Newton’s $E = mc^2$ Two Hundred Years Before Einstein?

Unification of Newton and Einstein.

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Abstract
(This is an unedited first draft. The next version will be properly edited and extended.). The most famous Einstein formula is $E = mc^2$ and the most famous Newton formula is $F = G \frac{mm}{r^2}$. Here we will show that there exist a simple relationship between the Einstein and Newton formula. They are closely connected when it comes to fundamental particles. Newton indirectly had $E = mc^2$ two hundred years before Einstein without knowing about it.

Key words: $E = mc^2$, energy, kinetic energy, mass, gravity, relativity, Newton and Einstein.

1 Did Newton “Discover” $E = mc^2$ Two Hundred Years Before Einstein?

The Italian geologist and industrialist, Olinto de Pretto, speculated three years before Einstein that the old, well-known\(^1\) formula of $E = mv^2$ had to be equal to $E = mc^2$ when something moved at the speed of light, where $v$ is the speed of the object, $c$ is the speed of light, $m$ is the mass, and $E$ is the energy. Thomson [1] in his book “Electricity and Matter” published in 1904 present what he call a kinetic energy formula for light that he describe\(^2\) as $E = \frac{1}{2} mc^2$.

Einstein [2] is still likely the first to mathematically “prove” the $E = mc^2$ relationship between energy and mass. However, here we will show that Newton basically had the same mathematical relationship between energy and matter hidden in his gravity formula (see [3]) two hundred years before Einstein, if he just had known the “radius” of a fundamental particle.

The rest-mass of any fundamental particle can be written as

$$m = \frac{\hbar}{\bar{\lambda} c} \quad (1)$$

where $\bar{\lambda}$ is the reduced Compton wavelength. For example the rest mass of an electron is given by

$$m_e = \frac{\hbar}{\bar{\lambda}_e c} \approx 9.10938 \times 10^{-31} \text{ kg} \quad (2)$$

Based on this we have the following relationship between energy mass

$$E = mc^2 = \frac{G m_p m_p}{\bar{\lambda}} \quad (3)$$

where $m_p$ is the Planck mass, see [4], and $\bar{\lambda}$ is the reduced Compton wavelength of the fundamental particle $m$ (this is actually the “extended” “radius” of the particle, see [5, 7]). That is the rest energy embedded in any fundamental particle is equal to the gravity (energy) between two Planck masses separated by the reduced Compton wavelength of the fundamental particle of interest.

Further the Kinetic energy of a fundamental particle is given by

$$E_k = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2 = G \frac{m_p m_p}{\bar{\lambda} \sqrt{1 - \frac{v^2}{c^2}}} - G \frac{m_p m_p}{\bar{\lambda}}. \quad (4)$$

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\(^1\) $E = mv^2$ was suggested by Gottfried Leibniz in the period 1676-1689.

\(^2\)In the original formula he uses notation $E = MV^2$, but he Thomson states “where $V$ is velocity with which light travels through the medium....”
And when \( v << c \) we can use the first term of a series expansion and very well approximate the kinetic energy by

\[
E_k \approx \frac{1}{2}mv^2 = \frac{1}{2}G \frac{m_p m_p}{c^2} \frac{v^2}{c^2}
\]  

Further we must have

\[
\frac{mc^2}{c^2} = G \frac{m_p m_p}{c^2 \lambda^2} = m = G \frac{m_p m_p}{c^2 \lambda^2}
\]  

And what we can call the rest-force of a fundamental particle is given by

\[
F = \frac{mc^2}{\lambda} = \frac{G m_p m_p}{\lambda^2}
\]  

This means the relativistic force must be

\[
F = \frac{mc^2}{\lambda \sqrt{1 - v^2}} = G \frac{m_p m_p}{\lambda^2 \sqrt{1 - \frac{v^2}{c^2}}}
\]  

Based on Haug’s recent research the maximum velocity of a Planck mass particle (mass gap particle) surprisingly is zero. The Planck mass particle is at rest in any reference frame and therefore the only invariant particle. This can only happen if the Planck mass particle only last an instant (see [6, 8]). The Planck mass particle is the collision points between two photons, and it only last one Planck second. This means the force for any particle moving at its maximum velocity must be

\[
F = \frac{m_p c^2}{l_p} = \frac{G m_p m_p}{l_p^2}
\]  

A Planck mass particle should not be confused with two Planck masses, we can have a heap of proton that makes up a Planck mass, this is not a Planck mass particle. A Planck mass particle has a reduced Compton wavelength of \( l_p \). For non Planck masses it seems one possibly need to do a relativistic adjustment for the gravity, this is not the case for gravity between light particles (photons).

Table 1 summarize the mathematical relationship between special relativity and Newton gravity (Newton inspired formulas).

<table>
<thead>
<tr>
<th></th>
<th>Einstein = Newton</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>( m = G \frac{m_p m_p}{c^2 \lambda^2} )</td>
</tr>
<tr>
<td>Relativistic mass</td>
<td>( \sqrt{\frac{m}{1 - \frac{v^2}{c^2}}} = \frac{G m_p m_p}{c^2 \lambda \sqrt{1 - \frac{v^2}{c^2}}} )</td>
</tr>
<tr>
<td>Energy</td>
<td>( E = mc^2 = G \frac{m_p m_p}{\lambda^2} )</td>
</tr>
<tr>
<td>Relativistic energy</td>
<td>( E = \frac{mc^2}{\lambda \sqrt{1 - \frac{v^2}{c^2}}} = \frac{G m_p m_p}{\lambda^2 \sqrt{1 - \frac{v^2}{c^2}}} )</td>
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<td>Kinetic energy</td>
<td>( E_k = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2 = G \frac{m_p m_p}{\lambda \sqrt{1 - \frac{v^2}{c^2}}} - G \frac{m_p m_p}{\lambda^2} )</td>
</tr>
<tr>
<td>Kinetic energy</td>
<td>( E_k \approx \frac{1}{2}mv^2 = \frac{1}{2}G \frac{m_p m_p}{\lambda \sqrt{1 - \frac{v^2}{c^2}}} )</td>
</tr>
<tr>
<td>Force</td>
<td>( F = \frac{mc^2}{\lambda \sqrt{1 - \frac{v^2}{c^2}}} = G \frac{m_p m_p}{\lambda^2 \sqrt{1 - \frac{v^2}{c^2}}} )</td>
</tr>
</tbody>
</table>

**Table 1:** The table shows some simple mathematical relationships between the Einstein special relativity formulas and Newton inspired formulas.

Table 2 show the relativistic limit for the Einstein and Newton formula. This is based on the maximum velocity for anything with rest-mass being \( v_{max} = c \sqrt{1 - \frac{v^2}{c^2}} \).
Einstein = Newton

<table>
<thead>
<tr>
<th>Relativistic mass</th>
<th>( m_{\text{max}} = \frac{m}{\sqrt{1 - \frac{v^2}{c^2}}} = G \frac{m_p m_p}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{G m_p m_p}{c^2 l_p} = m_p )</th>
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<td>( E_{k,\text{max}} = \frac{mc^2}{\sqrt{1 - \frac{v^2}{c^2}}} - mc^2 = G \frac{m_p m_p}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{G m_p m_p}{c^2 l_p} = m_p c^2 - mc^2 )</td>
</tr>
<tr>
<td>Relativistic force</td>
<td>( F_{\text{max}} = \frac{mc^2}{\sqrt{\lambda^2 - \frac{v^2}{c^2}}} = G \frac{m_p m_p}{\sqrt{\lambda^2 - \frac{v^2}{c^2}}} = \frac{G m_p m_p}{c^2 l_p} = \frac{m_p c^2}{l_p} )</td>
</tr>
</tbody>
</table>

Table 2: The table show the relativistic maximum limit for energy, kinetic energy and force based on Haug’s maximum velocity formula.

## 2 Conclusion

We have presented some interesting mathematical relationships between Einstein’s special relativity formulas and Newton’s formulas. It seems like the Newton gravity formulas likely are non-relativistic. Still they are more than that, as they also seems to be the relativistic limit based on Haug’s maximum velocity formula. We do not claim that Newton knew about \( E = mc^2 \) directly, but indirectly Newton basically had an energy mass relationship formula that was valid for any fundamental particle if he just had known the concept of reduced Compton wavelength and used that as the radius in his formula for fundamental particles.

## References


