Thermodynamics of Gravity and a Statistical Model of Black Holes

Risto Raitio *
02230 Espoo, Finland

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Abstract

I propose a model for quantum black holes and gravity based on a harmonic oscillator representing the black hole horizon covered by Planck length sized squares carrying soft hair. An analogical entropy function is constructed to the null surfaces of spacetime. Extremizing this entropy leads to the equation for the background metric of the spacetime with the cosmological constant as an integration constant. Secondly, I redefine the partition function sum over horizon squares by a sum over black hole stretched horizon constituents, which are black holes themselves. Based on this partition function Bekenstein-Hawking entropy law, Hawking radiation and Unruh effects are predicted.

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*E-mail: risto.raitio@gmail.com

1 Introduction

Thermodynamical properties, like entropy and temperature, of black holes have been established about four decades ago [1, 2]. More recently thermodynamics has been considered as the major agent behind general relativity [3]. Thermodynamical concepts have been applied to black holes as well as to local Rindler, or acceleration, frames [4]. In [5] acceleration frame considerations have been applied to a model of stretched horizon black holes [1] calculating the partition function of the system.

I propose first a simple model for the structure of quantum black holes. The black hole horizon is a spherical membrane covered with l_{Pl}^2 size squares each of which can be in k states. The membrane dynamics is represented by a two dimensional harmonic oscillator. I expand the model of black hole to the entropy function leading to equation of the metric of general relativity by a null surface extremization method [6].

Secondly, I redefine the membrane partition function as a sum over black hole stretched horizon constituents based on [5]. This method gives as predictions Bekenstein-Hawking entropy, Hawking radiation and Unruh effects.

Large amounts of the results of research by the various schools of thought towards quantum gravity are widely scattered around in various journals in the literature. The motivation of this note is to compile together some of them as I see appropriate with a some thoughts of mine.

This note is organized as follows. After the Introduction the simple oscillator model for black holes is described in section 2. The presentation is very concise in all sections. In section 3 the Hamiltonian of section 2 is generalized into Riemann geometry. This section relies on the work of done in [3, 6, 7]. In section 4 I present the main points of the stretched horizon black hole model [5]. Finally in section 5 I give a brief discussion of results and conclusions.

2 Membrane Model of Horizon

As the first model for black holes of any size I assume the picture of a hole as a spherical horizon covered with l_{Pl}^2 size squares. The minimal horizon radius is of the order of l_{Pl}. All physics takes place on the surface of the sphere, and tentatively, none inside. Suppose there are n squares on the horizon and each square can be in k soft hair states [8]. Then the total number of states is k^n. This gives for entropy S of the sphere the well known result

$$S = k_B \; log \; k^n = k_B \; n \; ln \; k \propto \frac{A}{l_{\mathrm{Pl}}^2} \tag{1}$$

whre k_B is the Boltzmann constant and A is the area of the horizon.

The vibrations of an oscillator can be calculated in normal way. The geometry is a two dimensional sphere

$$H = \frac{-\hbar^2}{2m}\nabla^2 + \frac{1}{2}m\omega^2\vec{x}^2 \tag{2}$$

The energy eigenvalues for a square are given $E = \hbar\omega(n+1)$ where $n = 0, 1, 2, \ldots$.

The partition function Q is (to be used in section 4)

$$Z = \Sigma_i g_j exp(E_i/kT) \tag{3}$$

where g_i (= k in (3)) is the degeracy of the ith state and E_i its energy and T the temperature.

[1] The author considers them atoms of spacetime upon which I kindly disagree.

3 Entropy and Einstein Equation

In [6] it is shown that there is an analogical model of (2) leading to classical dynamics of spacetime, the Einstein equation. The analogy is taken from solid state physics' atoms. I start directly from the model horizon (2). The entropy density function can then have quadratic terms in both $\nabla_a \xi^b$ and ξ^a

$$S[\xi] = \int_V d^4x \sqrt{-g}\left(4P_{ab}^{cd} \nabla_c \xi^a \nabla_d \xi^b - T_{ab}\xi^a \xi^b\right) \tag{4}$$

where the fourth rank tensor tensor P_{abcd} should have the algebraic symmetries similar to the Riemann tensor R_{abcd} and T_{ab} turns out to be the energy momentum tensor of matter.

The field equations are obtained from extremizing the entropy. The entropy functional in (4) is well defined for any displacement vector field ξ^a. One can therefore associate an entropy functional with any hypersurface in the spacetime, by choosing the normal to the hypersurface as ξ^a. The null hypersurfaces will play a key role since they act as one-way membranes which block information for a specific class of observers. One now extremizes $S[\xi]$ with respect to variations of the null vector field ξ^a and demands that the resulting condition holds for all null vector fields. The equilibrium configurations of the spacetime are the ones in which the entropy associated with every null vector is extremized. Varying the null vector field ξ^a after adding a Lagrange multiplier λ for imposing the null condition $\xi_a \delta \xi^a = 0$, the authors [6] find to the lowest order the equation reduces to:

$$\frac{1}{8\pi} R_b^a - T_b^a = F(g)\delta_b^a \tag{5}$$

where F is an arbitrary function of the metric. Writing this equation as

$$(G_b^a - 8\pi T_b^a) = Q(g)\delta_b^a \tag{6}$$

with $Q = 8\pi F - (1/2)R$ and using $\nabla_a G_b^a = 0, \nabla_a T_b^a = 0$ one gets

$$\partial_b Q = \partial_b[8\pi F - (1/2)R] = 0 \tag{7}$$

so that Q is an undetermined integration constant, say Λ, and F must have the form $8\pi F = (1/2)R + \Lambda$. The resulting equation is

$$R_b^a - (1/2)R\delta_b^a = 8\pi T_b^a + \Lambda\delta_b^a \tag{8}$$

which leads to Einstein's theory if one identifies T_{ab} as the matter energy momentum tensor with a cosmological constant appearing as an integration constant. In [7] a small value of $\Lambda \propto exp(-36\pi^2)$ is indicated which can be transformed, together with the experimental value of Λ, to a reasonable prediction for the inflationary scale between $(1-6)\times 10^{15}$ GeV.

A key feature of the functional in (4) is that the entropy associated with null vector fields is invariant under the shift $T_{ab} \to T_{ab} + \rho g_{ab}$ where ρ is a scalar. This fact will play an important role, its is a symmetry in quantum theory but not in general relativity.

4 Model of Stretched Horizon

I consider a micro black hole dressed by a (virtual reality [9]) stretched horizon, which is a membrane hovering about a Planck length outside the event horizon and which is both physical and hot. A treatment of the stretced horizon has been done in [5] where it is assumed that the stretched horizon consists of finite number of discrete constituents each contributing to the stretched horizon an area of a non-negative integer times a constant

$$A = \alpha l_{\rm Pl}^2 (n_1 + n_2 + ... + n_N) \tag{9}$$

where N is the number of constituents, the n_i define their area quantum states and α is a number of the order unity, to be determined later. For the constituents themselves one assumes simply black holes of size $l_{\rm Pl}$. It is supposed that each stationary quantum state of a black hole is determined by the quantum numbers $n_1, n_2, ..., n_N$ of its stretched horizon.

To calculate the partition function of a Schwarzschild black hole one needs to know the energy states of the system. The energy of the hole from the point of view of an observer on its stretched horizon is called Brown-York energy [10]

$$E = \frac{ac^2}{8\pi G} A \tag{10}$$

where a is the (constant) proper acceleration of an observer on the stretched horizon and A is the area of the horizon. The possible energy values of a black hole are, from the point of view of an observer located on its stretched horizon, in terms of the acceleration

$$E_n = n\alpha \frac{\hbar a}{8\pi c} \tag{11}$$

where $n = n_1 + n_2 + ... + n_N$. The number of microscopic states associated with energy E_n is the number of ways of writing a given positive integer n as a sum of exactly N positive integers, whith $N \leq n$, which is given by the binomial coefficient

$$\Omega_N(n) = \left(\begin{array}{c} n-1 \\ N-1 \end{array} \right). \tag{12}$$

For instance, there are $\left(\begin{array}{c} 5-1 \\ 3-1 \end{array} \right) = \left(\begin{array}{c} 4 \\ 2 \end{array} \right) = 6$ ways to express a number 5 as a sum of exactly 3 positive integers:

$$5 = 3+1+1 = 1+3+1 = 1+1+3 = 1+2+2 = 2+1+2 = 2+2+1. \tag{13}$$

It depends on n and N only, and it gives the degeneracy function $g(E_n)$ needed to calculate the partition function

$$Z(\beta) = \Sigma_n g(E_n) e^{-\beta E_n} \tag{14}$$

The resulting partition function $Z(\beta)$ of the Schwarzschild black hole may be calculated explicitly yielding a simple expression [5]:

$$Z(\beta) = \frac{1}{2^{\beta T_C} - 2} \left[1 - \left(\frac{1}{2^{\beta T_C} - 1} \right)^{N+1} \right] \tag{15}$$

where the temperature

$$T_C = \frac{\alpha \hbar a}{4(\ln 2)\pi k_B c} \tag{16}$$

is called the characteristic, or critical, temperature of the hole.

From the partition function one can calculate the average energy

$$E(\beta) = -\frac{\partial}{\partial \beta} \ln Z(\beta) \tag{17}$$

of the hole at temperature $T = 1/\beta$ which yields

$$E(\beta) = \left[\frac{2^{\beta T_C}}{2^{\beta T_C} - 2} - \frac{(N+1)2^{\beta T_C}}{(2^{\beta T_C} - 1)^{N+2} - 2^{\beta T_C} + 1} \right] T_C \ln 2 \tag{18}$$

The average energy per constituent is

$$\bar{E}(\beta) = \frac{E(\beta)}{N} \tag{19}$$

and one gets for large N

$$\bar{E}(\beta) = \bar{E}_1(\beta) + \bar{E}_2(\beta) \tag{20}$$

where

$$\bar{E}_1(\beta) = \frac{1}{N} \frac{2^{\beta T_C}}{2^{\beta T_C} - 2} T_C \ln 2, \tag{21a}$$

$$\bar{E}_2(\beta) = -\frac{2^{\beta T_C}}{(2^{\beta T_C} - 1)^{N+2} - 2^{\beta T_C} + 1} T_C \ln 2. \tag{21b}$$

where $(N+1)/N \approx 1$ has been used.

It has been shown in [5] that when $T = T_C$ the average energy per a constituent of the stretched horizon is, in SI units,

$$\bar{E} = k_B T_C \ln 2 \tag{22}$$

and that

$$\frac{d\bar{E}}{dT}|_{T=T_C} = \frac{1}{6} k_B (\ln 2)^2 N + \mathcal{O}(1) \tag{23}$$

where $\mathcal{O}(1)$ denotes the terms, which are of the order N^0, or less. When the number N of constituents becomes large increase of energy does not change the temperature of the hole at $T = T_C$. So the hole undergoes a phase transition at $T = T_C$. When $T < T_C$, $\bar{E}$ is nearly zero. When $T = T_C$, the curve $\bar{E} = \bar{E}(T)$ becomes practically vertical. When T is slightly greater than T_C, $\bar{E}(T)$ is approximately $1.4 k_B T_C$, which is about the same as $2 \ln 2$. Finally, the dependence of $\bar{E}(T)$ on T becomes approximately linear when $T \gg T_C$.

The most important implication of the observed phase transition at the characteristic temperature T_C is that it predicts the Hawking effect: the result that $\bar{E}(T)$ is practically zero, when $T < T_C$, and then suddenly jumps to $\bar{L} = 2k_B T_C \ln 2$, when $T = T_C$, indicates that the characteristic temperature T_C is the lowest possible temperature a black hole may have. If the temperature T of the black hole were less than its characteristic temperature T_C, all of the constituents of its stretched horizon, except one, would be in vacuum, and there would be no black hole. The characteristic temperature T_C may be written in terms of the Schwarzschild mass M and the Schwarzschild radial coordinate r of an observer on the stretched horizon as:

$$T_C = \frac{\alpha}{8\pi \ln 2} \left(1 - \frac{2M}{r}\right)^{-1/2} \frac{M}{r^2} \tag{24}$$

With some more effort one can obtain the Bekenstein-Hawking entropy law for the Schwarzschild black hole from its partition function which, in turn, followed from the specific microscopic model of its stretched horizon [5]

$$S(A) = \frac{1}{4}\frac{k_B c^3}{\hbar G}A \tag{25}$$

When $T = T_C$, the energy of the hole from the point of view of an observer on its stretched horizon is exactly

$$E = (N+2)k_B T_C \ln 2 \tag{26}$$

It is interesting that, up to an unimportant numerical factor $2\ln 2$, this expression for energy is the same as the one used as a starting point in the scenario for an entropic theory of gravity in [11].

It is shown in [5] that when $T = T_C$ the entropy of the Schwarzschild black hole may be written in terms of N, the number of the constituents of the stretched horizon, as:

$$S = k_B \ln(2^{N+2}). \tag{27}$$

Putting in another way, this means that when the temperature T of the hole is exactly its characteristic temperature T_C, which means that its temperature from the point of view of a faraway observer agrees with its Hawking temperature T_H, each constituent of the stretched horizon carries, on the average, exactly one bit of information. In this sense model [5] reproduces in some respects Wheeler's famous "it from bit" proposal.

5 Discussion and Conclusions

Any non-inertial observer who perceives a horizon will attribute to it the Unruh temperature (16)

$$T = \frac{\hbar}{k_B c}\frac{\kappa}{2\pi} \tag{28}$$

where κ is the acceleration of the observer, which is predicted in [5]. This result makes the notion of temperature and all of thermodynamics observer dependent phenomena.

It has turned out that horizons have profound importanece in gravity both on thermodynamical and statistical levels. There are interesting questions of heat as inertial effect and static observer's virtual reality in [9]. The origin of an acceleration surface is the stretched horizon structure of black holes presented in section 4.

The stretched horizon structure can perhaps be described by a bootstrap sum equation, or integral in the large i limit, for physical black hole states $|O\rangle$ (from opi, hole in Greek) in terms of bare black holes $|o\rangle$

$$|O\rangle = |o\rangle_0 + \Sigma_i |O_i\rangle \tag{29}$$

where i is the number of horizon constituent and $i = 0$ refers to ground state.

In the UV black holes cannot be probed deeper than l_{Pl}. With increasing energy the hole begins to grow approaching the classical regime. This model is therefore consistent with the concept of self-completeness [12].

The regime of real quantum gravity is limited to the vicinity of mini black holes and very early universe. Otherwise classical theory is accurate, see however for Lanczos-Lovelock theories in [6].

Having the entropy, rather than the metric, as central concept in the model of section 2 has lead to an interesting role for the cosmological constant: a constant of integration.

Its numerical value has been determined in [7]. This may help in dealing with possible problems of infinities, they can subtracted away.

A model of decay and radiation of black holes has been proposed in [13, 14]. The lightest black hole state $E_{n=0}$, the gravon, is expected to decay via a grand unified theory phase finally into standard model particles. Otherwise black holes radiate by the Hawking mechanism and by a classical no-hair theorem based mechanism producing non-thermal particles, dominantly light leptons.

There are at present a number competing theoretical schemes for quantum gravity like string theory, loop quantum gravity, causal dynamical triangulation, and others. The model of section 4 goes very deep into the structure of the physical universe and can be considered a promising candidate.

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