A Suggested Boundary for Heisenberg's Uncertainty Principle

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Abstract

In this paper we are combining Heisenberg's uncertainty principle with Haug's suggested maximum velocity for anything with rest-mass; see [1, 2, 3]. This leads to a suggested exact boundary condition on Heisenberg's uncertainty principle. The uncertainty in position at the potential maximum momentum for subatomic particles (as derived from the maximum velocity) is half of the Planck length.

Perhaps Einstein was right after all when he stated, "God does not play dice." Or at least the dice may have a stricter boundary on possible outcomes than we have previously thought.

We also show how this suggested boundary condition seems to make big G consistent with Heisenberg's uncertainty principle. We obtain a mathematical expression for big G that is fully in line with empirical observations.

Hopefully our analysis can be a small step in better understanding Heisenberg's uncertainty principle and its interpretations and by extension, the broader implications for the quantum world.

Key words: Heisenberg's uncertainty principle, maximum velocity matter, point particle, boundary condition, big G, Planck mass particle, Planck length, reduced Compton wavelength.

1 Introduction

Haug [1, 2, 3] has recently introduced a new maximum velocity for subatomic particles (anything with mass) that is just below the speed of light. The formula is given by

$$v_{max} = c\sqrt{1 - \frac{l_p^2}{\bar{\lambda}^2}} \tag{1}$$

where $\bar{\lambda}$ is the reduced Compton wavelength of the particle we are trying to accelerate and l_p is the Planck length [4]. This formula can for example be derived from special relativity by simply assuming that the maximum frequency one can have is the Planck frequency $\frac{c}{l_p}$, or that the shortest wavelength possible is the Planck length. We will also obtain the same formula if we assume that the ultimate fundamental particle has a spatial dimension equal to l_p and always is traveling at the speed of light, a model outlined by [1, 5].

This maximum velocity puts an upper boundary condition on the kinetic energy, the momentum, and the relativistic mass, as well as on the relativistic Doppler shift in relation to subatomic particles. Basically, no fundamental particle can attain a relativistic mass higher than the Planck mass, and the shortest reduced Compton wavelength we can observe from length contraction is the Planck length. In addition, the maximum frequency is limited to the Planck frequency. Here we will combine this equation with Heisenberg's uncertainty principle.

2 Heisenberg's Uncertainty Principle in Relation to Maximum Momentum

Heisenberg's uncertainty principle [7] is given by[1]

$$\sigma_x \sigma_p \geq \frac{\hbar}{2} \tag{2}$$

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[1] See also Kennard [8] who was the first to "prove" this modern inequality based on the work of Heisenberg.

where σ_x is considered to be the uncertainty in the position, σ_p is the uncertainty in the momentum, and $\hbar$ is the reduced Planck constant.

Haug [1] has suggested that the maximum momentum for a fundamental particle likely is given by

$$\begin{aligned} p_{max} &= \frac{mv_{max}}{\sqrt{1-\frac{v_{max}^2}{c^2}}} \\ p_{max} &= \frac{mv_{max}}{\sqrt{1-\frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}} \\ p_{max} &= \frac{mv_{max}}{\sqrt{1-\frac{\left(c^2-c^2\frac{l_p^2}{\bar{\lambda}^2}\right)}{c^2}}} \\ p_{max} &= \frac{mc\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}}{\frac{l_p}{\bar{\lambda}}} \\ p_{max} &= m_pc\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}} \end{aligned} \tag{3}$$

Based on this we can find a lower boundary in the uncertainty of the position, σ_x, for of any fundamental particle when assuming the σ_p is limited to the maximum momentum for the subatomic particle in question. From this we get

$$\begin{aligned} \sigma_x\sigma_p &\geq \frac{\hbar}{2} \\ \sigma_x\frac{mv_{max}}{\sqrt{1-\frac{v_{max}^2}{c^2}}} &\geq \frac{\hbar}{2} \\ \sigma_x m_p v_{max} &\geq \frac{\hbar}{2} \\ \sigma_x m_p c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}} &\geq \frac{\hbar}{2} \\ \sigma_x &\geq \frac{\hbar}{2m_pc\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}} \end{aligned} \tag{4}$$

and since the Planck mass can be written as $m_p = \frac{\hbar}{l_p}\frac{1}{c}$, we can rewrite this as

$$\begin{aligned} \sigma_x &\geq \frac{\hbar}{2\frac{\hbar}{l_p}\frac{1}{c}c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}} \\ \sigma_x &\geq \frac{l_p}{2\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}} \end{aligned} \tag{5}$$

For any known fundamental particle, $\bar{\lambda} >> l_p$, so we can use the first term of a series expansion: $\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}} \approx 1-\frac{1}{2}\frac{l_p^2}{\bar{\lambda}^2}$. This gives us

$$\begin{aligned} \sigma_x &\geq \frac{l_p}{2\left(1-\frac{1}{2}\frac{l_p^2}{\bar{\lambda}^2}\right)} \\ \sigma_x &\geq \frac{l_p}{2-\frac{l_p^2}{\bar{\lambda}^2}} \end{aligned} \tag{6}$$

and when $\bar{\lambda} >> l_p$, we have a very good approximation by

$$\sigma_x \geq \frac{l_p}{2} \tag{7}$$

In other words, the maximum uncertainty in the position of any fundamental subatomic particle (when assuming σ_p is equal to the maximum momentum of the particle) is half the Planck length. This lies in strong contrast to standard physics, where there is basically no boundary on the maximum momentum a fundamental particle can achieve as long as it is below infinite. Therefore, in the standard theory there is no limit on how close σ_x can be, relative to zero. As [9] recently has shown, this leads to absurd possibilities for relativistic mass, kinetic energy, and momentum. Under the standard theory, an electron could attain a relativistic mass equal to the rest-mass of the Moon, the Earth, the Sun, and even the whole observable universe while still traveling below the speed of light.

In the new theory presented by Haug no fundamental particle can attain a relativistic mass larger than the Planck mass. Under our new interpretation of Heisenberg's principle there is an exact upper limit on the momentum equal to the Planck momentum, and it is identical for all subatomic fundamental particles. Naturally this will only hold true because their maximum velocity is not the same and is dependent on their reduced Compton wavelength. Our theory gives an exact limit on how close v can get to c. To give an example, for an electron this maximum velocity is

$$\Delta v_{max} = c\sqrt{1-\frac{l_p^2}{\bar{\lambda}_e^2}} \approx c \times 0.999999999999999999999999999999999999999999999124 \tag{8}$$

This is the same maximum velocity as given by [1, 2]. These calculations require very high precision and were calculated in Mathematica.[2]

In our view, one possible interpretation is that the reduced Compton wavelength of the electron is contracted down to the Planck length at this maximum velocity, as discussed by [6]. In this case, we cannot claim that the electron is at an exact point location $\sigma_x \approx 0$, simply because it is not a point particle. The reduced Compton wavelength is, in our view, the distance from center to center between two indivisible particles that make up the electron, traveling back and forth counter-striking. When they are ultimately compressed (due to length contraction of the void in between the indivisibles making up the fundamental particle), the particles must lie side by side. The reduced Compton wavelength is now l_p. And our best estimate of where the electron is now would be half the Planck length, that is to say, in the middle of its contracted reduced Compton wavelength. Heisenberg's uncertainty principle combined with our maximum velocity formula possibly indicates that there can be no point particles. Alternatively, one can just interpret this as if there is a known maximum momentum for a fundamental particle. Then this must be the maximum uncertainty in momentum and from the Heisenberg principle there must be a limitation on how low the uncertainty in that location can be.

In the special case of a Planck mass particle, we find that $\sigma_x = \infty$. This may sound dramatic, but one possible interpretation is simply that the momentum of a Planck mass particle always is zero, since the Planck mass particle is standing still as observed from any reference frame; see [1]. I would also claim that Heisenberg's uncertainty principle may not be ideally suited for describing the situation for any particle that is merely standing still (at rest).

Based on this maximum velocity Haug claims that the Planck mass particle and the Planck length is the same and is invariant as seen from any reference frame. This can only hold true of the Planck mass only lasts for an instant. The Planck mass can be seen as the collision of two light particles, and therefore constitutes the turning point of light. When a photon turns direction by 180 degrees (backscattering) does it not, at the very turning point, stand still for an instant? The collision of two photons to create matter was first suggested by Breit and Wheeler 1934; see [10]. The implications drawn from the instances where light is colliding with light have recently received increased attention; see [11, 12, 13], for example.

Again, the shortest σ_x we can have in relation to a momentum is $\frac{1}{2}l_p$, which can be used to find the maximum momentum for any subatomic particle.

[2] We used several different set-ups in Mathematica; here is one of them: $N[\mathrm{Sqrt}[1-(1616199*10\hat{}(-41))\hat{}2/(3861593*10\hat{}(-19))\hat{}2],50]$, where $1616199*10\hat{}(-41)$ is the Planck length and $3861593*10\hat{}(-19)$ is the reduced Compton wavelength of the electron. An alternative way to write it is: $N[\mathrm{Sqrt}[1-(\mathrm{SetPrecision}[1.616199*10\hat{}(-35))\hat{}2,50]/(\mathrm{SetPrecision}[3.861593*10\hat{}(-13))\hat{}2,50]],50]$.

$$
\begin{aligned}
\sigma_x\sigma_p &\geq \frac{\hbar}{2} \\
\sigma_p &\geq \frac{\hbar}{2\sigma_x} \\
\sigma_p &\geq \frac{\hbar}{2\frac{1}{2}l_p} \\
\frac{mv}{\sqrt{1-\frac{\Delta v^2}{c^2}}} &\geq \frac{\hbar}{l_p} \\
\frac{v}{\sqrt{1-\frac{v^2}{c^2}}} &\geq \frac{\hbar}{l_p m} \\
\frac{v^2}{1-\frac{v^2}{c^2}} &\geq \frac{\hbar^2}{l_p^2 m^2} \\
v^2 &\geq \frac{\hbar^2}{l_p^2 \frac{\hbar^2}{\bar{\lambda}^2}\frac{1}{c^2}}\left(1-\frac{v^2}{c^2}\right) \\
v^2 &\geq \frac{\bar{\lambda}^2 c^2}{l_p^2}\left(1-\frac{v^2}{c^2}\right) \qquad (9)
\end{aligned}
$$

This leads to a quadratic equation with negative and positive solutions for v, where only the positive solution seems to make practical sense[3], namely that $v = c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}$. This is the maximum uncertainty in velocity for a subatomic particle with known mass or known reduced Compton wavelength. This gives us the maximum momentum for any subatomic particle equal to $p_{max} = m_p c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}$. And when $\bar{\lambda} >> l_p$, this is approximately equal to the Planck momentum, $p_{max} \approx m_p c$.

We are not the only ones to suggest an absolute minimum uncertainty in the position of any particle, such as an electron. Adler and Santiago [14] have, based on assumed gravitational interaction of the photon and the particle being observed, modified the uncertainty principle with an additional term. By doing this they find a minimum uncertainty in the position that is not far from our prediction. The strength in our result is that no additional terms in the Heisenberg principle are needed to get a minimum uncertainty in the position of any particle, and thereby also a maximum limit in the uncertainty of the momentum.

3 Time and Energy

Heisenberg's uncertainty principle in terms of time and energy can be written as

$$\sigma_t\sigma_E \geq \frac{\hbar}{2} \qquad (10)$$

Haug [1] has shown that the maximum kinetic energy of a fundamental particle with reduced Compton wavelength of $\bar{\lambda}$ is given by

[3] Or the negative solution could be interpreted as a particle traveling in the opposite direction of the positive solution.

$$
\begin{aligned}
E_{k,max} &= \frac{mc^2}{\sqrt{1-\frac{v_{max}^2}{c^2}}} - mc^2 \\
E_{k,max} &= \frac{mc^2}{\sqrt{1-\frac{\left(c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}}\right)^2}{c^2}}} - mc^2 \\
E_{k,max} &= \frac{mc^2}{\sqrt{1-\frac{c^2\left(1-\frac{l_p^2}{\bar{\lambda}^2}\right)}{c^2}}} - mc^2 \\
E_{k,max} &= \frac{mc^2}{\frac{l_p}{\bar{\lambda}}} - mc^2 \\
E_{k,max} &= \frac{\bar{\lambda}}{l_p}mc^2 - mc^2 \\
E_{k,max} &= \frac{\bar{\lambda}}{l_p}\frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2 - \frac{\hbar}{\bar{\lambda}}\frac{1}{c}c^2 \\
E_{k,max} &= \frac{\hbar}{l_p}c - \frac{\hbar}{\bar{\lambda}}c \\
E_{k,max} &= \hbar c\left(\frac{1}{l_p} - \frac{1}{\bar{\lambda}}\right) \qquad (11)
\end{aligned}
$$

We can use this result in Heisenberg's time energy uncertainty inequality equation

$$
\begin{aligned}
\sigma_t\sigma_E &\geq \frac{\hbar}{2} \\
\sigma_t\hbar c\left(\frac{1}{l_p} - \frac{1}{\bar{\lambda}}\right) &\geq \frac{\hbar}{2} \\
\sigma_t &\geq \frac{\hbar}{2\hbar c\left(\frac{1}{l_p} - \frac{1}{\bar{\lambda}}\right)} \\
\sigma_t &\geq \frac{1}{2c\left(\frac{1}{l_p} - \frac{1}{\bar{\lambda}}\right)} \qquad (12)
\end{aligned}
$$

and when $\bar{\lambda} >> l_p$, we have a very good approximation by

$$\sigma_t \geq \frac{1}{2}\frac{l_p}{c} \qquad (13)$$

Which is half a Planck second. It is worth mentioning that the half Planck second and half Planck length found as boundary conditions here are exactly the same as the results we obtained when looking at the Lorentz transformation in the limit of the maximum velocity of mass [15].

4 Big G and Heisenberg's Uncertainty Principle

As shown in [3], the maximum velocity can also be written as

$$v_{max} = c\sqrt{1-\frac{l_p^2}{\bar{\lambda}^2}} = c\sqrt{1-\frac{Gm^2}{\hbar c}} \qquad (14)$$

where G is Newton's gravitational constant [16] and m is the mass of a fundamental particle. It is important to understand that m in this context is not just any mass; this mass must have a reduced Compton wavelength. In other words, it is the mass of fundamental particles. Based on this observation, we can assess whether or not we can use this in combination with Heisenberg's uncertainty principle to derive a theoretical value of big G. We are not the first to suggest that Heisenberg's uncertainty principle could be related to Newtonian gravity. McCulloch [17] has shown that Newton's gravity formula basically can be derived from Heisenberg's uncertainty principle. However, he has not shown how big G also can be derived from it.

We could also say that this is just another way to show the maximum velocity for matter may be consistent with Heisenberg's uncertainty principle, although this should not be considered as evidence that we will get big G from Heisenberg's uncertainty principle. We have

$$\begin{aligned}
\sigma_x\sigma_p &\geq \frac{\hbar}{2} \\
\sigma_x \frac{m v_{max}}{\sqrt{1-\frac{v_{max}^2}{c^2}}} &\geq \frac{\hbar}{2} \\
\sigma_x m_p v_{max} &\geq \frac{\hbar}{2} \\
\sigma_x m_p c\sqrt{1-\frac{Gm^2}{\hbar c}} &\geq \frac{\hbar}{2} \\
\sqrt{1-\frac{Gm^2}{\hbar c}} &\geq \frac{\hbar}{2\sigma_x m_p c} \\
1-\frac{Gm^2}{\hbar c} &\geq \frac{\hbar^2}{4\sigma_x^2 m_p^2 c^2} \\
G &\geq \frac{\hbar c}{m^2} - \frac{\hbar^2 \hbar c}{4\sigma_x^2 m_p^2 c^2 m^2} \\
G &\geq \frac{\hbar c}{\frac{\hbar^2}{\bar{\lambda}^2}\frac{1}{c^2}} - \frac{\hbar^2\hbar c}{4\frac{\hbar^2}{4m_p^2 v_{max}^2} m_p^2 c^2 m^2} \\
G &\geq \frac{\bar{\lambda}^2 c^3}{\hbar} - \frac{\hbar c v_{max}^2}{m^2 c^2} \\
G &\geq \frac{\bar{\lambda}^2 c^3}{\hbar} - \frac{\hbar c c^2\left(1-\frac{l_p^2}{\bar{\lambda}^2}\right)}{m^2 c^2} \\
G &\geq \frac{\bar{\lambda}^2 c^3}{\hbar} - \frac{\hbar c\left(1-\frac{l_p^2}{\bar{\lambda}^2}\right)}{\frac{\hbar^2}{\bar{\lambda}^2}\frac{1}{c^2}} \\
G &\geq \frac{\bar{\lambda}^2 c^3}{\hbar} - \frac{\bar{\lambda}^2 c^3\left(1-\frac{l_p^2}{\bar{\lambda}^2}\right)}{\hbar} \\
G &\geq \frac{l_p^2 c^3}{\hbar} \approx 6.67384\times 10^{-11} \qquad (15)
\end{aligned}$$

To write the gravitational constant as $G=\frac{l_p^2c^3}{\hbar}$ has already been suggested by Haug [18, 19] in order to simplify a series of expressions in Newton and Einstein gravity end results. It has also been derived by dimensional analysis [3] and used to simplify the equation form of the Planck units. Further, Haug has suggested that the Planck length (at least in a thought experiment) can be found independent of G based on the maximum velocity formula.

Since v_{max} here is a function of the universal constants G, $\hbar$ and c, one could try to argue that this provides evidence that l_p must be a function of G and $\hbar$ and c and not that G is a function of l_p. In other words, G must be a universal constant and l_p is just a derived constant. However, the beauty of the maximum velocity formula is that G and $\hbar$ cancel out and that we are left with v_{max} only as a function of c, l_p and the reduced Compton wavelength of the particle in question, $\bar{\lambda}$, and not of G and $\hbar$. It is worth pointing out that the reduced Compton wavelength of an electron can be found completely independent of any knowledge of G; see [20]. To find l_p we need the reduced Compton wavelength that can be found independently of G, as well at the maximum velocity for an electron, v_{max}. This maximum velocity has to be found experimentally. This maximum velocity for an electron is very close to c, but it is still higher than the velocities that are in operation at LHC. Even so, the fact that something is predicted and not found yet is not a good enough argument to completely reject a theory yet.

Our formula for big G gives the same value as the gravitational constant, as it is known from experiments, and it can actually be calibrated to the experiments. There is still considerable uncertainty about the exact measurement of the gravitational constant.

Experimentally, substantial progress has been made in recent years based on various methods. See [21, 22, 23, 24, 25], for example. In the formula presented here, the uncertainty lies in the exact value of the Planck length, as well as in $\hbar$; the speed of light $c = 299792458$ is exact by definition. At the moment, the Planck length can only be found from G, but if we had access to much more advanced particle accelerators than the Large Hadron Collider, we could expect to detect v_{max} and then back the Planck length out from there. We claim that big G is indeed a universal constant, but it is a composite constant that is dependent on three even more fundamental constants, namely $\hbar$, l_p, and c.

5 Conclusion

By combining Heisenberg's uncertainty principle with the newly introduced maximum velocity on mass, we have shown that the smallest locational uncertainty of a fundamental particle is related to half the Planck length, and that the shortest time interval is related to half the Planck time. This is the "same" finding that we obtained when combining this maximum velocity with the Lorentz transformation [15].

References

[1] E. G. Haug. The Planck mass particle finally discovered! Good bye to the point particle hypothesis! *http://vixra.org/abs/1607.0496*, 2016.

[2] E. G. Haug. A new solution to Einstein's relativistic mass challenge based on maximum frequency. *http://vixra.org/abs/1609.0083*, 2016.

[3] E. G. Haug. The gravitational constant and the Planck units: A simplification of the quantum realm. *Physics Essays Vol 29, No 4*, 2016.

[4] M. Planck. *The Theory of Radiation.* Dover 1959 translation, 1906.

[5] E. G. Haug. *Unified Revolution: New Fundamental Physics.* Oslo, E.G.H. Publishing, 2014.

[6] E. G. Haug. Deriving the maximum velocity of matter from the Planck length limit on length contraction. *http://vixra.org/abs/1612.0358*, 2016.

[7] W. Heisenberg. Über den anschaulichen inhalt der quantentheoretischen kinematik und mechanik. *Zeitschrift für Physik*, (43):172–198, 1927.

[8] E. H. Kennard. Zur quantenmechanik einfacher bewegungstypen". *Zeitschrift für Physik*, (44):326–352, 1927.

[9] E. G. Haug. Modern physics' incomplete absurd relativistic mass interpretation. and the simple solution that saves Einstein's formula. *http://vixra.org/abs/1612.0249*, 2016.

[10] G. Breit and J. A. Wheeler. Collision of two light quanta. *Physical Review*, 46, 1934.

[11] B. King and C. H. Keitel. Photon–photon scattering in collisions of intense laser pulses. *New Journal of Physics*, 14, 2012.

[12] O. J. Pike, F. Mackenroth, E. G. Hill, and R. S. J. A photon–photon collider in a vacuum hohlraum. *Nature Photonics*, 8, 2014.

[13] D. E. Chang, V. Vuletić, and M. D. Lukin. Quantum nonlinear optics – photon by photon. *Nature Photonics*, 8, 2014.

[14] R. J. Adler and D. I. Santiago. On gravity and the uncertainty principle. *Modern Physics Letters A*, 14.

[15] E. G. Haug. The Lorentz transformation at the maximum velocity for a mass. *http://vixra.org/abs/1609.0151*, 2016.

[16] I. Newton. *Philosophiae Naturalis Principia Mathematica.* London, 1686.

[17] M. E. McCulloch. Gravity from the uncertainty principle. *Astrophysics and Space Science*, 349, 2014.

[18] E. G. Haug. Planck quantization of Newton and Einstein gravitation. *International Journal of Astronomy and Astrophysics*, 6(2), 2016.

[19] E. G. Haug. Newton and Einstein's gravity in a new perspective for Planck masses and smaller sized objects. *http://vixra.org/abs/1610.0328*, 2016.

[20] S. Prasannakumar, S. Krishnaveni, and T. K. Umesh. Determination of rest mass energy of the electron by a Compton scattering experiment. *European Journal of Physics*, 33(1), 2012.

[21] G. S. Bisnovatyi-Kogan. Checking the variability of the gravitational constant with binary pulsars. *International Journal of Modern Physics D*, 15(07), 2006.

[22] B. Fixler, G. T. Foster, J. M. McGuirk, and M. A. Kasevich. Atom interferometer measurement of the Newtonian constant of gravity. *Science*, 315, 2007.

[23] S. Galli, A. Melchiorri, G. F. Smoot, and O. Zahn. From Cavendish to Planck: Constraining Newton's gravitational constant with CMB temperature and polarization anisotropy. *Physical Review D*, 80, 2009.

[24] G. Rosi, F. Sorrentino, L. Cacciapuoti, M. Prevedelli, and G. M. Tino. Precision measurement of the Newtonian gravitational constant using cold atoms. *Nature*, 510, 2014.

[25] S. Schlamminger. A fundamental constant: A cool way to measure big G. *Nature*, 510, 2014.