Scientia Magna
Vol. 11 (2016), No. 1, 1-3

A note on the Smarandache cyclic geometric determinant sequences

A. A. K. Majumdar

APU, 1–1 Jumonjibaru, Beppu-shi 875–8577, Oita-ken, Japan
E-mail: majumdar@apu.ac.jp/aakmajumdar@gmail.com

Abstract This paper gives an alternative approach to find the determinant of the right circular matrix with geometric sequence, using the known results of the circulant matrix.

Keywords circulant matrix, right circulant matrix with geometric sequence.

§1. Introduction

In a recent paper, Bueno[1] has introduced the concept of the right circulant geometric matrix with geometric sequence, defined as follows:

Definition 1.1. *A right circulant matrix (of order n) with geometric sequence, denoted by RCIRC(n), is a matrix of the form*

$$RCIRC(n) = \begin{pmatrix} 1 & r & r^2 & \cdots & r^{n-2} & r^{n-1} \\ r^{n-1} & 1 & r & \cdots & r^{n-3} & r^{n-2} \\ r^{n-2} & r^{n-1} & 1 & \cdots & r^{n-4} & r^{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ r^2 & r^3 & r^4 & \vdots & 1 & r \\ r & r^2 & r^3 & \vdots & r^{n-1} & 1 \end{pmatrix}.$$

Using the elementary properties of matrices and determinants, Bueno[1] has found an explicit form of the associated determinant.

In this paper, we follow an alternative approach to derive the determinant of the matrix RCIRC(n). This is given in Section 3. Some preliminary results are given in Section 2.

§2. Some preliminary results

In this section, we give some well-known results that would be needed later in proving the main results of this paper in Section 3. We start with the following definition.

Definition 2.1. *The circulant matrix with the vector* $\boldsymbol{C} = (c_0, c_1, \ldots, c_{n-1})$, *denoted* *by C_n, is the matrix of the form*

$$C(n) = \begin{pmatrix} c_0 & c_1 & c_2 & \cdots & c_{n-2} & c_{n-1} \\ c_{n-1} & c_0 & c_1 & \cdots & c_{n-3} & c_{n-2} \\ c_{n-2} & c_{n-1} & c_0 & \cdots & c_{n-4} & c_{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ c_2 & c_3 & c_4 & \vdots & c_0 & c_1 \\ c_1 & c_2 & c_3 & \vdots & c_{n-1} & c_0 \end{pmatrix}.$$

Lemma 2.1. *For any n (≥ 2),*

$$\begin{vmatrix} c_0 & c_1 & c_2 & \cdots & c_{n-2} & c_{n-1} \\ c_{n-1} & c_0 & c_1 & \cdots & c_{n-3} & c_{n-2} \\ c_{n-2} & c_{n-1} & c_0 & \cdots & c_{n-4} & c_{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ c_2 & c_3 & c_4 & \cdots & c_0 & c_1 \\ c_1 & c_2 & c_3 & \vdots & c_{n-1} & c_0 \end{vmatrix} = \prod_{j=0}^{n-1} (c_0 + c_1 \omega_j + c_2 \omega_j^2 + \ldots + c_{n-1} \omega_j^{n-1}),$$

where $\omega_0 \equiv 0$, $\omega_j = e^{\frac{2\pi i}{n} j}$ ($1 \leq j \leq n - 1$) are the n^{th} roots of unity.

§3. Main result

We now give the main result of this paper in the following theorem.

Lemma 3.1. *For $n \geq 1$, $\det(RCIRC(n)) = (1 - r^n)^{n-1}$.*

Proof. From Lemma 2.1 with $c_j = r^j$ ($0 \leq j \leq n - 1$), we see that

$$\det(RCIRC(n)) = \prod_{j=0}^{n-1} (1 + r\omega_j + r^2 \omega_j^2 + \ldots + r^{n-1} \omega_j^{n-1}).$$

But, for any j with $0 \leq j \leq n - 1$,

$$1 + r\omega_j + r^2 \omega_j^2 + \ldots + r^{n-1} \omega_j^{n-1} = \frac{1 - (r\omega_j)^n}{1 - r\omega_j} = \frac{1 - r^n}{1 - r\omega_j}. \tag{1}$$

Again, since

$x^n - 1 = (x - \omega_0)(x - \omega_1)(x - \omega_2) \ldots (x - \omega_{n-1})$,

for $x = \frac{1}{r}$, we get

$$\frac{1 - r^n}{r^n} = \frac{(1 - r\omega_0)(1 - r\omega_1)(1 - r\omega_2)\ldots(1 - r\omega_{n-1})}{r^n},$$

so that

$$(1-\omega_0)(1-\omega_1)(1-\omega_2)\ldots(1-\omega_{n-1}) = 1-r^n. \quad (2)$$

The lemma now follows by virtue of (1) and (2).

References

[1] A. C. F. Bueno, Smarandache Cyclic Geometric Determinant Sequences, *Sci. Magna,* 8 (2012), pp. 88–91.

[2] D. Geller, I. Kra, S. Popescu and S. Simanca, On Circulant Matrices, *Preprint,* pp. 1–13. Retrieved on June 9, 2014.

[3] Circulant Matrix, *http://en.wikipedia.org/wiki/Circulant_matrix*, pp. 1–3. Retrieved on June 9, 2014.