Rough Neutrosophic Hyper-complex Set and its Application to Multi-attribute Decision Making

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Abstract

This paper presents multi-attribute decision making based on rough neutrosophic hyper-complex sets with rough neutrosophic hyper-complex attribute values. The concept of neutrosophic hyper-complex set is a powerful mathematical tool to deal with incomplete, indeterminate and inconsistent information. We extend the concept of neutrosophic hyper-complex set to rough neutrosophic hyper-complex environ-ment. The ratings of all alternatives are expressed in terms of the upper / lower approximations and pairs of neutrosophic hyper-complex sets, which are characterized by two hyper-complex functions and an indeterminacy component. We also define cosine function based on rough neutrosophic hyper-complex set to determine the degree of similarity between rough neutrosophic hyper-complex sets. We establish a new decision making approach based on rough neutrosphic hyper-complex set. Finally, a numerical example is provided to prove the applicability of the proposed approach.

Keywords

neutrosophic set, rough neutrosophic set, rough neutrosophic hyper-complex set, cosine function, decision making.

1 Introduction

The concept of rough neutrosophic set has been introduced by Broumi et al. [1, 2]. It has been derived as a combination of the concepts of rough set proposed by Z. Pawlak [3] and of neutrosophic set introduced by F. Smarandache [4, 5]. The rough sets and the neutrosophic sets and are both capable of dealing with partial information and uncertainty.

To deal with real world problems, Wang et al. [6] introduced the single valued netrosophic sets (SVNSs).

Recently, Mondal and Pramanik proposed a few decision making models in rough neutrosophic environment.

Mondal and Pramanik [7] applied the concept of grey relational analysis based rough neutrosophic set in multi-attribute decision making.

Mondal and Pramanik [8] also studied the cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis.

The same authors [9] proposed the multi-attribute decision making using rough accuracy and score function.

The same authors [10] also proposed the cotangent similarity measure under rough neutrosophic sets.

The same authors [11] further studied some similarity measures, namely Dice similarity measure [12] and Jaccard similarity measure [12] in rough neutrosophic sets.

The rough neutrosophic hyper-complex set is the generalization of rough neutrosophic set and of neutrosophic hyper-complex set [13].

S. Olariu [14] introduced the concept of hyper-complex number, and studied some of its properties.

Mandal and Basu [15] studied hyper-complex similarity measure for SVNS and its application in decision making.

Mondal and Pramanik [16] studied tri-complex rough neutrosophic similarity measure and presented an application in multi-attribute decision making.

In this paper, we have defined the rough neutrosophic hyper-complex set and the rough neutrosophic hyper-complex cosine function (RNHCF).

We have also proposed a multi-attribute decision making process in rough neutrosophic hyper-complex environment.

The paper is organized in the following way: *Section 2* presents preliminaries of neutrosophic sets and of single valued neutrosophic sets, and some basic ideas of hyper-complex sets. *Section 3* gives the definition of the rough neutro-sophic hyper-complex sets. *Section 4* gives the definition of the rough neutrosophic hyper-complex cosine function. *Section 5* introduces the multi-attribute decision-making method based on rough neutrosophic hyper-complex cosine function. *Section 6* offers a numerical example of the proposed approach. Finally, *Section 7* produces the concluding remarks and some aims of future research.

2 Neutrosophic Preliminaries

Neutrosophic set is derived from neutrosophy [4].

2.1 Neutrosophic Set

Definition 2.1 [4, 5]

Let U be a universe of discourse. Then a neutrosophic set A can be presented in the form:

$$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in U\}, \quad (1)$$

where the functions T, I, F: U→ $]^{-}0,1^{+}[$ represent respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in U$ to the set Asatisfying the following the condition.

$$^{-}0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^{+} \quad (2)$$

Wang et al. [6] mentioned that the neutrosophic set assumes the values from the real standard or non-standard subsets of $]^{-}0, 1^{+}[$ based on philosophical point of view. So instead of $]^{-}0, 1^{+}[$ Wang et al. [6] consider the interval [0, 1] for technical applications, because $]^{-}0, 1^{+}[$ is difficult to apply in the real applications such as scientific and engineering problems. For two netrosophic sets (NSs),

$$A_{NS} = \{<x: T_A(x), I_A(x), F_A(x)> \mid x \in X\} \quad (3)$$

and

$$B_{NS} = \{< x, T_B(x), I_B(x), F_B(x)> \mid x \in X\}, \quad (4)$$

the two relations are defined as follows:

(1) $A_{NS} \subseteq B_{NS}$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$

(2) $A_{NS} = B_{NS}$ if and only if $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$.

2.2 Single valued neutrosophic sets (SVNS)

Definition 2.2 [6]

Assume that X is a space of points (objects) with generic elements in X denoted by x. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity membership function $F_A(x)$, and for each point x in X, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. When X is continuous, a SVNS A can be written as:

$$A = \int_X \frac{<T_A(x), I_A(x), F_A(x)>}{x} : x \in X. \quad (5)$$

When X is discrete, a SVNS A can be written as:

$$A = \sum_{i=1}^{n} \frac{<T_A(x_i), I_A(x_i), F_A(x_i)>}{x_i} : x_i \in X. \quad (6)$$

For two SVNSs,

$$A_{SVNS} = \{<x: T_A(x), I_A(x), F_A(x)> \mid x \in X\} \quad (7)$$

and

$$B_{SVNS} = \{<x, T_B(x), I_B(x), F_B(x)> \mid x \in X\}, \quad (8)$$

the two relations are defined as follows:

(i) $A_{SVNS} \subseteq B_{SVNS}$ if and only if $T_A(x) \leq T_B(x)$, $I_A(x) \geq I_B(x)$, $F_A(x) \geq F_B(x)$;

(ii) $A_{SVNS} = B_{SVNS}$ if and only if $T_A(x) = T_Q(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$,

for any $x \in X$.

2.3 Basic concept of Hyper-complex number of dimension *n* [12]

The hyper-complex number of dimension *n* (or *n*-complex number) was defined by S. Olariu [13] as a number of the form:

$$u = x_0 + h_1x_1 + h_2x_2 + \ldots + h_{n-1}x_{n-1}, \quad (9)$$

where $n \geq 2$, and the variables $x_0, x_1, x_2, \ldots, x_{n-1}$ are real numbers, while $h_1, h_2, \ldots, h_{n-1}$ are the complex units, $h_0 = 1$, and they are multiplied as follows:

$$h_jh_k = h_{j+k} \text{ if } 0 \leq j+k \leq n-1, \text{ and } h_jh_k = h_{j+k-n} \text{ if } n \leq j+k \leq 2n-2. \quad (10)$$

The above complex unit multiplication formulas can be written in a simpler form as:

$$h_jh_k = h_{j+k}\ (mod\ n) \quad (11)$$

where *mod n* means *modulo n*. For example, if n = 5, then:

$$h_3h_4 = h_{3+4}(\text{mod } 5) = h_7\ (mod\ 5) = h_2. \quad (12)$$

The formula above allows us to multiply many complex units at once, as follows:

$$h_{j1}h_{j2}\ldots h_{jp} = h_{j1+j2+\ldots+jp}\ (mod\ n), \quad (13)$$

for $p \geq 1$.

The Neutrosophic hyper-complex number of dimension n [12] which is a number and it can be written of the form:

$$u + vI, \tag{14}$$

where u and v are n-complex numbers, and I is the indeterminacy.

3 Rough Neutrosophic Hyper-complex Set in Dimension n

Definition 3.1

Let Z be a non-null set and R be an equivalence relation on Z. Let A be a neutrosophic hyper-complex set of dimension n (or neutrosophic n-complex number), and its elements of the form u+vI, where u and v are n-complex numbers and I is the indeterminacy.

The lower and the upper approximations of A in the approximation space (Z, R) denoted by $\underline{N}(A)$ and $\overline{N}(A)$ are respectively defined as follows:

$$\underline{N}(A) = \left\langle < x, [u+vI]_{\underline{N}(A)}(x) > / z \in [x]_R, x \in Z \right\rangle \tag{15}$$

$$\overline{N}(A) = \left\langle < x, [u+vI]_{\overline{N}(A)}(x) > / z \in [x]_R, x \in Z \right\rangle, \tag{16}$$

where

$$[u+vI]_{\underline{N}(A)}(x) = \wedge_z \in [x]_R \, [u+vI]_A(z), \tag{17}$$

$$[u+vI]_{\overline{N}(A)}(x) = \vee_z \in [x]_R \, [u+vI]_A(z). \tag{18}$$

So, $[u+vI]_{\underline{N}(A)}(x)$ and $[u+vI]_{\overline{N}(A)}(x)$ are neutrosophic hyper-complex number of dimension n.

Here $\vee$ and $\wedge$ denote 'max' and 'min' operators respectively. $[u+vI]_A(z)$ and $[u+vI]_A(z)$ are the neutrosophic hyper-complex sets of dimension n of z with respect to A. $\underline{N}(A)$ and $\overline{N}(A)$ are two neutrosophic hyper-complex sets of dimension n in Z.

Thus, NS mappings $\underline{N}$, $\overline{N}$: $N(Z) \to N(Z)$ are respectively referred to as the lower and upper rough neutrosophic hyper-complex approximation operators, and the pair $(\underline{N}(A), \overline{N}(A))$ is called the rough neutrosophic hyper-complex set in (Z, R).

Based on the above mentioned definition, it is observed that $\underline{N}(A)$ and $\overline{N}(A)$ have constant membership on the equivalence clases of R, if $\underline{N}(A) = \overline{N}(A)$; i.e. $[u+vI]_{\underline{N}(A)}(x) = [u+vI]_{\overline{N}(A)}(x)$.

Definition 3.2

Let N(A) = ($\underline{N}(A), \overline{N}(A)$) is a rough neutrosophic hyper-complex set in (Z, R). The rough complement of N(A) is denoted by $\sim N(A) = (\underline{N}(A)^c, \overline{N}(A)^c)$, where $\underline{N}(A)^c$ and $\overline{N}(A)^c$ are the complements of neutrosophic hyper-complex set of $\underline{N}(A)$ and $\overline{N}(A)$ respectively.

$$\underline{N}(A)^c = \left\langle < x, [u + v(1-I)]_{\underline{N}(A)}(x) > /, x \in Z \right\rangle, \quad (19)$$

and

$$\overline{N}(A)^c = \left\langle < x, [u + v(1-I)]_{\overline{N}(A)}(x) > /, x \in Z \right\rangle \quad (20)$$

Definition 3.3

Let N(A) and N(B) be two rough neutrosophic hyper-complex sets, respectively in Z, then the following definitions holds:

$$N(A) = N(B) \Leftrightarrow \underline{N}(A) = \underline{N}(B) \wedge \overline{N}(A) = \overline{N}(B) \quad (21)$$

$$N(A) \subseteq N(B) \Leftrightarrow \underline{N}(A) \subseteq \underline{N}(B) \wedge \overline{N}(A) \subseteq \overline{N}(B) \quad (22)$$

$$N(A) \cup N(B) = < \underline{N}(A) \cup \underline{N}(B), \overline{N}(A) \cup \overline{N}(B) > \quad (23)$$

$$N(A) \cap N(B) = < \underline{N}(A) \cap \underline{N}(B), \overline{N}(A) \cap \overline{N}(B) > . \quad (24)$$

If A, B, C are the rough neutrosophic hyper-complex set in (Z, R), then the following propositions are stated from definitions:

Proposition 1

I. $\sim A(\sim A) = A$ (25)

II. $\underline{N}(A) \subseteq \overline{N}(B)$ (26)

III. $< \sim (\underline{N}(A) \cup \underline{N}(B)) = \sim (\underline{N}(A)) \cap \sim (\underline{N}(B))$ (27)

IV. $< \sim (\underline{N}(A) \cap \underline{N}(B)) = \sim (\underline{N}(A)) \cup \sim (\underline{N}(B))$ (28)

V. $< \sim (\overline{N}(A) \cup \overline{N}(B)) = \sim (\overline{N}(A)) \cap \sim (\overline{N}(B))$ (29)

VI. $< \sim (\overline{N}(A) \cap \overline{N}(B)) = \sim (\overline{N}(A)) \cup \sim (\overline{N}(B))$ (30)

Proof I

If N(A) = $[\underline{N}(A), \overline{N}(A)]$ is a rough neutrosophic hyper-complex set in (Z, R), the complement of N(A) is the rough neutrosophic hyper-complex set defined as follows:

$$\underline{N}(A)^c = \left\langle < x, [u + v(1-I)]_{\underline{N}(A)}(x) > /, x \in Z \right\rangle, \quad (31)$$

and

$$\overline{N}(A)^c = \left\langle < x, [u + v(1-I)]_{\overline{N}(A)}(x) > /, x \in Z \right\rangle \quad (32)$$

From this definition, we can write:

$$\sim A(\sim A) = A. \quad (33)$$

Proof II

The lower and the upper approximations of A in the approximation (Z, R) denoted by $\underline{N}(A)$ and $\overline{N}(A)$ are respectively defined as follows:

$$\underline{N}(A)^c = \left\langle < x, [u + v(1-I)]_{\underline{N}(A)}(x) > /, x \in Z \right\rangle, \quad (34)$$

and

$$\overline{N}(A)^c = \left\langle < x, [u + v(1-I)]_{\overline{N}(A)}(x) > /, x \in Z \right\rangle, \quad (35)$$

where

$$[u + vI]_{\underline{N}(A)}(x) = \wedge_z \in [x]_R \, [u + vI]_A(z), \quad (36)$$

$$[u + vI]_{\overline{N}(A)}(x) = \vee_z \in [x]_R \, [u + vI]_A(z). \quad (37)$$

So,

$$\underline{N}(A) \subseteq \overline{N}(A). \quad (38)$$

Proof III

Consider:

$$x \in \sim(\underline{N}(A) \cup \underline{N}(B))$$

$$\Rightarrow x \in \sim \underline{N}(A) \text{ and } x \in \sim \underline{N}(B)$$

$$\Rightarrow x \in \sim(\underline{N}(A)) \cap \sim(\underline{N}(B))$$

$$\Rightarrow x \in \sim(\underline{N}(A)) \cap \sim(\underline{N}(B))$$

$$\Rightarrow \sim(\underline{N}(A) \cup \underline{N}(B)) \subseteq \sim((\underline{N}(A)) \cap \sim(\underline{N}(B))). \quad (39)$$

Again, consider:

$$y \in \sim((\underline{N}(A)) \cap \sim(\underline{N}(B)))$$

$$\Rightarrow y \in \sim \underline{N}(A) \text{ or } y \in \sim \underline{N}(B)$$

$$\Rightarrow y \in \Rightarrow \sim(\underline{N}(A) \cup \underline{N}(B))$$

$$\Rightarrow \sim(\underline{N}(A) \cup \underline{N}(B)) \supseteq \sim((\underline{N}(A)) \cap \sim(\underline{N}(B))). \quad (40)$$

Hence,

$$\sim\left(\underline{N}(A)\cup\underline{N}(B)\right)=\sim\left(\left(\underline{N}(A)\right)\cap\sim\left(\underline{N}(B)\right)\right). \quad (41)$$

Proof IV

Consider:

$$x \in \sim\left(\underline{N}(A)\cap\underline{N}(B)\right)$$

$$\Rightarrow x \in \sim\underline{N}(A) \text{ or } x \in \sim\underline{N}(B)$$

$$\Rightarrow x \in \sim\left(\underline{N}(A)\right)\cup\sim\left(\underline{N}(B)\right)$$

$$\Rightarrow x \in \sim\left(\underline{N}(A)\right)\cup\sim\left(\underline{N}(B)\right)$$

$$\Rightarrow \sim\left(\underline{N}(A)\cap\underline{N}(B)\right)\subseteq\sim\left(\left(\underline{N}(A)\right)\cup\sim\left(\underline{N}(B)\right)\right) \quad (42)$$

Again, consider:

$$y \in \sim\left(\left(\underline{N}(A)\right)\cup\sim\left(\underline{N}(B)\right)\right)$$

$$\Rightarrow y \in \sim\underline{N}(A) \text{ and } y \in \sim\underline{N}(B)$$

$$\Rightarrow y \in \sim\left(\underline{N}(A)\cap\underline{N}(B)\right)$$

$$\Rightarrow \sim\left(\underline{N}(A)\cap\underline{N}(B)\right)\supseteq\sim\left(\left(\underline{N}(A)\right)\cup\sim\left(\underline{N}(B)\right)\right). \quad (43)$$

Hence,

$$\sim\left(\underline{N}(A)\cap\underline{N}(B)\right)=\sim\left(\left(\underline{N}(A)\right)\cup\sim\left(\underline{N}(B)\right)\right). \quad (44)$$

Proof V

Consider:

$$x \in \sim\left(\overline{N}(A)\cup\overline{N}(B)\right)$$

$$\Rightarrow x \in \sim\overline{N}(A) \text{ and } x \in \sim\overline{N}(B)$$

$$\Rightarrow x \in \sim\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)$$

$$\Rightarrow x \in \sim\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)$$

$$\Rightarrow \sim\left(\overline{N}(A)\cup\overline{N}(B)\right)\subseteq\sim\left(\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)\right). \quad (45)$$

Again, consider:

$$y \in \sim\left(\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)\right)$$

$$\Rightarrow y \in \sim\overline{N}(A) \text{ or } y \in \sim\overline{N}(B)$$

$$\Rightarrow y \in \sim\left(\overline{N}(A)\cup\overline{N}(B)\right)$$

$$\Rightarrow \sim\left(\overline{N}(A)\cup\overline{N}(B)\right)\supseteq\sim\left(\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)\right). \quad (46)$$

Hence,

$$\sim\left(\overline{N}(A)\cup\overline{N}(B)\right)=\sim\left(\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)\right). \quad (47)$$

Proof VI

Consider:

$$x\in\sim\left(\overline{N}(A)\cap\overline{N}(B)\right)$$
$$\Rightarrow x\in\sim\overline{N}(A) \text{ or } x\in\sim\overline{N}(B)$$
$$\Rightarrow x\in\sim\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)$$
$$\Rightarrow x\in\sim\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)$$
$$\Rightarrow \sim\left(\overline{N}(A)\cap\overline{N}(B)\right)\subseteq\sim\left(\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)\right). \quad (48)$$

Again, consider:

$$y\in\sim\left(\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)\right)$$
$$\Rightarrow y\in\sim\overline{N}(A) \text{ and } y\in\sim\overline{N}(B)$$
$$\Rightarrow y\in\sim\left(\overline{N}(A)\right)\cap\sim\left(\overline{N}(B)\right)$$
$$\Rightarrow \sim\left(\overline{N}(A)\cap\overline{N}(B)\right)\supseteq\sim\left(\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)\right). \quad (49)$$

Hence,

$$\sim\left(\overline{N}(A)\cap\overline{N}(B)\right)=\sim\left(\left(\overline{N}(A)\right)\cup\sim\left(\overline{N}(B)\right)\right). \quad (50)$$

Proposition 2

$$\text{I. } \sim[N(A)\cup N(B)] = (\sim N(A))\cap(\sim N(B)); \quad (51)$$
$$\text{II. } \sim[N(A)\cap N(B)] = (\sim N(A))\cup(\sim N(B)). \quad (52)$$

Proof I

$$\sim[N(A)\cup N(B)]$$
$$= \sim<\underline{N}(A)\cup\underline{N}(B),\ \overline{N}(A)\cup\overline{N}(B)>$$
$$= <\sim\left(\underline{N}(P)\cap\underline{N}(Q)\right),\ \sim\left(\overline{N}(P)\cap\overline{N}(Q)\right)>$$
$$= (\sim N(A))\cap(\sim N(B)) \quad (53)$$

Proof II

$$\sim[N(A)\cap N(B)]$$
$$= \sim<\underline{N}(A)\cap\underline{N}(B),\ \overline{N}(A)\cap\overline{N}(B)>$$

$$= <\sim (\underline{N}(A) \cup \underline{N}(B)), \sim (\overline{N}(A) \cup \overline{N}(B))>$$

$$= (\sim N(A)) \cup (\sim N(B)) \quad (54)$$

4 Rough neutrosophic hyper-complex cosine function (RNHCF)

The cosine similarity measure is calculated as the inner product of two vectors divided by the product of their lengths. It is the cosine of the angle between the vector representations of two rough neutrosophic sets. The cosine similarity measure is a fundamental measure used in information technology. Now, a new cosine function between rough neutrosophic hyper-complex sets has been proposed as follows.

Definition 4.1

Assume that there are two rough neutrosophic hyper-complex sets

$$A = \left\langle [u + vI]_{\underline{N}(A)}(x), [u + vI]_{\overline{N}(A)}(x) \right\rangle, \quad (55)$$

and

$$B = \left\langle [u + vI]_{\underline{N}(B)}(x), [u + vI]_{\overline{N}(B)}(x) \right\rangle \quad (56)$$

in $X = \{x_1, x_2, \ldots, x_n)$.
Then rough neutrosophic hyper-complex cosine function between two sets A and B is proposed as follows:

$$C_{RNHCF}(A,B) =$$

$$\frac{1}{n}\sum_{i=1}^{n} \frac{\Delta u_A(x_i).\Delta u_B(x_i) + \Delta v_A(x_i).\Delta v_B(x_i) + \Delta I_A(x_i).\Delta I_B(x_i)}{\sqrt{(\Delta u_A(x_i))^2 + (\Delta v_A(x_i))^2 + (\Delta I_A(x_i))^2}\sqrt{(\Delta u_B(x_i))^2 + (\Delta v_B(x_i))^2 + (\Delta I_B(x_i))^2}} \quad (57)$$

where

$$\Delta u_A(x_i) = 0.5.\left| u_{\underline{N}(A)(x_i)} + u_{\overline{N}(A)(x_i)} \right|, \quad (58)$$

$$\Delta u_B(x_i) = 0.5.\left| u_{\underline{N}(B)(x_i)} + u_{\overline{N}(B)(x_i)} \right|, \quad (59)$$

$$\Delta v_A(x_i) = 0.5.\left| v_{\underline{N}(A)(x_i)} + v_{\overline{N}(A)(x_i)} \right|, \quad (60)$$

$$\Delta v_B(x_i) = 0.5.\left| v_{\underline{N}(B)(x_i)} + v_{\overline{N}(B)(x_i)} \right|, \quad (61)$$

$$\Delta I_A(x_i) = 0.5.\left| I_{\underline{N}(A)(x_i)} + I_{\overline{N}(A)(x_i)} \right|, \quad (62)$$

$$\Delta I_B(x_i) = 0.5.\left| I_{\underline{N}(B)(x_i)} + I_{\overline{N}(B)(x_i)} \right|. \quad (63)$$

Proposition 3

Let A and B be rough neutrosophic sets; then:

I. $0 \leq C_{RNHCF}(A,B) \leq 1$ (64)

II. $C_{RNHCF}(A,B) = C_{RNHCF}(B,A)$ (65)

III. $C_{RNHCF}(A, B) = 1$, if and only if A = B (66)

IV. If C is a RNHCF in Y and $A \subset B \subset C$ then, $C_{RNHCF}(A, C) \leq C_{RNHCF}(A, B)$, and $C_{RNHCF}(A, C) \leq C_{RNHCF}(B,C)$. (67)

Proofs

I. It is obvious because all positive values of cosine function are within 0 and 1

II. It is obvious that the proposition is true.

III. When A = B, then obviously $C_{RNHCF}(A, B) = 1$. On the other hand if $C_{RNHCF}(A, B) = 1$ then, $\Delta T_A(x_i) = \Delta T_B(x_i)$, $\Delta I_A(x_i) = \Delta I_B(x_i)$, $\Delta F_A(x_i) = \Delta F_B(x_i)$.

This implies that A = B.

IV. If $A \subset B \subset C$ then we can write:

$$u_{\underline{N}(A)}(x_i) \leq u_{\underline{N}(B)}(x_i) \leq u_{\underline{N}(C)}(x_i), \quad (68)$$

$$u_{\overline{N}(A)}(x_i) \leq u_{\overline{N}(B)}(x_i) \leq u_{\overline{N}(C)}(x_i), \quad (69)$$

$$v_{\underline{N}(A)}(x_i) \leq v_{\underline{N}(B)}(x_i) \leq v_{\underline{N}(C)}(x_i), \quad (70)$$

$$v_{\overline{N}(A)}(x_i) \leq v_{\overline{N}(B)}(x_i) \leq v_{\overline{N}(C)}(x_i), \quad (71)$$

$$I_{\underline{N}(A)}(x_i) \geq I_{\underline{N}(B)}(x_i) \geq I_{\underline{N}(C)}(x_i), \quad (72)$$

$$I_{\overline{N}(A)}(x_i) \geq I_{\overline{N}(B)}(x_i) \geq I_{\overline{N}(C)}(x_i) \quad (73)$$

The cosine function is decreasing function within the interval $\left[0, \frac{\pi}{2}\right]$. Hence we can write $C_{RNHCF}(A, C) \leq C_{RNHCF}(A, B)$, and $C_{RNHCF}(A, C) \leq C_{RNHCF}(B, C)$.

If we consider the weights of each element x_i, a weighted rough neutrosophic hyper-complex cosine function (WRNHCF) between two sets A and B can be defined as follows:

$$C_{WRNHCF}(A,B) =$$

$$\sum_{i=1}^{n} W_i \frac{\Delta u_A(x_i).\Delta u_B(x_i) + \Delta v_A(x_i).\Delta v_B(x_i) + \Delta I_A(x_i).\Delta I_B(x_i)}{\sqrt{(\Delta u_A(x_i))^2 + (\Delta v_A(x_i))^2 + (\Delta I_A(x_i))^2}\sqrt{(\Delta u_B(x_i))^2 + (\Delta v_B(x_i))^2 + (\Delta I_B(x_i))^2}} \quad (74)$$

where

$$\Delta u_A(x_i) = 0.5.\left|u_{\underline{N}(A)(x_i)} + u_{\overline{N}(A)(x_i)}\right|, \quad (75)$$

$$\Delta u_B(x_i) = 0.5.\left|u_{\underline{N}(B)(x_i)} + u_{\overline{N}(B)(x_i)}\right|, \quad (76)$$

$$\Delta v_A(x_i) = 0.5.\left|v_{\underline{N}(A)(x_i)} + v_{\overline{N}(A)(x_i)}\right|, \quad (77)$$

$$\Delta v_B(x_i) = 0.5.\left|v_{\underline{N}(B)(x_i)} + v_{\overline{N}(B)(x_i)}\right|, \quad (78)$$

$$\Delta I_A(x_i) = 0.5.\left|I_{\underline{N}(A)(x_i)} + I_{\overline{N}(A)(x_i)}\right|, \quad (79)$$

$$\Delta I_B(x_i) = 0.5.\left|I_{\underline{N}(B)(x_i)} + I_{\overline{N}(B)(x_i)}\right|. \quad (80)$$

$w_i \in [0,1]$, i = 1, 2,..., n and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \frac{1}{n}$, i = 1, 2,..., n, then:

$$C_{WRNHCF}(A, B) = C_{RNHCF}(A, B) \quad (81)$$

The weighted rough neutrosophic hyper-complex cosine function (WRNHCF) between two rough sets A and B also satisfies the following properties:

I. $0 \le C_{WRNHCF}(A, B) \le 1$ (82)

II. $C_{WRNHCF}(A, B) = C_{WRNHCF}(B, A)$ (83)

III. $C_{WRNHCF}(A, B) = 1$, if and only if A = B (84)

IV. If C is a WRNHCF in Y and $A \subset B \subset C$ then, $C_{WRNHCF}(A, C) \le C_{WRNHCF}(A, B)$, and $C_{WRNHCF}(A, C) \le C_{WRNHCF}(B, C)$ (85)

5 Decision making procedure based on rough hyper-complex neutrosophic function

In this section, we apply rough neutrosophic hyper-complex cosine function between RNHSs to the multi-attribute decision making problem. Let A_1, A_2, ..., A_m be a set of alternatives and C_1, C_2, ..., C_n be a set of attributes. The proposed decision making method is described using the following steps.

Step1: Construction of the decision matrix with rough neutrosophic hyper-complex number

The decision maker considers a decision matrix with respect to *m* alternatives and *n* attributes in terms of rough neutrosophic hyper-complex numbers, as follows:

$DM = \left\langle \underline{dm}_{ij}, \overline{dm}_{ij} \right\rangle_{m\times n} =$ (86)

	C_1	C_2	...	C_n
A_1	$\langle \underline{dm}_{11}, \overline{dm}_{11} \rangle$	$\langle \underline{dm}_{12}, \overline{dm}_{12} \rangle$	...	$\langle \underline{dm}_{1n}, \overline{dm}_{1n} \rangle$
A_2	$\langle \underline{dm}_{21}, \overline{dm}_{21} \rangle$	$\langle \underline{dm}_{22}, \overline{dm}_{22} \rangle$	...	$\langle \underline{dm}_{2n}, \overline{dm}_{2n} \rangle$
.	...	...	...	...
.	...	...	...	...
A_m	$\langle \underline{dm}_{m1}, \overline{dm}_{m1} \rangle$	$\langle \underline{dm}_{m2}, \overline{dm}_{m2} \rangle$	...	$\langle \underline{dm}_{mn}, \overline{dm}_{mn} \rangle$

Table 1. Rough neutrosophic hyper-complex decision matrix.

Here $\langle \underline{dm}_{ij}, \overline{dm}_{ij} \rangle$ is the rough neutrosophic hyper-complex number according to the *i*-th alternative and the *j*-th attribute.

Step2: Determination of the weights of attribute

Assume that the weight of the attributes C (j = 1, 2, … , n) considered by the decision-maker be w_j (j = 1, 2, … , n) such that $\forall w_j \in [0, 1]$ (j = 1, 2, …, n) and $\sum_{j=1}^{n} w_j = 1$.

Step 3: Determination of the benefit type attribute and cost type attribute

Generally, the evaluation attribute can be categorized into two types: benefit attribute and cost attribute. Let K be a set of benefit attribute and M be a set of cost attribute. In the proposed decision-making method, an ideal alternative can be identified by using a maximum operator for the benefit attribute and a minimum operator for the cost attribute to determine the best value of each criterion among all alternatives. Therefore, we define an ideal alternative as follows:

$$A^* = \{C_1^*, C_2^*, \dots, C_m^*\}, \tag{87}$$

Benefit attribute:

$$C_j^* = \left[\max_i u_{C_j}^{(A_i)}, \max_i v_{C_j}^{(A_i)}, \min_i I_{C_j}^{(A_i)} \right]. \tag{88}$$

Cost attribute:

$$C_j^* = \left[\min_i T_{C_j}^{(A_i)}, \min_i I_{C_j}^{(A_i)}, \max_i F_{C_j}^{(A_i)} \right] \tag{89}$$

where

$$u_{C_j}^{(A_i)} = 0.5.\left| \left(u_{C_j}\right)_{\underline{N}(A_i)} + \left(u_{C_j}\right)_{\overline{N}(A_i)} \right|, \tag{90}$$

$$v_{C_j}^{(A_i)} = 0.5.\left| \left(v_{C_j}\right)_{\underline{N}(A_i)} + \left(v_{C_j}\right)_{\overline{N}(A_i)} \right|, \tag{91}$$

and

$$I_{C_j}^{(A_i)} = 0.5.\left| \left(I_{C_j}\right)_{\underline{N}(A_i)} + \left(I_{C_j}\right)_{\overline{N}(A_i)} \right|. \tag{92}$$

Step4: Determination of the over all weighted rough hyper-complex neutrosophic cosine function (WRNHCF)of the alternatives

Weighted rough neutrosophic hyper-complex cosine function is given as follows:

$$C_{WRNHCF}(A, B) = \sum_{j=1}^{n} w_j C_{WRNHCF}(A, B) \tag{93}$$

Step5: Ranking the alternatives

Using the weighted rough hyper-complex neutrosophic cosine function between each alternative and the ideal alternative, the ranking order of all alternatives can be determined and the best alternative can be easily selected with the highest similarity value.

Step 6: End

6 Numerical Example

Assume that a decision maker (an adult man/woman who is eligible to marry) intends to select the most suitable life partner for marriage from the three initially chosen candidates (S_1, S_2, S_3) by considering five attributes, namely: physical and mental health C_1, education and job C_2, management power C_3, family background C_4, risk factor C_5.

Based on the proposed approach discussed in section 5, the considered problem has been solved using the following steps:

Step1: Construction of the decision matrix with rough neutrosophic hyper-complex numbers

The decision maker considers a decision matrix with respect to three alternatives and five attributes in terms of rough neutrosophic hyper-complex numbers shown in the Table 2.

$$DM = \left\langle \underline{dm}_{ij}, \overline{dm}_{ij} \right\rangle_{3\times 5} =$$

	C_1	C_2	C_3	C_4	C_5
A_1	$\left\langle (i+0.6(1+i)), (2i+0.4(2+i)) \right\rangle$	$\left\langle ((1+i)+0.65(2i)), ((1+2i)+0.55(3i)) \right\rangle$	$\left\langle ((1+i)+0.4(2+i)), ((1+2i)+0.2(2+3i)) \right\rangle$	$\left\langle (4i+0.55(1+i)), ((4+i)+0.45(2+i)) \right\rangle$	$\left\langle (3i+0.78(2+3i)), ((1+3i)+0.72(3+3i)) \right\rangle$
A_2	$\left\langle (i+0.6(1+2i)), (3i+0.5(1+3i)) \right\rangle$	$\left\langle ((1+i)+0.55(i)), ((1+2i)+0.45(3i)) \right\rangle$	$\left\langle (2i+0.3(2+i)), ((2+i)+0.2(1+3i)) \right\rangle$	$\left\langle (i+0.52(2+3i)), (2i+0.48(4+3i)) \right\rangle$	$\left\langle ((1+i)+0.82(2+i)), (2i+0.78(4+3i)) \right\rangle$
A_3	$\left\langle (2i+0.5(1+i)), (3i+0.4(1+3i)) \right\rangle$	$\left\langle ((2+i)+0.69(5i)), ((2+i)+0.51(6i)) \right\rangle$	$\left\langle (i+0.6(1+i)), (2i+0.4(3+2i)) \right\rangle$	$\left\langle ((1+i)+0.48(3+4i)), ((1+2i)+0.42(5+3i)) \right\rangle$	$\left\langle ((1+i)+0.9(i)), ((1+2i)+0.7(2+3i)) \right\rangle$

(94)

where, $i=\sqrt{-1}$.

Table 2. Decision matrix with rough neutrosophic hyper-complex number.

Step 2: Determination of the weights of the attributes

The weight vectors considered by the decision maker are 0.25, 0.20, 0.25, 0.10, and 0.20 respectively.

Step 3: Determination of the benefit attribute and cost attribute

Here four benefit types attributes C_1, C_2, C_3, C_4 and one cost type attribute C_5. Using equations (12) and (13) we calculate A^* as follows:

$$A^* = [(5.00, 2.69, 0.45), (4.47, 5.50, 0.50), (3.60, 2.83, 0.25), (6.40, 5.30, 0.45), (3.16, 2.24, 0.80)]$$

Step 4: Determination of the over all weighted rough hyper-complex neutrosophic similarity function (WRHNSF)of the alternatives

We calculate weighted rough neutrosophic hyper-complex similarity values as follows:

$$S_{WRHCF}(A_1, A^*) = 0.9622;$$

$$S_{WRHCF}(A_2, A^*) = 0.9404;$$

$$S_{WRHCF}(A_3, A^*) = 0.9942.$$

Step 5: Ranking the alternatives

Ranking the alternatives is prepared based on the descending order of similarity measures. Highest value reflects the best alternative.
Here,

$$S_{WRHCF}(A_3, A^*) \prec S_{WRHCF}(A_1, A^*) \prec S_{WRHCF}(A_2, A^*). \quad (95)$$

Hence, the decision maker must choose the candidate A_3 as the best alternative for marriage.

Step 6: End

7 Conclusion

In this paper, we have proposed the rough neutrosophic hyper-complex set and the rough neutrosophic hyper-complex cosine function, and proved some of their basic properties.

We have also proposed the rough neutrosophic hyper-complex similarity measure based multi-attribute decision making.

We have presented an application, namely selection of best candidate for marriage for Indian context.

The concept presented in this paper can be applied for other multiple attribute decision making problems in rough neutrosophic hyper-complex environments.

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