# A Mechanistic Model for the Hydrogen Atom

Norman Graves
192 Old Woosehill Lane
Wokingham
RG41 3HQ
UK
+44 7785 305 291
normangraves@btinternet.com

**Abstract**

A model is proposed for the hydrogen atom in which the orbiting electron is seen as an objectively real particle. The model is based on the postulate that certain velocity terms can be treated as being affected by relativity. The model provides a mechanism which drives the quantization process and so leads to the discrete energy levels of the atom.

The Rydberg formula, being empirically derived, represents the yardstick by which any model for the hydrogen atom must be judged. Rather than develop a model and test it against the Rydberg formula, the approach taken here is to use the Rydberg formula itself as the basis for such a model.

The model effectively unifies quantum mechanics with classical mechanics as well as providing a simple mechanical explanation of the Somerfield Fine Structure Constant.

**Keywords**

Hydrogen atom, fine structure constant, unification, quantization, wave particle duality,

## Introduction

The quantization of matter and of electric charge are simple concepts to grasp since they involve merely the absence or presence of a integer number of discrete particles. Particles, like grains of sand, can simply be counted to give the total amount of matter in any given volume. Electric charge is only a little more complicated since it involves the arithmetic sum of particles which can contain unit charge which can be either positive or negative.

The discrete energy levels of the hydrogen atom on the other hand are a completely different matter. Here it is the energy carried by the particle which is somehow constrained to only take on certain discrete values. There is no particle of energy which can simply be counted. Energy is a compound value involving the interactions of several variables. There must be some sort of interplay between the various quantities involved which serves to constrain the overall energy in this particular way. In the past it was deemed that this was because angular momentum was somehow quantized and can only occur in discrete chunks or quanta. However there is no particle of angular momentum and angular momentum is itself a compound value dependent on three variables. No explanation has ever been proposed or found as to how these three variables might interact with one another to produce this quantization effect.

It is not sufficient to simply declare that this or that variable is quantized without any proof or justification. Neither is it sufficient to use this declaration as the basis for justifying the discrete energy levels of the atom. What is necessary is to show that there is some sort of mechanism or process which can cause a variable to be quantized and which in turn leads to the discrete energy levels of the atom.

## The Rydberg Formula

The Rydberg formula[i] was developed in the late 19th century based on observations of the absorption spectrum of hydrogen[1] and on earlier work by Balmer. It was not based on any theoretical model, but derived empirically from observations of the emissions and absorption of the hydrogen atom. As such it can be regarded as a sort of gold standard against which any theoretical model for the hydrogen atom must be judged. The atom is seen as occupying one of a number of discrete energy states, that energy being carried by the orbiting electron. Transitions between a high energy state and a low energy state result in the release of energy in the form of a photon. Those from a low energy state to a high energy state are the result of energy being absorbed from an incident photon.

The Rydberg formula tells us the wavelengths of the photons emitted or absorbed by a hydrogenic atom and in particular the hydrogen atom. The formula for hydrogen is most often quoted as

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad \text{Equation 1}$$

[1] In fact the Rydberg formula works for any so called hydrogenic atom, that is an atom which has been ionized to the extent that it has only one orbiting electron. The value of R is then specific to each type of atom.

Where $n_1$ and $n_2$ are the respective energy states for a particular energy transition and $R_H$ is a constant, now known as the Rydberg constant, in this case for hydrogen.

In this form the formula tells us little of what is happening within the atom, largely because it is expressed in terms of *1/λ*, the wavenumber, which has little direct physical significance. However if we multiply both sides of the formula first by *c*, the velocity of light, to convert the wavenumber into frequency and then by $h$, Planck's constant, to turn this frequency into energy, we get an expression for the energy associated with each type of transition

$$E_{n_1,n_2} = hcR_H\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad \text{Equation 2}$$

Based on his model for the hydrogen atom, Niels Bohr was able to determine an analytical expression for the value of $R_H$[ii]

$$R_H = \frac{mq^4}{8h^3c\,\varepsilon_0^2} \qquad \text{Equation 3}$$

Substituting this into Equation 2 and recognizing that

$$K = \frac{1}{4\pi\varepsilon_0} \qquad \text{Equation 4}$$

$$h = 2\pi\hbar \qquad \text{Equation 5}$$

And

$$\alpha = \frac{Kq^2}{\hbar c} \qquad \text{Equation 6}$$

We obtain the much more useful form

$$E_{n_1,n_2} = \frac{1}{2}mc^2\alpha^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad \text{Equation 7}$$

Where α is the Fine Structure Constant of which Richard Feynman once said[iii]

*"It has been a mystery ever since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it. Immediately you would like to know where this number for a coupling comes from: is it related to pi or perhaps to the base of natural logarithms? Nobody knows. It's one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man. You might say the "hand of God" wrote that number, and "we don't know how He pushed his pencil." We know what kind of a dance to do experimentally to measure this number very*

*accurately, but we don't know what kind of dance to do on the computer to make this number come out, without putting it in secretly!".*

The Rydberg formula tells us the amount of energy released when the electron orbiting the hydrogen nucleus makes a transition from the $n_1^{th}$ energy state to the $n_2^{th}$ energy state, or conversely the amount of energy absorbed if the transition is in the other direction. By letting $n_2 = \infty$ we obtain the energy associated with a transition to or from the maximum possible energy state and its energy in the $n^{th}$ energy state, that is we obtain the energy potential of the atom in the $n^{th}$ energy state. Doing so leads to the Rydberg Series

$$\Delta E_n = \frac{1}{2}mc^2\frac{\alpha^2}{n^2} \qquad \text{Equation 8}$$

The Rydberg Series is particularly useful because it allows us to easily calculate the energy associated with any energy transition, simply by taking the difference between two values in the series.

$\Delta E_n$ represents the difference between the energy of the electron in the $n^{th}$ energy state and the most energetic energy state possible, the $\infty$ energy state or energy ceiling of the atom. The energy ceiling of the atom represents the maximum energy that an orbiting electron could ever possibly have. Since nothing can ever travel faster than the speed of light, the energy ceiling is limited by the speed of light to be

$$e_{max} = \frac{1}{2}mc^2 \qquad \text{Equation 9}$$

It is reasoned here that the electron orbiting the atomic nucleus must do so at the constant radius, that is at the same orbital radius for every energy state. Anything other than this would imply the existence of the physically impossible 'quantum leap', the ability to move from A to B without occupying anywhere in between. This in turn means that there can be no change in potential energy when the electron transitions from one energy state to another energy state. Hence the energy of the electron in the $n^{th}$ energy state must be

$$e_n = \frac{1}{2}mv_n^2 \qquad \text{Equation 10}$$

Where $v_n$ is the orbital velocity in the $n^{th}$ energy state.

Combining Equation 8, Equation 9 and Equation 10 to calculate the energy potential in the $n^{th}$ energy state gives

$$\frac{1}{2}mc^2 - \frac{1}{2}m\,v_n^2 = \frac{1}{2}mc^2\frac{\alpha^2}{n^2} \qquad \text{Equation 11}$$

Equation 11 can be simplified to give

$$c^2 - v_n^2 = c^2\frac{\alpha^2}{n^2} \qquad \text{Equation 12}$$

And further simplified to give

$$\frac{1}{\sqrt{1-\frac{v_n^2}{c^2}}}=\frac{n}{\alpha} \qquad \text{Equation 13}$$

The term on the left of Equation 13 will be recognized as the Lorentz factor Gamma (γ) and hence

$$\gamma_n=\frac{n}{\alpha} \qquad \text{Equation 14}$$

Equation 13 can be solved for *v* in the base energy state where $n = 1$. Doing so reveals it to have a value of 0.999973371c. This means that the dynamic range of *v* is very small, ranging from 99.9973371% of *c* to 100% of *c*.

The angular momentum of the orbiting electron is equal to Planck's constant. The electron is seen to orbit at more or less constant radius given by

$$R=\frac{\hbar}{mc} \qquad \text{Equation 15}$$

Because *R* is the same for all energy states and both *m* and *c* are constants it can be seen that the angular momentum is the same in all energy states.

From Equation 14 it is evident that, in an atom where the electron is considered to be an objectively real particle orbiting at near light speed, the variable of quantization is γ and not angular momentum as in the Bohr model and other subsequent models based on the Bohr model.

There is however one important difference to note. In these earlier models angular momentum is taken to be quantum arithmetic value, that is it can only ever take on discrete values which are an integer multiple of Planck's constant. Here the situation is somewhat different. It is quite evident that γ is a continuous variable, ranging from unity to a theoretical upper limit of infinity. There are numerous examples of circumstances where γ has a value which is not related to α in any way whatsoever. It must therefore be the case that there is something about the dynamics of the atom that cause this otherwise continuous variable to only be capable of taking on one of a series of discrete values. In other words there has to be a mechanism or process which drives the quantization in the context of the dynamics of the hydrogen atom. Equation 14 shows that relativity has a role to play in the dynamics of the atom and it will be shown here that it is indeed instrumental in causing the atom to take on its discrete energy levels.

**Relativity**

The year 1905 was an eventful one for Albert Einstein. In that year, he not only published his paper on the discrete nature of the photon for which he later received the Nobel Prize but he also published two further seminal works as well as submitting his PhD thesis. The most famous of these other papers concerned the dynamics of moving bodies[iv]. This is the paper whose later editions contained the equation $e=mc^2$. The paper was based on a thought experiment and concerned the perception of time, distance and mass as experienced by two observers, one a stationary observer and one moving relative to the stationary observer at speeds approaching that of light.

What Einstein showed was that time elapsed more slowly for a moving observer, that distances measured in the direction of travel by a moving observer were foreshortened relative to those same distances measured by a stationary observer and that a stationary observer's perception of the mass of a moving object was that it had increased. All three effects occur to the same extent and are governed by a factor γ (Gamma). The time between two events observed by a stationary observer as time $t$ is seen by a moving observer as time $T=t/\gamma$. Similarly the distance between two point measured by a stationary observer as distance $d$ is seen by a moving observer as distance $D=d/\gamma$. And as far as a stationary observer is concerned the mass of the moving object is seen to increase by this same factor γ.

Gamma is referred to as the Lorentz factor and is given by the formula

$$\gamma = \frac{c}{\sqrt{c^2 - v^2}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad \text{Equation 16}$$

Both observers agree on their relative velocity, but go about calculating it in different ways. For the stationary observer the velocity of the moving observer is the distance travelled divided by the time taken as measured in his stationary domain. For the stationary observer the velocity is

$$v = \frac{d}{t} \qquad \text{Equation 17}$$

For the moving observer the distance as measured in his or her own domain is foreshortened by the factor Gamma, but the time taken to cover that distance reduced by the same factor Gamma hence

$$v = \frac{D}{T} = \frac{d/\gamma}{t/\gamma} = \frac{d}{t} \qquad \text{Equation 18}$$

A slightly different way to view the effect of relativity on distance, rather than imagine that the distance between points changes, is to imagine instead that the scale on which distance is measured changes. It is as if the measurements are made with a tape measure made of elastic, the faster one travels; the more the elastic tape measure is stretched and the further apart the scale markings appear, but only when making measurements in the direction of travel.

There is a great deal of experimental evidence to support Einstein's Special Theory. One of the more convincing experiments was carried out at CERN in 1977 and involved measuring the lifetimes of particles called muons in an apparatus called the muon storage ring[v]. The muon is an atomic particle which carries an electric charge, much like an electron, only more massive. It has a short lifetime of around 2.2 microseconds before it decays into an electron and two neutrinos.

In the experiment muons are injected into a 14m diameter ring at a speed of 99.94% of the speed of light. At this speed Gamma has a value of 29.33. The muons, which should normally live for 2.2 microseconds, were seen to have an average lifetime of 64.5 microseconds; that is the lifetime of the muon was extended by the factor Gamma. This comes about because the processes which take place inside the muon and which eventually lead to its decay are taking place in an environment which is moving relative to us at 99.94% of the speed of light and in which time, relative to us, is running 29.33 times slower. Hence the muon, in its own domain, still has a lifetime of 2.2 microseconds, it's just that to us, who are not moving, this appears as 64.5 microseconds.

Travelling at almost the speed of light a muon would normally be expected to cover a distance of 660 metres or roughly 7.5 times around the CERN ring during its 2.2 microsecond lifetime, but in fact the muons travelled almost 20,000 metres or 220 times around the ring. This is because distance in the domain of the muon is compressed so what we stationary observers see as being 20,000 metres the muon sees as being just 660 metres.

The muon ring experiment demonstrates two further important characteristics associated with orbiting objects traveling at near light speed.

The orbital radius is measured at right angles to the direction of travel of the muon and is therefore unaffected by relativity. This means that the angular displacement perceived by the muon must differ from that perceived by the stationary observer. For the stationary observer the muon travels a total of 20,000 m at a radius of 7 m, a total angle of 2857 radians. For the muon however the distance traveled is only 660 m but the radius is still 7 m and so the muon's perception of the angular displacement is 94 just radians.

Both parties agree that during its lifetime the muon completes some 220 turns around the ring. We stationary observers see this as having taken place in some 64.5 microseconds, corresponding to an orbital frequency of 3.41MHz, while the muon sees these 220 turns as having been completed in just 2.2 microseconds corresponding to an orbital frequency of 100MHz. Hence for the muon orbital frequency is multiplied by a factor $\gamma$ relative to that seen by a stationary observer.

**Harmonic Series**

The idea that the discrete energy levels of the hydrogen atom are associated with a harmonic series was first proposed by the French physicist Louis de Broglie. He suggested that the electron had an associated wavelength that was equal to Planck's constant divided by its linear momentum, effectively this is a restatement of Bohr's earlier assumption that angular momentum is quantized.

Here we see a different situation, Equation 14 tells us that each energy state is associated with a value of $\gamma$ that is an integer multiple of that of the base energy state and as we have seen, orbital frequency for a moving object is multiplied by $\gamma$ relative to that seen by a stationary observer. So while we stationary observers see the orbital frequency as being more or less constant, the orbiting electron sees the orbital frequency as being one of a series of frequencies which form a harmonic series.

For the stationary observer the orbital frequency is approximately

$$\omega = \frac{c}{R} = \frac{mc^2}{\hbar} \qquad \text{Equation 19}$$

For the moving electron however the orbital frequency is seen as

$$\omega_e = \frac{mc^2}{\hbar}\frac{n}{\alpha} \qquad \text{Equation 20}$$

Forming a harmonic series in which the orbital frequency in the $n^{th}$ energy state is the $n^{th}$ multiple of that of the base energy state.

**Sampling process**

Wherever we see a harmonic series in nature there must always be a corresponding sampling process. This becomes evident if we consider the Fourier representation of a harmonic series. Such a Fourier representation comprises a series of spikes equally spaced along the frequency axis. For a real function these are disposed equally on both the positive and negative frequency axes. These spikes are referred to as Dirac functions and such a collection of equally spaced Dirac functions is referred to as a Dirac comb.

The inverse Fourier transform of a Dirac comb in the frequency domain is itself another Dirac comb in the time domain[vi]. Such a Dirac comb in the time domain can be regarded as a sampling function, since if it is multiplied by any other signal it effectively takes a sample at regular intervals in time. All of this points to the idea that somewhere within the dynamics of the atom we can expect to find a sampling process. It means that there is something within the atom that happens or can happen only once per orbit of the orbiting electron.

Sampling Theory is the branch of mathematics which deals with continuous variables and discrete solutions. It was developed in the 1930's and 1940's at Bell labs in order to deal with capacity problems on the telephone network.

At that time telecommunications engineers were concerned to increase the capacity of the telephone network. One of the ideas that surfaced was called Time Division Multiplexing. In this each of a number of incoming telephone lines is sampled by means of a switch, the resulting samples are sent over a trunk line and are decoded by a similar switch at the receiving end before being sent on their way. This allowed the trunk line to carry more telephone traffic without the expense of increasing the number of cables or individual lines. The question facing the engineers at the time was to determine the minimum frequency at which the incoming lines needed to be sampled in order that the telephone signal can be correctly reconstructed at the receiving end.

The solution to this problem was arrived at independently by a number of investigators, but is now largely credited to two engineers. The so called Nyquist-Shannon sampling theorem is named after Harry Nyquist[vii] and Claude Shannon[viii] who were both working at Bell Labs at the time. The theorem states that in order to reproduce a signal with no loss of information, then the sampling frequency must be at least twice the highest frequency of interest in the signal itself. The theorem forms the basis of modern information theory and its range of applications extends well beyond transmission of analog telephone calls, it underpins much of the digital revolution that has taken place in recent years.

What concerned Shannon and Nyquist was to sample a signal and then to be able to reproduce that signal at some remote location without any distortion, but a corollary to their work is to ask what happens if the frequency of interest extends beyond this Shannon limit? In this condition, sometimes called 'under sampling', there are frequency components in the sampled signal that extend beyond the Shannon limit and maybe even beyond the sampling frequency itself.

The following example serves to illustrate the phenomenon. Suppose there is a cannon on top of a hill, some distance away an observer is equipped with a stopwatch. The job of the observer is to calculate the distance from his current location to the cannon. Sound travels in air at roughly 340 m/s. So it is simply a matter of the observer looking for the flash as the cannon fires and timing the interval until he hears the bang. Multiplying the result by 340 gives the distance D to the cannon in metres.

This is fine if the cannon just fires a single shot, but suppose the cannon is rigged to fire at regular intervals, T seconds apart. For the sake of argument and to simplify things, we can make T equal to 1. If the observer knows he is less than 340 m from the cannon there is no problem. He makes the measurement and calculates the distance D as before. If on the other hand he is free to move anywhere with no restriction placed on his distance to the cannon then there is a problem. There is no way that the observer can know which bang is associated with which flash, so he might be located at any one of a number of different discrete distances from the cannon. Not just any old distance will do however. The observer must be at a distance of D or D + 340 or D + 680 and so on, in general D + 340n. The distance calculated as a result of measuring the time interval between bang and flash is ambiguous. In fact there are an infinite number of discrete distances which could be the result of any particular measured value. This phenomenon is known as aliasing. The term comes about because each possible distance is an alias for the measured distance.

Restricting the observer to be within 340 m of the cannon is simply a way of imposing Shannon's sampling limit and by removing this restriction we open up the possibility of ambiguity in determining the position of the observer due to aliasing.

Turning the problem around slightly; instead of measuring the distance to the cannon the position of the observer is fixed. Once again, to make things simpler, we can choose a distance of 340m. This time however we are able to adjust the rate of fire of the cannon until the observer hears the bang and sees the flash as occurring simultaneously. If the rate of fire is one shot per second then the time taken for the slower bang to reach the observer exactly matches the interval between shots and so the two events, the bang and the flash are seen as being synchronous. Notice that the bang relates, not to the current flash, but to the previous flash.

If the rate of fire is increased then at first, for a small increment, the bang and the flash are no longer in sync. However they come back into sync again when the rate of fire is exactly two shots per second, and again when the rate is three shots per second. If we had a fast enough machine gun this sequence would extend to infinity but only for a rate of fire which is an integer number of shots per second. Notice that now the bang no longer relates to **the** previous flash, but to **a** previous flash. The fact that there are intermediate bangs and flashes is irrelevant. If we look at any arbitrary flash then there will be a synchronous bang provided the rate of fire is an integer number of shots per second.

It is interesting to note also that if the rate of fire is reduced from once per second then the observer will never hear and see the bang and the flash in sync with one another and so once per second represents the minimum rate of fire which will lead to a synchronous bang and flash. In fact what we have is a system that has as its solutions a base frequency and an infinite set of harmonic frequencies.

Here is a system which can cause a variable, in this case the rate of fire of the gun, to take on a series of discrete values even though, in theory at least, the rate of fire can vary continuously. Equally important is that if the system is capable of syncing to the lowest such frequency then all the multiples of this frequency are also solutions, in other words if the base frequency is a solution then so are harmonics of the base frequency. It is suggested here that this is precisely the type of mechanism that occurs inside the atom and leads to its discrete energy levels.

**Relativistic Velocity**

We saw in Equation 18 that velocity is generally taken to be invariant with respect to relativity. Indeed this idea is axiomatic in the derivation of special relativity. Hence for the moving observer both the distance and the time are scaled by the same factor $\gamma$ relative to those seen by the stationary observer and these cancel such that the velocity is the same for both observers.

In order to measure the speed of an object moving at close to the speed of light in real time it is necessary for a stationary observer to use two clocks, at least conceptually. One clock must be set up at the point of departure and another at the point of arrival. The two clocks must then be synchronized before the measurement can begin. The time that the moving object leaves the point of departure is noted on the departure clock and the time of its arrival is noted on the arrival clock. At least one of these measurements must then be transmitted to the other location before the difference can be taken and the speed calculated. Any attempt to measure such a velocity in real time is thwarted by the fact that the clock would have to move with the moving object and so would itself be slowed down due to the effects of relativity.

There is however one circumstance where this is not the case and that is when the moving object is in orbit. Under this circumstance the object returns to its point of origin once per orbit and so it is possible conceptually at least, to measure its orbital velocity in real time using a single clock. Such measurement is only possible when the object returns to its point of departure that is once per complete orbit. This then is the sampling process of which I spoke earlier. The orbital velocity of the electron is such that it can only be determined once per orbit.

It is thus possible to define a velocity term which couples the two domains, that of the stationary observer and that of the moving electron. Such a velocity is calculated as the distance as measured by the moving object divided by the time as measured by the stationary observer, this latter can only meaningfully be measured for one or more complete orbits. For obvious reasons I have called this type of velocity Relativistic Velocity as opposed to the Actual Velocity and propose that it is this Relativistic Velocity that applies to phenomena associated with objects in orbit, specifically to centrifugal and centripetal force and acceleration and to angular momentum. Relativistic Velocity has the important characteristic that it gets smaller as the actual velocity approaches the speed of light.

$$v_r = \frac{D}{t} = \frac{d}{\gamma t} = \frac{v}{\gamma}$$ Equation 21

**Synchrotron Radiation**

Returning to Equation 15 which gives us the radius of the atom (reproduced here)

$$R = \frac{\hbar}{mc}$$ Equation 15

In this form the equation fails to take account of relativity. For a stationary observer located at the atomic nucleus electron is seen to be traveling at near light speed and so the mass term should be multiplied by γ. However it is argued here that the velocity term should be considered to be affected by relativity meaning that this should use the term for Relativistic Velocity, which would then mean that it should be divided by γ and hence

$$R = \frac{\hbar}{(m\gamma)}\left(\frac{\gamma}{c}\right)$$ Equation 22

Although in strict mathematical terms the two γ terms could cancel to return to Equation 15, it is important to note that there is a subtle difference between these two equations. In Equation 22 the value of the radius is actively driven to have the value *ħ/mc* and so it is perhaps more meaningful to state that *R* is identically equal to *ħ/mc*, rather than simply being equal to it.

$$R \equiv \frac{\hbar}{mc}$$ Equation 23

This provides an explanation as to why the orbiting electron does not decay due to the emission of synchrotron radiation. Rather than being driven in any conventional manner to adopt a circular orbit, here the atom is constrained by the combined effects of relativity and Planck's constant to always have a constant value. It is as if the electron is orbiting on a hard surface, one which it cannot penetrate and from which it cannot depart. This is more akin to the way in which we view general relativity, where objects move in straight lines on a curved space.

**The Base Energy State**

For the hydrogen atom to be stable it is necessary that the forces acting on the electron be in balance. The electrical force tending to pull the electron towards the nucleus must balance the centrifugal force tending to throw it off.

The electron is orbiting at near light speed where the effects of relativity must be taken into account. The mass term is affected by relativity and once again it is argued here that the velocity term should be based on the relativistic velocity and so for the base energy state

$$\frac{Kq^2}{R^2} = \frac{(m\gamma)}{R}\left(\frac{c}{\gamma}\right)^2 \qquad \text{Equation 24}$$

This can be combined with Equation 23 and simplified to give

$$\frac{Kq^2}{\hbar c} = \frac{1}{\gamma} \qquad \text{Equation 25}$$

The term on the LHS of Equation 25 is recognized as α, the Somerfield fine structure constant and so in the base energy state

$$\gamma = \frac{1}{\alpha} \qquad \text{Equation 26}$$

The forces are in balance because the Relativistic Velocity term causes the centrifugal force to decrease as the Actual Velocity increases eventually reaching the point where it exactly matches the attractive electrical force. At this point $\gamma$ has a value of approximately 137 and the orbital path length has been reduced by a factor $\gamma = 1/\alpha$ to $2\pi R\alpha = 1.77065 * 10^{-14}$while the orbital period is $2\pi R/c = 8.093 * 10^{-21}$and the Relativistic Velocity is $c\alpha$ .

Using the analogy of the elastic tape measure and taking a tape measure of natural length $2\pi R\alpha$ the tape measure has been stretched as the Actual Velocity increases. Stability is achieved when this tape measure is stretched sufficiently to encircle the Actual orbit of the electron exactly once.

It is important to note however that while $2\pi R\alpha$ is the distance as measured around the orbital circumference by the electron, it is not the only possible distance. Every integer multiple of this distance is an alias for this distance, so the distance could be interpreted as $4\pi R\alpha$ or as $6\pi R\alpha$ or in general as $2\pi R\alpha n$ where $n = 1,2,3,4, \ldots$ As far as the electron is concerned all of these possible distances are indistinguishable from one another and any one of these distances could be the distance travelled since it was overhead its point of departure on the orbital circumference. In the base state however none of these other aliased distances correspond to the atom being in a stable state, in much the same way as the point at which the cannon described earlier comes first into sync at 1 shot per second.

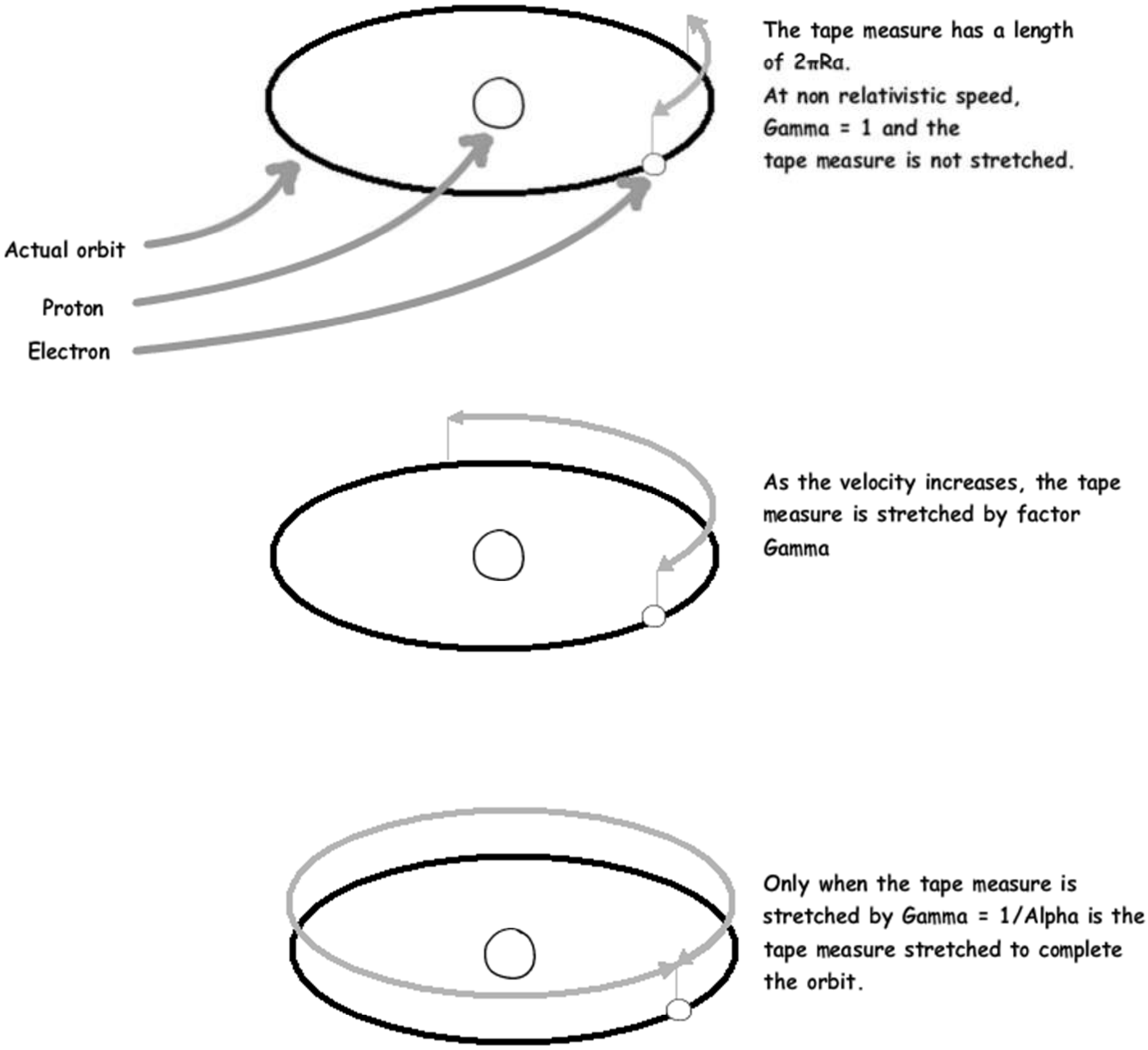


Figure 1  Orbital path length for a stable orbit in the base state

Here also we see the true nature of the Somerfield Fine Structure Constant. It is seen as the extent to which the orbital path length must be foreshortened due to the effects of relativity in order to produce a stable atom. Conversely it can be seen as the extent to which our elastic tape measure must be stretched under relativity to describe one Actual orbit.

## Higher Energy States

For a small increase in orbital velocity the forces are no longer in balance and the atom would be unstable. They next come into balance when $\gamma = 2/\alpha$ or approximately 274. At this Actual velocity the Actual orbital period remains substantially unaltered at $2\pi R/c$. The relativistic path length as seen by the electron is however halved over that of the base state, although again using the analogy of the elastic tape measure which is now stretched by a factor of 274 the electron is has completed exactly one orbit during this period. When the Actual orbital velocity is such that $\gamma = 2/\alpha$ distance traveled by the electron is $\pi R\alpha$.

Once again however the perceived distance travelled by the electron is not only this shortest distance, every $n^{\text{th}}$ multiple of this distance is an alias for this distance. Because of the effects of aliasing, the electron can perceive the distance it has travelled since it was overhead its point of departure as being any one of $\pi R\alpha$, $2\pi R\alpha$ $3\pi R\alpha$ or in general $n\pi R\alpha$. Any one of these distances can be interpreted by the electron as the distance it has traveled since it was overhead its point of departure and one of them, when $n = 2$ results in a path length of $2\pi R\alpha$ and gives a stable atom. It is as if the electron had completed two orbits in the relativistic domain for each orbit in the actual domain.

This situation repeats again in an exactly similar manner when $n = 3$ only this time the alias that results in a stable atom is the one corresponding to three orbits around the nucleus. The situation repeats for every integer value of $n$.

In effect therefore the distance around the orbital path in the $n^{\text{th}}$ energy state is the $n^{\text{th}}$ alias of the distance around a single orbit as perceived by the electron or $2\pi Rn\alpha/n$ for integer $n$. The period in the domain of the observer is always roughly the same at $2\pi R/c = h/mc$ and therefore the relativistic velocity is always the same $c\alpha n/n$ for integer $n$.

In general therefore each successive stable state occurs as $\gamma$ equals an integer multiple of $1/\alpha$, so

$$\gamma_n = \frac{n}{\alpha} \qquad \text{Equation 27}$$

Just as with the cannon, if the base frequency is a solution, then so are all the harmonics a solution. And just as with the cannon where multiple bangs and flashes occurring between the ones of interest here we see that the electron perceives itself as having completed more than one orbit in order to achieve stability which leads directly to the idea of frequency multiplication and the stable states of the atom corresponding to a harmonic series.

From this we can calculate the actual orbital velocity in the $n$th energy state as

$$v_n = c\sqrt{1 - \frac{\alpha^2}{n^2}} \qquad \text{Equation 28}$$

And from this we can calculate the various energy levels and their differences, which exactly match those of the Rydberg Formula.

## Conclusions

The energy levels for the hydrogen atom predicted by the model exactly match those of the Rydberg formula. The electron orbits at a constant radius and at a velocity very close to the speed of light hence changes in energy level are accomplished by a change in orbital velocity with no change in orbital radius.

The model for the hydrogen atom described here effectively unifies classical mechanics with quantum mechanics. It does so by showing the mechanism which causes the electron orbiting the hydrogen nucleus to do so only at certain very particular velocities, each associated with its respective energy level. The model is based on a single postulate, that certain orbital velocity terms should be considered as themselves being affected by relativity. In doing so the model takes full account of the effects of relativity on the various components that make up the atom. Indeed relativity is seen as being at the very heart of the model, not just an adjunct to be added later.

The discrete energy levels of the atom are associated with a series of harmonic frequencies which are experienced by the moving electron, but are not directly experienced by external stationary observers. These harmonics are in turn associated with the quantization of the variable Gamma, which is constrained to only take on certain values each of which is an integer multiple of the reciprocal of the Fine Structure Constant. It is important to understand that Gamma is not itself inherently quantized. The model shows that Gamma is quantized only in the context of the dynamics of the atom and that this comes about because orbital velocity as it affects centrifugal and centripetal force and acceleration and angular momentum is itself affected by relativity causing the effective velocity to reduce by the factor Gamma. In other contexts Gamma is free to take on any value over the dynamic range of 1 to infinity.

The electron is seen as a particle in the classical sense, a point particle of almost infinitesimal size and having deterministic position and velocity. This is not to say that the uncertainty principle does not exist, but rather that uncertainty is not an inherent property of the particle. It is instead a practical difficulty of measurement which occurs when the object being measured and the tools used to measure it are of the same order of magnitude - the so called Observer Effect.

The electron orbits at a constant radius irrespective of energy level. It should be noted that this is a necessary condition for the electron to be considered an objectively real particle, since anything else implies the existence of the physically unrealizable quantum leap or its latter day equivalents. Changes in energy level are then associated with changes in orbital velocity with no change in orbital radius. There is therefore no change in potential energy with change in energy level, merely a change in kinetic energy. Hence the morphology of the atom does not vary with energy level and so it is evident that such an atom would have the same physical and chemical properties irrespective of energy state which is what we observe in practice.

The constant orbital radius of the electron is driven by the combined effects of relativity on both the mass of the electron and its orbital velocity. Rather than simply cancelling one another out, these effectively constrain the orbital radius to have a constant value and it is this that explains why the orbiting electron does not emit synchrotron radiation.

The electron has wave like characteristics which derive directly from its orbital motion. We stationary observers, viewing the atom from an external viewpoint, see the frequency of this wavelike motion as being more or less constant. Viewed from the electron's point of view however, where time is slowed due to the effects of relativity, the orbital frequency of each successive energy state is an integer multiple of that seen by the electron in the base energy state forming a harmonic series with successive harmonics each being associated with a discrete energy level.

Louis de Broglie was the first to propose that the electron has a dual nature. In it the electron, is seen as being both a particle and a wave at the same time. De Broglie struggled to validate this idea and spent much of the last 40 years of his life trying and failing to do so. In de Broglie's duality the nature of the particle is seen as being split between that of a wave on the one hand and that of a particle on the other. De Broglie identified the wavelength of the particle with Planck's constant divided by its linear momentum and in doing so devised a set of wavelike properties for the electron which are not capable of physical realization and in fact amount to little more than a euphemism for the equally unrealizable quantum leap of the earlier Bohr model.

In developing his ideas about the wave particle duality, De Broglie made two key observations. First he proposed that the discrete energy levels of the atom were in some way associated with a harmonic sequence and secondly his proposed waves were at a frequency higher than that of the orbiting electron, implying that some sort of frequency multiplication process is taking place within the atom.

Here we find that both of these conditions are met but not quite in the way that de Broglie envisaged. An object orbiting at near light speed experiences time as passing at a slower rate than does a stationary observer. However the number of orbits in any given period is the same for both observers meaning that the moving observer sees the orbital frequency as being higher than does the stationary observer, the same number of cycles having occurred in a shorter period for the moving observer. Hence the moving observer sees the orbital frequency as having been multiplied by $\gamma$.

De Broglie understood that the dynamics of the hydrogen atom required some sort of dual solution and chose to identify these separately with the wavelike properties of the electron and its particle like properties. For de Broglie therefore the duality existed **between** the wave and the particle.

Here the situation is somewhat different. The electron still has wavelike properties but these derive directly from its orbital motion as an objectively real discrete particle in the classical sense having both deterministic position and deterministic velocity in much the same way as any object in orbit will display wavelike properties of wavelength, amplitude frequency and phase to an external observer. The relationship between the wave like properties of the particle and its orbital motion is unique in exactly the same way as it is on any other scale.

The duality applies separately, but not independently, to both the particle like properties and the wavelike properties. It stems from the fact that relativity means that both time and distance as far as the electron is concerned each have two different values depending on the perspective of the observer and on the velocity of the electron. For the particle the length of the orbital path seen by an external, stationary observer is Gamma times longer than that seen by the moving electron. Hence the orbital path length is considered to be $2\pi R$ by an external observer, but is perceived as being $2\pi R/\gamma$ when viewed from the perspective of the moving electron. This then is the dual nature of the particle.

Similarly for the wavelike characteristics, the orbital frequency is seen as having one value as far as an external stationary observer is concerned, but having Gamma times this value when viewed from the perspective of the moving electron. This is because time for the moving electron is slowed by the factor Gamma, but the number of orbits remains the same for both observers and hence the same number of orbits is completed in a shorter interval for the moving electron than for the stationary observer. This is the dual nature of the wavelike properties of the electron.

It is therefore not the case that the electron is either a particle or a wave. It always has both particle like and wave like properties, the former because it is a discrete point particle in the classical sense and the latter because it is following a circular orbit which subtends a wave to any external observer. It is the properties of both the particle and of the wave that each display a dual nature and that these are brought about by the effects of relativity. For the stationary observer where $v \approx c$ the frequency $\omega = c/R$. For the moving electron frequency is multiplied by Gamma due to the effects of relativity and so $\omega_e = cn/R\alpha$.

It is therefore appropriate to describe the wave/particle relationship not as the wave particle duality but as the wave particle identity, and to describe the particle as having a dual nature and to describe the wave as having a dual nature.

The model provides a simple physical interpretation of the physical nature of the Somerfield Fine Structure Constant. This constant is a pure number and therefore must be derived from the ratio of two quantities with similar Dimensions or units. Here it is seen as the ratio of the orbital velocity as experienced by the stationary observer to that experienced by the moving electron observer in relation to its orbital or Relativistic velocity. Since these share the same orbital period it can also be seen as the ratio of the distance around the orbital circumference foreshortened due to relativity to the actual distance around the orbital circumference as seen by a stationary observer.

Finally the model extends the laws of physics down to the scale of the atom and most likely beyond. It does however demand a subtle change to those laws which would apply equally on any scale, notably that certain orbital velocity terms should be taken as being affected by relativity.

---


[i] Heilbron, J. L. (1981), *Historical Studies in the Theory of Atomic Structure*, Arno Press

[ii] Niels Bohr (1913). *On the Constitution of Atoms and Molecules, Part I.* Philosophical Magazine 26: 1–24.

[iii] Feynman, Richard P., (1985) *QED: The Strange Theory of Light and Matter*, Princeton University Press, p. 129.

[iv] Einstein, Albert (1905), *Zur Elektrodynamik bewegter Körper* (*On the Electrodynamics of Moving Bodies)*, Annalen der Physik 17 (10): 891–921,

[v] Bailey, H.; Borer, K.; Combley F.; Drumm H.; Krienen F.; Lange F.; Picasso E.; Ruden W. von; Farley F. J. M. ; Field J. H.; Flegel W. & Hattersley P. M. (1977). *Measurements of relativistic time dilatation for positive and negative muons in a circular orbit*. Nature 268 (5618): 301–305

[vi] Bracewell, R.N. (1986), *The Fourier Transform and Its Applications (revised ed.)*, McGraw-Hill; 1st ed. 1965, 2nd ed. 1978

[vii] H. Nyquist, *Certain Topics in Telegraph Transmission Theory*, Trans. AIEE, vol 47, pp 617-644, Apr 1928

[viii] C. E Shannon, C*ommunication in the Presence of Noise*, Proc. Institute of Radio Engineers, vol 37 no. 1 pp 10-21, Jan 1949.