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DSmT Coupling with PCR5 for Mobile Robot's Map Reconstruction*

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***Abstract* - In this paper, considering the information fusion of multi-sensors on mobile robot, a new tool (the fusion machine of DSmT (*Dezert-Smarandache Theory*) coupling with PCR5 (*Proportional Conflict Redistribution Rule*)) is applied to autonomous mobile robot's map reconstruction from 16 sonar rangefinders. According to the physical characteristics of sonar rangefinder, general basic belief assignment functions are constructed respectively. By comparing the result of map reconstruction with this new tool from other approaches (i.e. Probability, Fuzzy, DST), when a virtual mobile robot evolves in the virtual environment with obstacles, we can adequately testify that the new tool owns better performance to rebuild map and even can solve the challenging SLAM problem more effectively.**

Index Terms - DSmT, PCR, Information Fusion, Mobile Robot, Map Reconstruction.

I. INTRODUCTION

The study on the exploration of entirely unknown environment for intelligent mobile robots has being a popular and difficult subject; especially SLAM is a challenging work in the autonomous mobile robot community, which is comparing as the puzzle of chicken and egg. This is because mobile robots are unknown about the environment around themselves, that is, they have no any experienced knowledge about the environment such as size, shape, layout of the environment, and also no any signs such as beacons, landmarks, let alone the determinate location about robot among the environment. Even if some intelligent sensors are installed to the mobile robot as if a person has perceptive organs, some uncertainty still occurs because of the physical limitation of sensors themselves. Map reconstruction is an important loop in SLAM. Especially, Grip map is the most popular method presently. How to manage and fuse the information of multi-sources (homogeneous or heterogeneous) successfully will help to build more precise map, and even solve the puzzle more effectively. Many approaches have been proposed, such as probability theory, Fuzzy theory, DST. In this paper, we will introduce and apply a new tool to map building, that is, DSmT[1] coupling with PCR5[2], which was proposed by Jean Dezert (France) and Florentin Smarandache (American) recently. We let the virtual autonomous mobile robot run in the virtual environment with obstacles, by comparing the result of map reconstruction from other methods, in order to testify the validity of the new tool.

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II. FUSION MACHINE OF DSMT COUPLING WITH PCR5

Here we introduce a new fusion machine of DSmT coupling with PCR5. DSmT (Dezert-Smarandache Theory) is a new, general and flexible arithmetic of fusion, which can solve the fusion problem of different tiers including data-tier, feature-tier and decision-tier, and even, not only can dispose the static problem of fusion, but also can dispose the dynamic one. Especially, it has a prominent merit that it can deal with the uncertain and highly conflicting information [1]. PCR5 goes backwards on the tracks of the conjunctive rule and redistributes the partial conflicting masses only to the sets involved in the conflict and proportionally to their masses put in the conflict, considering the conjunctive normal form of the partial conflict. PCR5 is quasi-associative and preserves the neutral impact of the vacuous belief assignment [2].

A. Basic of DSmT

1) Let $\Theta=\{\theta_1,\theta_2,\cdots,\theta_n\}$, here Θ is the frame of discernment, which includes n finite focal elements $\theta_i\,(i=1,\cdots n)$. Because the focal element is not precisely defined and separated, so that no refinement of Θ in a new larger set Θ_{ref} of disjoint elementary hypotheses is possible.

2) The hyper-power set D^Θ is defined as the set of all compositions built from elements of Θ with $\bigcup$ and $\bigcap$ (Θ generates D^Θ under operators $\bigcup$ and $\bigcap$) operators such that

a) $\phi,\theta_1,\theta_2,\theta_3\cdots\theta_n\in D^\Theta$

b) If $A,B\in D^\Theta$, then $A\bigcap B\in D^\Theta$ and $A\bigcup B\in D^\Theta$

c) No other elements belong to D^Θ , except those obtained by using rules a) or b).

3) General belief function and Plausibility function

Let $\Theta=\{\theta_1,\theta_2,\cdots,\theta_n\}$ is the general frame of discernment. For every evidential source S, let us define a set of map of $m(\cdot):D^\Theta\in[0,1]$ associated to it (abandoning Shafer's model) by assuming here that the fuzzy/ vague/ relative nature of elements $\theta_i\,(i=1,2,3\cdots n)$ can be non-exclusive, as well as no refinement of Θ into a new finer exclusive frame of discernment Θ_{ref} is possible. The mapping $m(\cdot)$ is called a generalized basic belief assignment function (gbbaf), if it satisfies

$$m(\phi)=0 \quad \text{and} \quad \sum_{A\in D^{\Theta}} m(A)=1$$

$m(A)$ is called A's generalized basic belief assignment function (gbbaf).The general belief function and plausibility function are defined respectively in almost the same manner as within the DST, i.e.

$$bel(A)=\sum_{B\in D^{\Theta},B\subseteq A} m(B) \tag{1}$$

$$Pl(A)=\sum_{B\cap A\neq\phi,B\in D^{\Theta}} m(B) \tag{2}$$

4) Classical (free) DSmT rule of combination

Let $M^f(\Theta)$ is a free model of DSmT, and then the classical (free) DSmT rule of combination for $k\geq 2$ sources is given as follows:

$$\forall A\neq\phi\in D^{\Theta},\quad m_{M^f(\Theta)}(A)\cong[m_1\oplus\cdots m_k](A) = \sum_{\substack{X_1,\cdots X_K\in D^{\Theta}\\ (X_1\cap\cdots X_K)=A}} \prod_{i=1}^{k} m_i(X_i) \tag{3}$$

B. Simple Review of PCR5

PCR5 fines the conflicting mass, and just redistribute the partial conflicting mass to the elements involved in the partial conflict, in order to improve the quality of fusion, and deal with the conflicting information again.

Here we introduce the PCR5 formula for $s=2$ sources: $\forall X\in G/\{\phi\}$,

$$m_{PCR5}(X)=m_{12}(X)+\sum_{\substack{Y\in G/\{X\}\\ c(X\cap Y)=\phi}}\left[\frac{m_1(X)^2 m_2(Y)}{m_1(X)+m_2(Y)}+\frac{m_2(X)^2 m_1(Y)}{m_2(X)+m_1(Y)}\right] \tag{4}$$

where, $c(X)$ represents the canonical form of X, $m_{12}(\bullet)$ corresponds to the conjunctive consensus, i.e. $m_{12}(X)\cong\sum_{\substack{X_1,X_2\in G\\ X_1\cap X_2}} m_1(X_1)m_2(X_2)$, and where all denominators are different from zero. If a denominator is zero, that fraction is discarded. For $s>2$, here we don't give, if readers have interest in it, please see also the reference [2].

. MODELING FOR SONAR GRID INFORMATION

Sonar sensors' working principle (shown as Fig 1) is: producing sheaves of cone-shaped wave, and detecting the objects by receiving the reflected wave. Due to the restriction of sonar physical characteristic, metrical data behaves out uncertainty as follows:

a) Beside its own error of making, the influence of external environment is also very great, for example, temperature, humidity, atmospheric pressure and so on.

b) Because the sound wave spreads outwards in the form of loudspeaker, and there exists a cone-shaped angle, we cannot know the true position of object detected among the fan-shaped area, with the enlargement of distance between sonar and it.

c) The use of lots of sonar sensors will result in interference each other. For example, when the *i*th sonar gives out detecting wave towards an object of irregular shape, if the angle of incidence is too large, the sonar wave might be reflected out of the receiving range of the *i*th sonar sensor or also might be received by other sonar sensors.

d) Because sonar sensors utilize the reflection principle of sound wave, if object absorbs very heavy sound wave, the sonar sensor might be invalid.

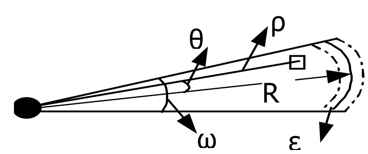

Fig.1: Sketch of the principle of sonar

A. Modeling Based on DSmT

Pointing to the characteristics of sonar's measurement, we construct a model of uncertain information acquired from grid map using sonar based on DSmT. Here we suppose there are two focal elements in system, that is, $\Theta=\{\theta_1,\theta_2\}$, here θ_1 means grid is empty, θ_2 means occupied, and then we can get its hyper-power set $D^{\Theta}=\{\phi,\theta_1\cap\theta_2,\theta_1,\theta_2,\theta_1\cup\theta_2\}$. Every grid in environment is scanned $k\geq 5$ times, each of which is viewed as source of evidence. Then we may define a set of map aiming to every source of evidence and construct the general basic belief assignment functions (gbbaf) as follows: $m(\theta_1)$ is defined as the gbbaf for grid-unoccupied (empty); $m(\theta_2)$ is defined as the gbbaf for grid-occupied; $m(\theta_1\cap\theta_2)$ is defined as the gbbaf for holding grid-unoccupied and occupied simultaneous (conflict). $m(\theta_1\cup\theta_2)$ is defined as the gbbaf for grid-ignorance due to the restriction of knowledge and experience presently (here referring to the gbbaf for these grids still not scanned presently), it reflects the degree of ignorance of grid-unoccupied or occupied.

The gbbaf of a set of map $m(\cdot):D^{\Theta}\to[0,1]$ is constructed by authors such as the formulae (5)~(8) according to sonar physical characteristics.

$$m(\theta_1)=E(\rho)E(\theta)=\begin{cases}\left(1-(\rho/R)^2\right)\cdot\lambda & \begin{cases}R_{\min}\leq\rho\leq R\leq R_{\max},\\ 0\leq\theta\leq\omega/2\end{cases}\\ 0 & \text{other}\end{cases} \tag{5}$$

$$m(\theta_2)=O(\rho)O(\theta)=$$
$$\begin{cases} \exp\left(-3\rho_v(\rho-\mathrm{R})^2\right)\cdot\lambda & \begin{cases} R_{\min}\le\rho\le R+\varepsilon\le R_{\max} \\ 0\le\theta\le\omega/2 \end{cases} \\ 0 & \text{other} \end{cases} \tag{6}$$
$$m(\theta_1\cap\theta_2)=$$
$$\begin{cases} \left(1-\left(2(\rho-(R-2\varepsilon))/R\right)^2\right)\cdot\lambda & \begin{cases} R_{\min}\le\rho\le R\le R_{\max} \\ 0\le\theta\le\omega/2 \end{cases} \\ 0 & \text{other} \end{cases} \tag{7}$$
$$m(\theta_1\cup\theta_2)=$$
$$\begin{cases} \tanh\left(2(\rho-R)\right)\cdot\lambda & \begin{cases} R\le\rho\le R_{\max}, \\ 0\le\theta\le\omega/2 \end{cases} \\ 0 & \text{other} \end{cases} \tag{8}$$

where $\lambda=E(\theta)=O(\theta)$ is given by(see ref.[3] for justification).

$$\lambda=E(\theta)=O(\theta)=\begin{cases} 1-(2\theta/\omega)^2 & 0\le|\theta|\le\omega/2 \\ 0 & other \end{cases} \tag{9}$$

Seen from Fig.2, gbbaf reflects really out the characteristics of sonar information with the shift of in the course of building grid map. Here we assume the range of sonar sensor from 0.2m~3m. High conflictive information happens about at the point of intersection of two curves between $m(\theta_1)$ and $m(\theta_2)$. The maximum $m(\theta_2)$ happens at R, while the maximum of $m(\theta_1)$ happens at R_{min}=0.2m. Of course, in order to satisfy the definition of DSmT, and assure the sum of all mass of $m(\cdot)$ to be one, we must renormalise them while acquiring sonar grip information.

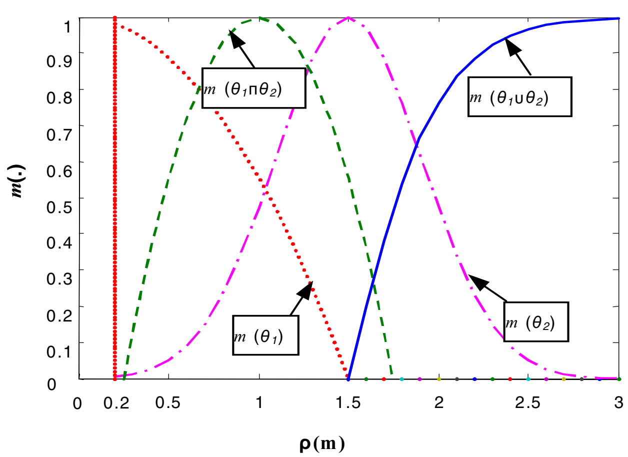

Fig.2: m(.) as function of given by Eq.(4~7)

B. Modeling Based on other methods

1) Probability Theory

Elfes and Moravec[3] firstly represented the probability of the grid occupied by obstacles with probability theory, here to avoid amount of computation, we suppose that all grids are independent. For every grid G_{ij}, let $s(G_{ij})=E$ represent the grid empty, while $s(G_{ij})=O$ represent the grid occupied, moreover, exists the restraint $P[s(G_{ij})=E]+P[s(G_{ij})=O]=1$ between the two events. According to the physical characteristics, we may get the probability model to map the sonar perception datum.

$$P[s(G_{ij})=O\mid R]=P[s(\rho,\theta)=O\mid R]$$
$$=\begin{cases} (1-\lambda')/2 & 0\le\rho<R-2\varepsilon \\ 0.5\left(1-\lambda'\left(1-\left(2+((\rho-r)/\varepsilon)\right)^2\right)\right) & R-2\varepsilon\le\rho<R-\varepsilon \\ 0.5\left(1+\lambda'\left(1-((r-\rho)/\varepsilon)^2\right)\right) & R-\varepsilon\le\rho<R+\varepsilon \\ 0.5 & \rho\ge R+\varepsilon \end{cases} \tag{10}$$

Where, $\lambda'=\Gamma(\theta)\cdot\Gamma(\rho)$ (see also ref[4]).

The fusion algorithm for multi-sensors is given according to the Bayesian estimate as follows:

$$P[s(G_{ij})=O\mid R_1,\cdots R_{k+1}]$$
$$=\frac{P[s(G_{ij})=O\mid R_{k+1}]\cdot P[s(G_{ij})=O\mid R_1,\cdots R_k]}{\sum_{X\in\{E,O\}}P[s(G_{ij})=X\mid R_{k+1}]\cdot P[s(G_{ij})=X\mid R_1,\cdots R_k]} \tag{11}$$

Remark: In order to make the equation (11) hold, we must suppose $P[s(G_{ij})=E]=P[s(G_{ij})=O]=0.5$, $\forall G_{ij}\in U$ at the beginning of map building,

2) Fuzzy Theory

Map building based on fuzzy logic is firstly proposed by Giuseppe Oriolo .etc [5], they define two fuzzy sets Ψ (represents grid empty) and Ω (represents grid occupied), which of size all are equal to U , correspondingly, their membership functions are μ_Ψ and μ_Ω. Similarly, we can get fuzzy model to map the sonar perception datum.

$$\mu^{S(R)}{}_\Psi(G_{ij})=\lambda'\cdot f_\Psi(\rho,R) \tag{12}$$
$$\mu^{S(R)}{}_\Omega(G_{ij})=\lambda'\cdot f_\theta(\rho,R) \tag{13}$$

Where,

$$f_\Psi(\rho,R)=\begin{cases} k_E & 0\le\rho<R-\varepsilon \\ k_E((R-\rho)/\varepsilon)^2 & R-\varepsilon\le\rho<\varepsilon, \\ 0 & \rho\ge R \end{cases}$$
$$f_\Omega(\rho,R)=\begin{cases} k_O & 0\le\rho<R-\varepsilon \\ k_O((R-\rho)/\varepsilon)^2 & R-\varepsilon\le\rho<R+\varepsilon \\ 0 & \rho\ge R+\varepsilon \end{cases}$$

Here, k_E, k_O are constant value, λ' is same as the definition in (10).

The fusion algorithm for multi-sensors is given according to the union operator in the fuzzy theory as follow:

$$
\begin{aligned}
&\mu_X^{S(R_1,\cdots R_{k+1})}\left(G_{ij}\right) \\
&= \mu_X^{S(R_1,\cdots R_k)}\left(G_{ij}\right) + \mu_X^{S(R_{k+1})}\left(G_{ij}\right) \\
&\quad - \mu_X^{S(R_1,\cdots R_k)}\left(G_{ij}\right) \bullet \mu_X^{S(R_{k+1})}\left(G_{ij}\right) \\
&\quad \forall X \in \{\Psi, \Omega\}
\end{aligned}
\tag{14}
$$

Remark:

Initially, we suppose $\mu_\Psi\left(G_{ij}\right) = \mu_\Omega\left(G_{ij}\right) = 0, \forall G_{ij} \in U$ According to membership of every grid, we can get the final map representation as follows:

$$M = \overline{\Psi^2 \cap \overline{\Omega} \cap \overline{A} \cap \overline{I}} \tag{15}$$

Here,

$$A = \Psi \cap \Omega, I = \overline{\Psi} \cap \overline{\Omega}.$$

3) DST

According to the requirement of sonar grid map building, here we also suppose two focal elements in system, that is, $\Theta = \{\theta_1, \theta_2\}$, here θ_1 means grid is empty, θ_2 means occupied, and then we can get its power set $2^\Theta = \{\phi, \theta_1, \theta_2, \theta_1 \cup \theta_2\}$. According to DST, let $m_{DST}(\phi) = 0$, here $m_{DST}(\theta_1)$ is defined as the basic belief assignment function (bbaf) for grid-empty, $m_{DST}(\theta_2)$ is defined as the basic belief assignment function(bbaf) for grid-occupied, $m_{DST}(\theta_1 \cup \theta_2)$ is defined as the basic belief assignment function for grid-ignorance. We may also construct basic belief assignment function such as $m_{DST}(\theta_1) = m(\theta_1)$, $m_{DST}(\theta_2) = m(\theta_2)$, $m_{DST}(\theta_1 \cup \theta_2) = m(\theta_1 \cup \theta_2)$. bbaf reflects still the characteristics of uncertainty for sonar grip map building in Fig.2. Though here we define the same bbaf as DSmT, because the definition of DST must be satisfied, we must renormalize them while acquiring sonar grip information [6].

IV. Simulation Experiment

The experiment consists in simulating the autonomous navigation of a virtual Pioneer II Robot carrying 16 simulated sonar detectors in a 5000mm× 5000mm square array with an unknown obstacle/object. The map building with sonar sensors on the mobile robot is done from the simulator of SRIsim (shown in Fig.3) of ActivMedia company and our self-developing experimental or simulation platform together. (shown in fig.4) together. Here the platform developed with the tool software of visual c++ 6.0 and OpenGL severs as a client end, which can connect the sever end (also developed by ourselves, which connects the SRIsim and the client). When the virtual robot runs in the virtual environment, the sever end can collect many information (i.e. the location of robot, sensors reading, velocity .etc) from the SRIsim. Through the protocol of TCP/IP, the client end can get any information from the sever end and fuse them. The Pioneer II Robot may begin to run at arbitrary location; here we choose the location (1500mm, 2700mm) with an 88 degrees angle the robot faces to. We let the robot move at speeds of trans-velocity 100mm/s and turning-velocity 50degree/s around the object in the world map plotted by the Mapper (a simple plotting software), which is opened in the SRIsim shown in fig.3.

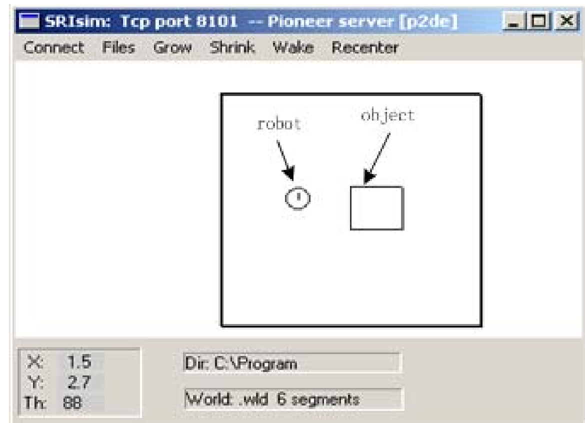

Fig.3 A world map opened in the SRIsim

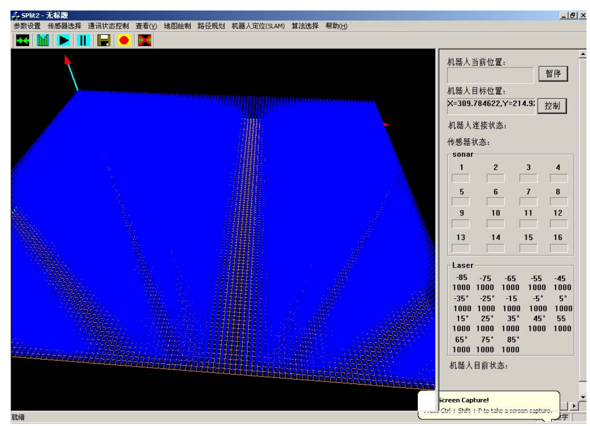

Fig.4 the platform for simulation or real experiment

We adopt grid method to build map. The global environment is divided into 50× 50 lattices (which of size are same). The object in fig.3 is taken as a regular rectangular box, when the virtual robot runs around the object, through its sonar sensors, we can clearly recognize the object and know its appearance, and even its location in the environment.

To describe the experiment clearly, the main steps of procedure based on the new tool are given as follows:

1) Initialize the parameter of robot (i.e. initial location, moving velocity, etc.).

2) Acquire 16 sonar readings, and robot's location, when the robot is running. (Here we set the first timer, of which interval is 100 millisecond)

3) Compute gabba of the fan-form area detected by each sonar sensor.
Whether some grids are scanned more than 5 times by sonar sensors (same sonar in deferent location, or different sonar sensors, of course, here we suppose each sonar sensor has the same characteristics)? if yes, go to next step, otherwise, go to step 2.

4) According to the combination rule and the PCR5 in (3) and (4) respectively, we can get the new basic belief masses, and redistribute the conflict mass to the new basic belief masses in the order of the sequential fusion, until all 5 times are over.

5) Compute the credibility of occupancy $bel(\theta_2)$ of some grids, which have been fused according to (1).

6) Update the map of the environment. (Here we set the second timer, of which interval is 100 millisecond) Whether all the grids have been fused? Yes, stop robot and exist. Otherwise, go to step 2.

Finally, we rebuild the map shown in the Fig.5 with the new tool. Because of the limitation of paper length, here we don't give map rebuilt by other methods, however we give the result of comparison in table.1. Seen from the Fig.6 and table.1, it can be concluded that:

Seen from the Fig.5, the new tool has a better performance than just DSmT in building map, (see also ref[7,8]), because of considering the conflict factor, and redistributing the conflict masses to other basic belief masses according to the PCR5.

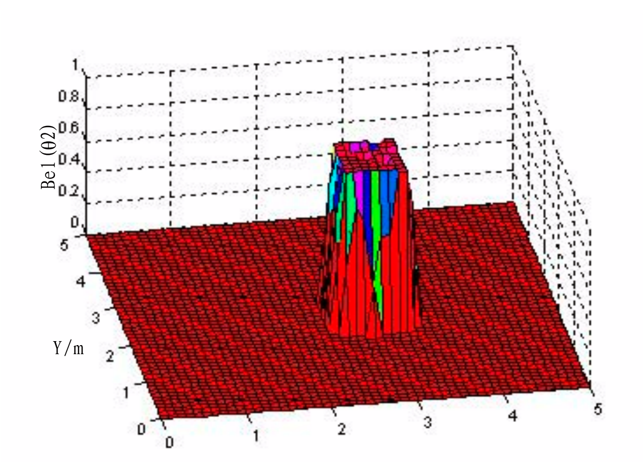

Fig.5 Map reconstruction with DSmT coupling with PCR5

Seen from the table.1, Probability theory spends lest time, while Fuzzy theory spends the most time. But Map reconstruction with probability theory has very low precision and high mistaken judging rate. Though the new tool spends a little more time than probability theory, however, it has very high precision the same as the fuzzy theory and very low mistaken judging rate. In fact, the comparison in map building between DST and DSmT have made in detail by ourselves in the reference [6], Through the analysis of comparison among the four tool, we testify the new tool to play a better role in map building.

TABLE Ⅰ Comparison of Four Tools in Building Map

	Probability	Fuzzy	DST	New tool
Total spent time (ms)	18146	35823	19673	19016
Map precision	Very low	Very high	High	Very high
Mistaken rate	High	Low	General	Very low

V. Conclusion

In this paper, we introduce a new tool to rebuild map for autonomous mobile robot. Through comparing the result among the four tools (i.e. probability theory, fuzzy theory, DST, DSmT coupling with PCR5), we may testify the new tool to have a better performance in map reconstruction. We expect in next works to improve the performances of map building using the PCR6 coupling with DSmT. Since the PCR6 fusion rule proposed recently by Martin and Osswald [9] has already been shown in another context of application more efficient than PCR5 and many other classical fusion rules (Dempster's rule, Dubois &Prade's, etc). Moreover, we are working at the particle filter with the new tool for the mobile robot's SLAM.

Acknowledgments

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