On the Logical Inconsistency of the Special Theory of Relativity

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ABSTRACT

Einstein's Special Theory of Relativity employs both clock synchronisation and the Lorentz Transformation. Without both the Theory of Relativity fails. Clock synchronisation is proven inconsistent with the Lorentz Transformation. The Theory of Relativity is therefore invalid due to an intrinsic logical contradiction.

1 Introduction

Physicists since Einstein have assumed compatibility of clock synchronisation with the Lorentz Transformation. Without both, the Theory of Relativity fails. Clock synchronisation is however, easily proven inconsistent with the Lorentz Transformation. Special Relativity is therefore logically inconsistent and consequently false.

2 Einstein's synchronisation of clocks

In §1 of his 1905 paper, Einstein [1] defined the 'common time' for the points A and B in a space:

"We have so far defined only an 'A time' and a 'B time.' We have not defined a common 'time' for A and B, for the latter cannot be defined at all unless we establish by definition *that the 'time' required by light to travel from A to B equals the 'time' it requires to travel from B to A. Let a ray of light start at the 'A time' t_A from A towards B, let it at the 'B time' t_B be reflected at B in the direction of A, and arrive again at A at the 'A time' t'_A.*

"In accordance with definition the two clocks synchronize if

$$t_B - t_A = t'_A - t_B."$$

Einstein [1, §3] then produced the Lorentz Transformation:

$$\tau = \beta\left(t - vx/c^2\right), \quad \xi = \beta\left(x - vt\right),$$
$$\eta = y, \quad \zeta = z, \tag{1}$$
$$\beta = 1/\sqrt{1 - v^2/c^2},$$

where x, y, z, t, pertain to the 'stationary system' and v is the uniform rectilinear speed between the two systems of coordinates in the direction of the positive x-axis.

Einstein [1, §3] synchronised his clocks for both his 'stationary system' K and his 'moving system' k:

"...let the time t of the stationary system be determined for all points thereof at which there are clocks by means of light signals in the manner indicated in §1 ; similarly let the time τ of the moving system be determined for all points of the moving system at which there are clocks at rest relatively to that system by applying the method, given in §1, of light signals between the points at which the latter clocks are located.

"To any system of values x, y, z, t, which completely defines the place and time of an event in the stationary system, there belongs a system of values ξ, η, ζ, τ, determining that event relatively to the system k".

Hence, for any given 'event', by his sychronisation method, the 'stationary system' is K, with coordinates x, y, z, t, and the 'moving system' is k, with corresponding coordinates ξ, η, ζ, τ. All points in Einstein's 'stationary system' K have the common time t and all points in his 'moving system' k have the common time τ.

3 The Lorentz Transformation

Sychronisation of clocks is an essential feature of Special Relativity. Einstein [1, §3] holds that the Lorentz Transformation associates coordinates x, y, z, t of the 'stationary system' K with the coordinates ξ, η, ζ, τ of the 'moving system' k. Synchronisation and the Lorentz Transformation are the basis for Einstein's time dilation and length contraction. It is regarded in general by physicists [2, §12.1] that clocks which are synchronised when at rest are not synchronised when they all move together with respect to the 'stationary system' K, as illustrated in figure 1.

Clocks to the left of the central clock in the 'moving system' k are 'ahead' of the central clock and those to the right of it 'lag' it, according to the 'stationary system' K where all the clocks therein always read the same time t. This is the so-called 'relativity of simultaniety' [1, §2] [2, §12.1]. After
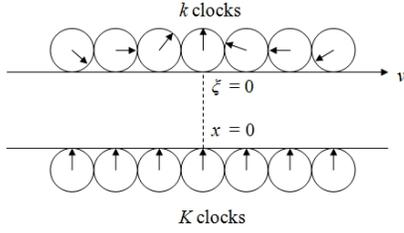
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Fig. 1: All the synchronised clocks in the 'stationary system' K read the same time at all positions in the K system. All the clocks in the 'moving system' k do not read the same time according to the K system, despite being synchronised with respect to the k system. Only at $x = \xi = 0$ do the clocks read the same time in both systems, where $t = \tau = 0$.

a time $t > 0$ the moving clocks advance to the right and the hands on the moving clocks advance, but they do not read the same time τ. As time t increases all the hands of the 'stationary' clocks advance by the same amount and all clocks in K still read the same time t - they are synchronised. However, for any x and t in the stationary system K there is in general a place $x^* \neq x$ with a clock that reads $t^* \neq t$, yet does not disturb the values of τ and ξ of the 'moving system' k, thereby contradicting the assumption of synchronisation of clocks. Recall the Lorentz Transformations equations for the time τ in the 'moving system' k according to the 'stationary system' K:

$$\tau = \beta\left(t - \frac{vx}{c^2}\right), \tag{2a}$$

$$\xi = \beta\left(x - vt\right). \tag{2b}$$

Assume all clocks in the 'stationary system' K to be synchronised as in figure 1. Then for any time t of K all the stationary clocks read the same time at every x in K. Similarly, assume all clocks in the 'moving system' k to be synchronised with respect to the 'moving system' k, just as Einstein prescribed. When the k-system of clocks is in motion its clocks are not synchronised with respect to the 'stationary system' K, as shown in figure 1. Now set,

$$x^* = \sigma x$$

$$t^* - \frac{v\sigma x}{c^2} = t - \frac{vx}{c^2} \tag{3}$$

where $0 \leq \sigma$. From the second of equations (3),

$$t^* = t + \frac{(\sigma - 1)vx}{c^2}. \tag{4}$$

The following is a table of sample values:

σ	x^*	t^*	τ
0	0	$t - vx/c^2$	$\beta\left(t - vx/c^2\right)$
1/2	$x/2$	$t - vx/2c^2$	$\beta\left(t - vx/c^2\right)$
1	x	t	$\beta\left(t - vx/c^2\right)$
2	$2x$	$t + vx/c^2$	$\beta\left(t - vx/c^2\right)$
3	$3x$	$t + 2vx/c^2$	$\beta\left(t - vx/c^2\right)$

Note that for any time $t > 0$ of the 'stationary system' K there is, in general, a place $x^* \neq x$ with a clock reading $t^* \neq t$, which does not alter the values of either τ or ξ, thereby contradicting the assumption of clock synchronisation in K. Hence, by *reductio ad absurdum*, synchronisation of clocks is inconsistent with the Lorentz Transformation. Conversely, the Lorentz Transformation is inconsistent with synchronisation of clocks - they are mutually exclusive.

4   Conclusions

By his clock synchronisation method Einstein attempted to ensure that time at all places within a given system is the same, despite subsequently invoking the Lorentz Transformation. Clock synchronisation is inconsistent with the Lorentz Transformation - they are mutually exclusive. Special Relativity is inconsistent with the Lorentz Transformation and therefore contains an insurmountable logical contradiction.

Einstein [1, §1] defined time by means of his clocks. However, time is no more defined by a clock than pressure is defined by a pressure gauge, speed by a speedometer, heat by a thermometer, or gravity by a spring. Measuring instruments are invented to measure something other than themselves. Einstein's clocks measure only themselves.

References

[1] Einstein, A., On the electrodynamics of moving bodies, *Annalen der Physik*, 17, 1905

[2] Griffiths, D.J., Introduction to Electrodynamics, 4th Ed., Pearson Education Inc., 2013

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