\mu\nu}$ is the energy tensor and J^{μ} is the charge and current vector. Two specific cases are for a region free from electrons, or $T^{\mu\nu}-E^{\mu\nu}=0$, or a region free of the gravitational potential or in the weak Weyl limit of the gravitational field, $\Box F_{\mu\nu}=J_{\mu\nu}-J_{\nu\mu}$ where $\Box$ is the 4-space D'Alembertian operator. The solution for this latter case is for the tensor potential $A_{\mu\nu}$,

$$F_{\mu\nu}=\frac{1}{4\pi\gamma}\left(A_{\mu\nu}-A_{\nu\mu}\right)\int\frac{de}{r} \tag{20}$$

if all parts of the electron are the same or uniform in charge. For the proper charge, ρ_0, we have $J^{\mu}=\rho_0 A^{\mu}$ where *de* is the differential charge.

In the limit of $A_{\mu}^{\mu}=0$, then ρ_0, the proper density, is given as

$\rho_0=-\frac{\gamma^2}{12\pi}J_{\mu}J^{\mu}$ for $\gamma=\left(1-\beta^2\right)^{-\frac{1}{2}}$ In Weyl's 4D world then, matter cannot be constituted without electric charge and current. But since the density of matter is always positive the electric charge and current inside an electron must be a space-like vector, thus the square of its length is negative. To quote Eddington:

> It would seem to follow that the electron cannot be built up of elementary electrostatic charges but resolves into something more akin to magnetic charges [12].

Perhaps we can use the structure of Maxwell's equations in complex form to demonstrate that this magnetic structure is indeed the complex part of the field and ask what the source is. A fundamental question is, what gives rise to charge? What attributes of matter and field give rise to charge? It is interesting to note that the charge on a proton and electron is exactly equal and opposite even though the protons mass, m_p is ~ 1860 times greater than the mass of the electron, m_e.

Considering $F_{\mu\nu}$ and $A_{\mu\nu}$ as complex entities rather than 4-space real forms, we may be required to have complex forms of the current density. The relationship between $F_{\mu\nu}$ and $A_{\mu\nu}$ has a spatial integral over charge. If we consider $F_{\mu\nu}$ and $A_{\mu\nu}$ as complex quantities, we deduce possible implications for the charge *e* or differential charge *de* being a complex quantity. Perhaps the expression $e=e_{\mathrm{Re}}+ie_{\mathrm{Im}}$ is not appropriate, but a form for the charge integral is, such as: $\int de_{\mathrm{Re}}de_{\mathrm{Im}}/\underline{r}$ where $r=r_{\mathrm{Re}}+ir_{\mathrm{Im}}$ is more appropriate. Fractional charges such as for quarks, give rise to the question of the source of charge in elementary particles and its fundamental relationship to magnetic phenomena (magnetic domains) are essential considerations and may be illuminated by this or similar formalisms. Neither the source of electrics or magnetics is known, although a great deal is known about their properties.

Faraday's conclusion of the identical nature of the magnetic field of a loadstone and a moving current may need reexamination as well as the issue of Hertzian and non-Hertzian waves. A possible description of such phenomena may come from a complex geometric model [13]. As discussed, one can generalize Maxwell's equations and examine real and imaginary components which comprise a symmetry in the form of the equations. We can examine in detail what the implications of the complex electric and magnetic components have in deriving a Coulomb equation and examine the possible way, given a rotational coordinate, this formalism relates to the 5D geometries of Kaluza and Klein. The approach we take in this chapter does not involve a compact rolled up 5th dimension of the original Kaluza-Klein approach which may lead to singularity problems.

Starting with $F^{\mu\nu}$, A_μ and J^μ, Maxwell's equations can be compactly written as $\dfrac{\partial F^{\mu\nu}}{\partial x_\nu} = J^\mu$ and again, $F_{\mu\nu} = \dfrac{\partial A_\mu}{\partial x_\nu} - \dfrac{\partial A_\nu}{\partial x_\mu}$ and $F_\nu^{\mu\nu} = J^\mu$. Suppose that an electron moves in such a way that its own field on the average just neutralizes an applied external field $F'_{\mu\nu}$ in the region occupied by the electron. The value of $F_{\mu\nu}$ averaged for all the elements of change constituting the electron is given by

$$eF_{\mu\nu} = \frac{1}{4\pi}\left(A_{\mu\nu} - A_{\nu\mu}\right)\iint \frac{de_1 de_2}{r_{12}}$$

and (21)

$$eF_{\mu\nu} = \frac{1}{4\pi}\left(A_{\mu\nu} - A_{\nu\mu}\right)\frac{e^2}{a}$$

where $1/a$ is the average value of $1/r_{12}$ for every pair of points in the electron and a will then be a length comparable to the radius of the sphere throughout which the charge is spread. The mass of the electron is $m = e^2/4\pi a.$ We thus have a form of Coulomb's law; and as we have shown, the complex form of $F^{\mu\nu}$ is consistent with Coulomb's law which is incorporated into Maxwell's equations in a manner that has both a real and an imaginary form of Coulomb's law.

Self-consistency can be obtained in the model by assuming that all physical variables are complex. Thus, as before, we assumed that space, time, matter, energy, charge, etc. are on an equal footing as coordinates of a Cartesian space quantized variable model. In [14] we present a 10D space applicable to quantum theory and cosmology in terms of standard physical quantities. It is reasonable then to complexity space and time as well as the electric and magnetic fields and to determine the relationship of the equations governing standard physical phenomena. Examined in detail is any unifying properties of the model in terms of complexifying physical quantities and examining any new predictions that can be made.

Faraday discusses some possible implications of considering $A_{\mu\nu}$, rather than $F^{\mu\nu}$ as fundamental in such a way that $A_{\mu\nu}$ may act in a domain where $F^{\mu\nu}$ is not observed [13]. In a later section we present a complexification of $A_{\mu\nu}$ rather than $\underline{E}$ and $\underline{B}$ (in $F^{\mu\nu}$). Continuing with the relationship of $F^{\mu\nu}$, the vector A^μ, and scalar potential, ϕ and the source terms of metric space, s^μ let us relate our complex electromagnetic field, $F^{\mu\nu}$, to complex spacetime. We have the volume element, $d\tau = \sqrt{g}\,dxdydz$ for

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \tag{22}$$

and for a particular vector component of $F_{\mu} = \sqrt{g_{\mu\nu}} f^{\mu}$.
Then we have

$$\nabla \cdot \underline{F} = \frac{1}{\sqrt{g}} \frac{\partial f^{\mu} \sqrt{g}}{\partial x^{\mu}} \tag{23}$$

For $F = \nabla \phi$ the function f^{μ} is related to the electromagnetic potential and gravitational potential as $f^{\mu} = g^{\mu\nu} \dfrac{\partial \phi}{\partial x^{\nu}}$. As before, $\dfrac{\partial F_{\mu\nu}}{\partial x_{\nu}} = J_{\mu}$ and $\gamma_{\mu\nu} \dfrac{\partial F_{\mu\nu}}{\partial x_{\nu}} = J_{\mu}$. As before we also had $\left(F_{41}, F_{42}, F_{43}\right) = i\underline{E}$ and $\left(F_{23}, F_{31}, F_{12}\right) = \underline{B}$ then the generalized complex form of $F^{\mu\nu}$ is

$$F^{\mu\nu} = \begin{pmatrix} 0 & B_z & -B_y & -\frac{i}{c}E_x \\ -B_z & 0 & B_x & -\frac{i}{c}E_y \\ B_y & -B_x & 0 & -\frac{i}{c}E_z \\ \frac{iE_x}{c} & \frac{iE_y}{c} & \frac{iE_z}{c} & 0 \end{pmatrix} \tag{24}$$

which can also be written as

$$\underline{F} = \left(B, -\frac{i}{c}\underline{E}\right) \text{ or } \underline{F}^{*} = \left(-\frac{iE}{c}, B\right). \tag{25}$$

We can now relate the complex *E* and *B* fields of the complex spacetime coordinates.

Returning to the compact notation for the two homogeneous equations, $\nabla \times \underline{E} + \frac{1}{c}\frac{\partial \underline{B}}{\partial t} = 0$ and $\nabla \cdot \underline{B} = 0$ as

$$\frac{\partial F_{\mu\nu}}{\partial x_{\kappa}} + \frac{\partial F_{\kappa\mu}}{\partial x_{\nu}} + \frac{\partial F_{\nu\kappa}}{\partial x_{\mu}} = 0 \tag{26}$$

It is very clear that introducing the imaginary components into these equations as $\partial/\partial\left(ix_{\mu}\right)$ and $\partial/\partial\left(it\right)$ leaves them unchanged. Examine the inhomogeneous equations $\nabla\cdot\underline{E}=4\pi\rho$ and $\nabla\times\underline{B}=\frac{1}{c}\frac{\partial\underline{E}}{\partial t}+\underline{J}_{e}$. Then

$$F_{\mu\nu}=\frac{\partial A_{\nu}}{\partial x^{\mu}}-\frac{\partial A_{\mu}}{\partial x^{\nu}}$$

or (27)

$$\underline{F}=\underline{\nabla}\times\underline{A}\ \text{ for }\ \underset{\sim}{A}_{\mu}=\left(\underline{A}_{j},\phi\right),$$

as before j runs 1 to 3 and all Greek indices run 1 to 4. Then the inhomogeneous equations become in general form, $\partial F^{\mu\nu}/\partial x^{\nu}=s^{\mu}$ which sets the criterion on s for using $\partial/\partial\left(ix_{\mathrm{Im}}\right)$; that is, $s'\rightarrow is$. To be consistent [15], we can use $A_{\mu}=\left(A_{j},\frac{1}{c}\phi\right)$.

We then consider the group of affine connections for a linear transformation from one system Σ to another Σ' where Σ and Σ' are two frames of reference and

$$x_{\mu}^{'}=a_{\mu\nu}x_{\nu} \tag{28}$$

where $a_{\mu\nu}a_{\mu}^{\kappa}=\delta_{\nu\kappa}$ and $\det\left|a_{\mu\nu}\right|=1$. In general we can form a 4 x 4 coefficient matrix for the usual diagonal condition where, $a_{11}=1$, $a_{22}=1, a_{33}=1$ and $a_{44}=-1$, all the other elements are zero, i.e. the signature (+++-). We can choose arrays of $a_{\mu\nu}$'s both real and imaginary for the general case so that we obtain forms for space and time components as being complex; for example,

$$x_{3}^{'}=\gamma\left(x_{3}+i\beta x_{4}\right) \tag{29}$$

for $x_{4}=t$, $\gamma=\left(1-\beta^{2}\right)^{-1/2}$ and $\beta=v/c$. Other examples involve other combinations of complex space and time which must also be consistent with unitarity. We have discussed an 8-space formalism for the usual diagonal conditions. See Table 1.

Let us examine the effect of a gravitational field on an electron. Then we discuss some multidimensional models in which attempts are made to relate the gravitational and electromagnetic forces. Some of these multidimensional models are real and some are complex. The structure of the metric may well be determined by the geometric constraints set up by the coupling of the gravitational and electromagnetic forces. These geometric constraints govern allowable conditions on such phenomena as types of allowable wave transmission and the manner in which remote spacetimes are connected. Nonlocality or remote spacetime connections have implications for electromagnetic phenomena such as Young's double slit experiment and Bell's theorem.

Table 1 Coefficient Matrix For The Affine Connection For The Transformation From Reference Frame Σ To Σ'

$$\begin{array}{c|cccc} & x_1 & x_2 & x_3 & x_4 \\ \hline x_1' & a_{11} & a_{12} & a_{12} & a_{12} \\ x_2' & a_{21} & a_{22} & a_{23} & a_{24} \\ x_3' & a_{31} & a_{32} & a_{33} & a_{34} \\ x_4' & a_{41} & a_{42} & a_{43} & a_{44} \end{array}$$

In fact, these experiments are more general than just the properties of the photon, that is, both experiments above can be and have been conducted with photons and other particles; and therefore what are exhibited are general quantum mechanical properties. Remote connection and/or transmission and nonlocality are more general than just electromagnetic phenomena but certainly have their application in electrodynamics and the nonlocal properties of the spacetime metric can be tested by experiments involving classical and quantum electro-dynamic properties.

2. Complex *E* and *B* in Real 4-Space and the Complex Lorentz Condition

Another approach to relate the relativistic and electromagnetic theories is the approach of Wyler in his controversial work at Princeton [16]. The model of Kaluza and Klein use a 5th rotational dimension to develop a model to relate electromagnetic and gravitational phenomena. This geometry is one-to-one mapable to our complex Minkowski space. Wyler introduces a complex Lorentz group with similar motives to those of Kaluza and Klein [17,8]. Wyler's formalism appears to relate to our complex Maxwell formalism and to that of Kaluza and Klein. The fundamental formalism for the calculation of the fine-structure constant, α is most interesting but perhaps not definitive.

$$\alpha = \frac{e^2}{\hbar c 4\pi\varepsilon_0} = \frac{e^2 c\mu_0}{2h} \tag{20}$$

where e is elementary charge, ε_0 vacuum permittivity and μ_0 the magnetic constant or vacuum permeability. An anthropic explanation has been given as the basis for the value of the fine-structure constant by Barrow and Tipler. They suggest that stable matter and intelligent living systems would not exist if α were much different because carbon would not be produced in stellar fusion [19].

Wyler [16] introduces a complex description of spacetime by introducing complex generators of the Lorentz group. He demonstrates that the Minkowski, M^n group is conformally isomorphic to the SO(n,2) group and then introduces a Lie algebra of M^4 which is isomorphic to SO(5,2). From his five and four spaces he generates a set of coefficients that generate the value of the fine structure constant, α. It is through introducing the complex form of the Lorentz group, L(T^n) that he forms an isomorphism to SO(n,2). Wyler calculates the electromagnetic coupling constant in terms of geometric group representations. He expands the generators of the set of linear transformations, T^n, of the group L(T^n). By definition, L(T^n) is isomorphic to the Poincairé group P(M^n), where M^n is the Minkowski space with signature (+++-) or, more generally, (1, n-l). The conformal group C(M^n) is then isomorphic to the SO(n,2) group, which is of quadratic form and signature (n,2).

Wyler then chooses the complex form

$$T^n = R^n + iV^n \tag{31}$$

where R^n represents T_{Re} , and V^n represents T_{Im} for $y \,\varepsilon\, R^n$, or y is an element of R^n and all y's are $y > 0$. The Poincairé group, P(M^n) is the semi-direct product of the Lorentz group SO(l, n-l) and the group of transformations R_n then is $g \,\varepsilon$ S0(n,2).

Then C(M^4)$\cong$ SO(4,2) is the invariance group of Maxwell's equations. The hyperboloids of the 4-mass shell momentum operators are $p_1^2,..., p_4^2 = m^2$ from the representation of the Lie group geometry of M^4 isomorphic to SO(5,2). The intersection of the D^5 (5D) hyperspace with D^4 (4D) gives a structure reduced on D^4 which is colinear to the reduction of a Casimir operator function, $f(z)$ harmonic in D^4.

The coefficients of the Poisson group D^n (n dimensional) as D^4 and D^5 give the value of $\alpha \sim$ 1/137.036. Actually, it is the coefficients of the Poisson nucleus $P^n(z, \xi)$ harmonic in D^n which gives the value of α in terms of z where z is, in general, a complex function and ξ is a spinor. The value is obtained from the isomorphic groups SO(5) x SO(2) and SO(4) x SO(2) which gives $(9/8\pi 4)\,(V(D^5))^{1/4}$ = 1/137.037 where V(D^5) is a Euclidean value of the D^5 domain [20].

The expression for the Poisson nucleus is given by Hau [20]. Note that the Wyler calculation is another example of the relationship between a fifth dimension and a complex "space" of Lorentz transformation. The Wyler theory appears to strongly support the fundamental nature of geometric models. If one can calculate the fine structure constant or any other force field coupling constants from first principles, this gives great impetus to the concept that geometric constraints are extremely significant and may potentially be able to explain the origin of scientific law. In particular, we may be able to at least describe the major force fields (nuclear, electromagnetic, weak, and gravitational in terms of a geometric structure and, perhaps, by this formalism demonstrate the unifying aspects of major forces of nature [14,21].

Wyler also associates the conformal group C(M^n)$\cong$ SO(4,2) with the invariant group of Maxwell equations. The 4-mass shell conditions on the hyperboloids of mass form the representation of the Lie algebras of M^4. Isomorphism to SO(5,2) and S(4,2) intersection lead to a model of the intersection of Maxwell's field and the elementary particle field, i.e. a possible unification of electromagnetic and weak interactions as another approach to the electroweak vector - axial vector model [22]. In the presence of an external gravitational field, the cosmological term is small and finite and depends on vacuum state polarization. In fact, the cosmological term is given by the sum of all vacuum diagrams. In supersymmetry the cosmological term vanishes and therefore the total zero-point energy density of the free fields vanishes [23].

We return to our complex $\underline{E}$ and $\underline{B}$ fields and suggest the relation of our formalism to the Wyler formulation. Using the invariance of line elements $s = X^2 - c^2t^2$ for $r = ct = \sqrt{X^2}$ for $X^2 = x^2 + y^2 + z^2$, to measure the distance from a test charge to an electron charge, we can write for the imaginary part of the complex Maxwell equation $\nabla \times \left(i\underline{E}_{\mathrm{Im}}\right) = \frac{1}{c}\frac{\partial\left(i\underline{B}_{\mathrm{Im}}\right)}{\partial t} + i\underline{J}_{\mathrm{Im}}$ then for $E_{\mathrm{Im}} = 0$.

$$\nabla \times \left(i\underline{E}_{\mathrm{Im}}\right) = 0 \quad \text{or} \quad \frac{1}{c}\frac{\partial\left(i\underline{B}_{\mathrm{Im}}\right)}{\partial t} = i\underline{J}_{\mathrm{Im}} \tag{32}$$

or

$$\frac{\partial\left(i\underline{B}_{\mathrm{Im}}\right)}{\partial r} = ic\underline{J}_{\mathrm{Im}} \quad \text{or} \quad \frac{\partial \underline{B}_{\mathrm{Im}}}{\partial r} = c\underline{J}_{\mathrm{Im}} \tag{33}$$

for the assumed imaginary, B_{Im} commutator relation.

The energy associated with the imaginary part of the magnetic field, $\underline{B}_{\mathrm{Im}}$ is of interest. We calculate an energy invariant by squaring and integrating the above equation as [1,4]

$$\mathcal{E} = -\int_\tau J_m^2 R d\tau = -\int_\tau \left(\frac{\partial B_z}{\partial r}\right)^2 R d\tau \leq 0 \tag{34}$$

The distance function, $R(r)$ over the volume element, $d\tau$ is assumed to be point-symmetrical and vanishes for positive real energy states. The volume, $d\tau$ is constructed to include a small real domain where a point charge is located, avoiding possible divergences. The negative value of the energy integral leads us to hypothesize about the possible source of this energy, such as arising from the vacuum. Perhaps it can be related to vacuum state polarization in a Fermi-Dirac sea model, as we have presented before [24]. Another possible association is with advanced potential models such as those of de Beauregard [25,26]. A third and perhaps the most interesting association would be with the complex coordinate space [27,28].

In Weyl's non-Riemannian geometry, [10,11] he presents a model that does not apply to actual spacetime but to a graphic representation of that relational structure, which is the basis in which both electromagnetic and metric variables are interrelated [12]. This is the deep significance of the geometry and relates to work of Hanson and Newman [29] and Rauscher [27,28] on the complex Minkowski space as well as Wyler's work [16] on complex group theories, such as complex Lorentz invariance, where he attempts to reconcile Maxwell's equations and relativity theory. The examination of the hyperspheres of the de Sitter space is presented by Ellis, where he attempts to unify electromagnetic and gravitational theory [30]. Eddington has suggested that the Weyl formalism, developed around 1923, is one of the major advances in the work of Einstein. The key is that if electromagnetism and QED can be reconciled with the gravitational field, along with the electroweak theory, a unity of the four forces can be made with a simpler and perhaps more reasonable model than the current Theories of everything (TOE). The strong force must also be included.

There is a significant difference between Einstein's generalization of Galilean geometry and Weyl's generalization of Riemannian geometry. The gravitational force field renders Galilean geometry useless and therefore required a move to Riemannian geometry. In terms of Weyl's geometry, we find that the electromagnetic force, $F_{\mu\nu}$, is comparable to the surface of an electron of 4 x 10^{18} volts/cm, [12] and the size of the charge was compatible with the radius of curvature of space.

For the electromagnetic mass, $m_e = e^2 / 4\pi a$, we have

$$m_g ds = \frac{1}{8\pi} G\sqrt{g}\, d\tau \tag{35}$$

where we denote the curvature R by G for the general case of both gravitational and electromagnetic field. The ratio of the masses m_g / m_e relates to the ratio of field strengths of about 10^{-37}.

3. Complex Electromagnetic Forces in a Gravitational Field

We used the weak Weyl limit of the gravitational force in previous calculations of this chapter. We will outline how the complexification of $F_{\mu\nu}$ can be formulated geometrically. We demonstrate that we obtain the same results for the relationship of mass and charge. Let v^{μ} denote the velocity vector as $v^{\mu} = dX_{\mu} / ds$ of the electron in the field, and ρ_0 denote the proper density of charge, *e*. The current is given by $J^{\mu} = \rho_0 v^{\mu}$. The fields, $F_{\mu\nu}$ refer to the applied external force of the electron. Returning to Eddington's approach [11], we then have

$$mA^{\nu} A_{\mu\nu} = -F_{\mu\nu}\rho_0 A^{\nu}. \tag{36}$$

We can also write ρ_0 as *e* in the above equation.

In the limit of our gravitational field we can neglect the gravitational field as an external field or also the gravitational energy of the electron. For an electron in a gravitational field we start from the field equations with the Ricci curvature tensor, R_{μ}^{ν} and the metric tensor, g_{μ}^{ν}. For the case where no matter is present we have:

$$G_{\mu}^{\nu} = R_{\mu}^{\nu} - \frac{1}{2} g_{\mu}^{\nu} R_{\mu}^{\nu} = -\frac{8\pi}{c^4} G E_{\mu}^{\nu} \tag{37}$$

using the scalar curvature, $R = \frac{8\pi G E}{c^4} = 0$, where $F = c^4 / G$ and *G* is the gravitational constant. This equation simplifies to

$$R_{\mu\nu} = -8\pi E_{\mu\nu} \tag{38}$$

and applies to certain regions that contain electromagnetic fields but no matter and no electron charges.

For the only surviving component in the energy considerations, we have

$$F_{41} = -F_{14} = \frac{\partial \phi}{\partial r} \tag{39}$$

where *r* is the radial separation. Then $F^{41} = g^{44} F_{41}$ and $\frac{\partial \phi}{\partial r} \propto \frac{e}{r^2}$ and

$$E_1^1 = +E_2^2 = E_3^3 = -E_4^4 = \frac{1}{2}\frac{\partial \phi}{\partial r} = \frac{1}{2}\frac{e^2}{r^4}. \tag{40}$$

One can associate m_e, the mass of the electron, with $4\pi e$, giving $\alpha = \frac{2\pi e^2}{m} \sim 1.5 \times 10^{-13}$ cm and justifies identifying $4\pi\varepsilon$ with the electrical charge *e* for $4\pi e$ or

$$F_{41} = \frac{\partial \phi}{\partial r} = \frac{1}{4\pi}\frac{e}{r^2} \tag{41}$$

We use

$$\square F_{\mu\nu} = J_{\mu\nu} - J_{\nu\mu} \text{ for } A^{\mu} = \frac{de}{4\pi r} \tag{42}$$

and then

$$F_{\mu\nu} = \int \frac{de\left(A_{\mu\nu} - A_{\nu\mu}\right)}{\left(4\pi\gamma\right)r} = \frac{1}{4\pi\gamma}\left(A_{\mu\nu} - A_{\nu\mu}\right)\int \frac{de}{r} \tag{43}$$

because all parts of the electron obey the same relativity where

$$\frac{\partial^2 A^{\mu}}{\partial t^2} - \nabla^2 A^{\mu} = J^{\mu}$$

and (44)

$$A^{\mu} = \frac{1}{4\pi}\frac{ds}{dt} v^{\mu} \int_{\tau} \frac{\rho d\tau}{r}.$$

for velocity, v^{μ}, we drop the γ since all measurements are assumed to be proper time measurements. Integrating over the electron between pairs of points on the electron surface,

$$eF_{\mu\nu} = \frac{1}{4\pi}\left(A_{\mu\nu} - A_{\nu\mu}\right)\iint \frac{de_1 de_2}{r_{12}} = \frac{1}{4\pi}\left(A_{\mu\nu} - A_{\nu\mu}\right)\frac{e^2}{a} \tag{45}$$

where 1/a is the average value of $1/r_{12}$. We can write Eq. (43) as

$$-eA^{\nu} F_{\mu\nu} = \frac{1}{4\pi} A^{\nu}\left(A_{\mu\nu} - A_{\nu\mu}\right)\frac{e^2}{a} \tag{46}$$

and using the equation from before, relating v^{ν}, $A_{\mu\nu}$, $F_{\mu\nu}$ and A^{ν}, $mv^{\nu}A_{\mu\nu} = -F_{\mu\nu}eA^{\nu}$, so that $m = e^2 / 4\pi a$ as before.

How does this relate to the deSitter spaces? In the deSitter algebras the proper time in all inertial frames of intervals is the same or equivalent. This is the powerful absolute of the deSitter space. The proper time interval, $d\tau$ on its geodesic world-line in the deSitter picture is given as

$$d\tau^2 = dt^2 - e^{2t}\left(dX^2\right) \tag{47}$$

for $dX^2 = dx^2 + dy^2 + dz^2$ in Euclidean coordinates and *t* is the cosmic time. The metric form of the deSitter universe represents the metric form consistent with the observed asymptotically flat, low density universe. The se Sitter space is constant with Einstein dynamic equations and is therefore consistent with the Hubble constant, H_R [30,31].

Ellis [32] suggests that geometry and electromagnetism can be unified by a rigorous analysis of time. The hyperspheres of deSitter space can be represented as a 5D metric manifold which tie the geometric models of gravity and electromagnetism to the structure of matter. Time is not primary but a property of the matter of elementary particles. If $\tau = t$ is allowed in the de Sitter space, then the typical geodesies represent what appears to be electromagnetic field lines. This is the manner in which Ellis attempts to describe the electromagnetic phenomena as geometric!

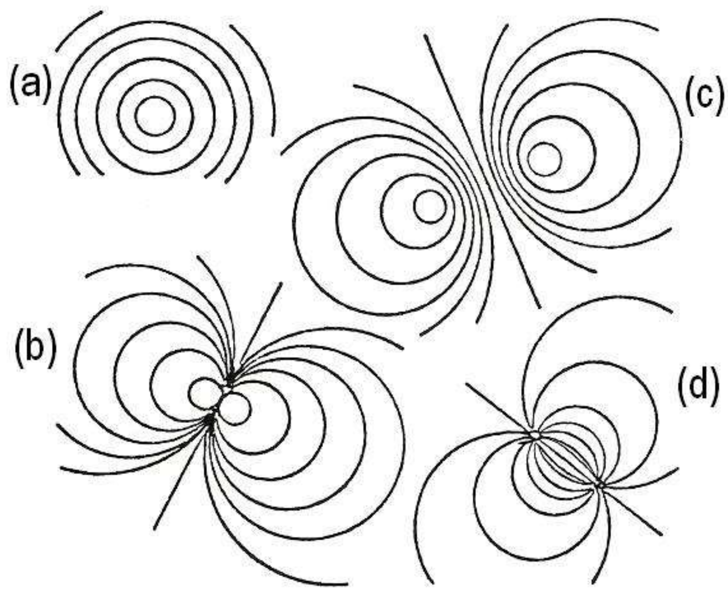

Figure 1 Geodesic plots of de Sitter space representing the field lines of the electromagnetic field. Various conditions for signal propagation are given.

The conformal invariant is given as

$$ds^2 = \frac{1}{R^2}\left(dx^2 + dy^2 + dz^2 - dR^2\right) \qquad (48)$$

which depends only on the ratios of distances and is thus independent of scale. Let $t = -\ell nR$, then $R = e^{-t}$ and $ds^2 = e^{2t}\left(dx^2 + dy^2 + dz^2\right) - dt^2$ which is the de Sitter metric element. Ellis' geodesies of his angle metric correspond to geodesics of the de Sitter space (Fig. 1). In 1b, time-like subluminal geodesies are represented, in 1c they are luminal, and in 1d these geodesics are space-like superluminal. (See Chap. 9) The figures also contain Euclidean space planes as spheres of infinite radii.

Feinberg [33] suggested that the first step in the test of multi-dimensional geometric models is to predict some simple phenomena such as the Coulomb attraction-repulsion; and that the geodesic form in Figure 3 may point a way to do this, because if we can relate this five-dimensional geometry to the complex geometry, then we can relate this complex geometry to Coulomb interactions. The curvature of space may then be related to a rotation or angular momentum component as a Kaluza-Klein 5$^{\text{th}}$ dimension. We form an isomorphism of this geometry to an 8D real-complex coordinate geometry which appears to not only unify electromagnetic theory and gravitational theory but may also resolve some other apparent paradoxes [34,35].

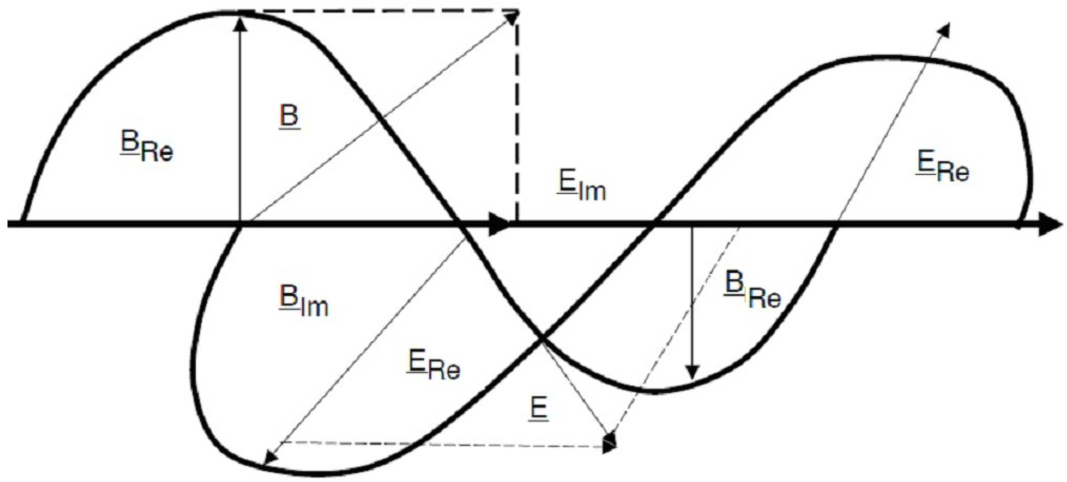

Figure 2 Hertzian and non-Hertzian waves. Where E and B are decoupled into two components $\underline{E}_{\mathrm{Re}}$ and $\underline{E}_{\mathrm{Im}}$ and $\underline{B}_{\mathrm{Re}}$ and $\underline{B}_{\mathrm{Im}}$.

We have observed that introducing complex E and B fields or complexifying the $F^{\mu\nu}$ field can be performed in such a way as to not distort the electric charge on the electron. We also find consistency with the 5D geometry of Kaluza and Klein, the 8D Minkowski space, and the de Sitter space where the geodesic represents the electromagnetic field lines. We can also maintain Lorentz invariance conditions for both real and complex transforms on the line element.

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