E_8: A GAUGE GROUP FOR THE ELECTRON

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ABSTRACT. The eight geometric objects of the impedance model of the electron, as fortuitous happenstance would have it, are those of the 3D Pauli subalgebra of the geometric interpretation of Clifford algebra. Given that impedance is a measure of the amplitude and phase of opposition to the flow of energy, and that quantum phase is the gauge parameter in quantum mechanics, one might consider an approach in which the elements of a gauge group of the electron would be impedances of interactions between these geometric objects. The resulting 4D Dirac subalgebra is briefly examined in relation to the E_8 exceptional Lie group.

INTRODUCTION

Pre-conference proceedings of the 2015 Barcelona conference on applications of the geometric interpretation [1, 2] of Clifford algebra are available [3]. The table of contents reveals incredible diversity in a budding field. Of particular interest to the particle physicist are two papers, one connecting Geometric Algebra (GA) with the geometry of the E_8 exceptional Lie group [4, 5], and the other [6] linking the impedance model of the elementary particle spectrum [7, 8] with GA. Taken together, these papers suggest the possibility of casting new light on efforts to connect E_8 and the particle spectrum [9].

Impedance (or more specifically impedance *matching*) governs the flow of energy. Defined to be a measure of the amplitude and phase of opposition to that flow [10, 11, 12, 13], impedance is a fundamental concept, universally valid. Familiar examples of its quantization include the photon far-field and quantum Hall impedances. Quantization can be generalized to the impedances of all interactions [7]. The goal of this note is to propose a gauge group consistent with an impedance model of the electron, and to explore the relationship between that group and E_8.

What is gauged in quantum mechanics is phase. Quantum impedances shift quantum phase. Gauge invariance is built in, natural. It follows that quantized impedances are natural choices for elements of the proposed gauge group.

GEOMETRIC OBJECTS OF THE IMPEDANCE MODEL

In the process of quantizing gauge theory gravity [6, 14, 15, 16], the impedance approach establishes a correspondence between geometric objects of the impedance model and those of GA. While the point particle model of the electron has proven incredibly precise in perturbative calculations of quantum electrodynamics, it is obvious that electrons are not point particles, that the physical electron must have some sort of structure. Considering electromagnetic fields only, taking maximal symmetry between electric and magnetic, and taking the simplest geometric objects needed for an arguably realistic model [7] gives:

- three objects - flux quantum (no singularity), monopole (one), and dipole (two)
- quantization of magnetic and electric flux, charge, and dipole moment
- confinement to a fundamental length, taken to be the electron Compton wavelength

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Group Generators and Elements

Six geometric objects, three magnetic and three electric, follow from the given impedance model. However, as explained in detail elsewhere [7, 17] the model yields not one but two electric flux quanta. The first is associated with the magnetic flux quantum (a fundamental constant) and quantization of magnetic flux in the photon, which by Maxwell's equations requires quantization of electric flux as well. The second follows from applying Gauss's law to the electron charge, and is a factor of 2α smaller than the first, where α is the fine structure constant. And similarly, there are not one but two electric dipole moments in the model.

As shown at the top of figure 1, and taking magnetic charge to be pseudoscalar rather than scalar, the resulting eight geometric objects of the impedance model comprise a complete 3D Pauli algebra of one scalar, three vectors, three bivectors, and one pseudoscalar [1].

Like the wave function, whose 'reality' is of interest in quantum interpretations [18], the primitive geometric objects of the electron model are not observable. Of interest here are impedances of the observables, taken to be impedances of interactions between the fundamental geometric objects - the mode impedances of the self-interacting electron model. If these impedances are to be group elements, then the fundamental geometric objects of the impedance model can be taken to be the group generators. With their interactions described by the geometric product of GA, the resulting impedance matrix is shown in figure 1. The algebra of the proposed group is the flat 4D SpaceTime subAlgebra (STA) of GA, the algebra of Minkowski space. The equivalent matrix algebra is that of the Dirac equation in the usual matrix basis (which unfortunately obscures the clarity of the geometric interpretation).

	electric charge e *scalar*	elec dipole moment 1 d_{E1} *vector*	elec dipole moment 2 d_{E2} *vector*	mag dipole moment μ_{Bohr} *vector*	elec flux quantum 1 ϕ_{E1} *bivector*	elec flux quantum 2 ϕ_{E2} *bivector*	mag flux quantum ϕ_B *bivector*	magnetic charge g *pseudoscalar*
e	ee 3.7 KeV *scalar*	ed_{E1} .511 MeV	ed_{E2} 3.7 KeV *vector*	$e\mu_B$ .511 MeV	$e\phi_{E1}$ .511 MeV	$e\phi_{E2}$ 3.7 KeV *bivector*	$e\phi_B$ null	eg .511 MeV *pseudovector*
d_{E1}	$d_{E1}e$ .511 MeV	$d_{E1}d_{E1}$ 70 MeV	$d_{E1}d_{E2}$ .511 MeV	$d_{E1}\mu_B$ 70 MeV	$d_{E1}\phi_{E1}$ 70 MeV	$d_{E1}\phi_{E2}$ .511 MeV	$d_{E1}\phi_B$ 70 MeV	$d_{E1}g$ 70 MeV
d_{E2}	$d_{E2}e$ 3.7 KeV *vector*	$d_{E2}d_{E1}$ .511 MeV	$d_{E2}d_{E2}$ 3.7 KeV *scalar + bivector*	$d_{E2}\mu_B$ .511 MeV	$d_{E2}\phi_{E1}$ .511 MeV	$d_{E2}\phi_{E2}$ 3.7 KeV *vector + pseudovector*	$d_{E2}\phi_B$ .511 MeV	$d_{E2}g$ .511 MeV *scalar + pscalar*
μ_B	$\mu_B e$ .511 MeV	$\mu_B d_{E1}$ 70 MeV	$\mu_B d_{E2}$ .511 MeV	$\mu_B\mu_B$ 70 MeV	$\mu_B\phi_{E1}$ 70 MeV	$\mu_B\phi_{E2}$ .511 MeV	$\mu_B\phi_B$ 70 MeV	$\mu_B g$ 70 MeV
ϕ_{E1}	$\phi_{E1}e$ .511 MeV *bivector*	$\phi_{E1}d_{E1}$ 70 MeV	$\phi_{E1}d_{E2}$ .511 MeV *vector + pseudovector*	$\phi_{E1}\mu_B$ 70 MeV	$\phi_{E1}\phi_{E1}$ 70 MeV	$\phi_{E1}\phi_{E2}$ .511 MeV *scalar + pseudoscalar*	$\phi_{E1}\phi_B$ 70 MeV	$\phi_{E1}g$ null *vector*
ϕ_{E2}	$\phi_{E2}e$ 3.7 KeV	$\phi_{E2}d_{E1}$ .511 MeV	$\phi_{E2}d_{E2}$ 3.7 KeV	$\phi_{E2}\mu_B$ .511 MeV	$\phi_{E2}\phi_{E1}$ .511 MeV	$\phi_{E2}\phi_{E2}$ 3.7 KeV	$\phi_{E2}\phi_B$ γ .511 MeV	$\phi_{E2}g$ null
ϕ_B	$\phi_B e$ null	$\phi_B d_{E1}$ 70 MeV	$\phi_B d_{E2}$ .511 MeV	$\phi_B\mu_B$ 70 MeV	$\phi_B\phi_{E1}$ 70 MeV	$\phi_B\phi_{E2}$ γ .511 MeV	$\phi_B\phi_B$ 70 MeV	$\phi_B g$ 70 MeV
g	ge .511 MeV *pseudovector*	gd_{E1} 70 MeV	gd_{E2} .511 MeV *scalar + pseudoscalar*	$g\mu_B$ 70 MeV	$g\phi_{E1}$ null	$g\phi_{E2}$ null *vector*	$g\phi_B$ 70 MeV	gg 70 MeV *scalar*

FIGURE 1. Proposed Gauge Group for the Electron

Ignoring for the moment the three orientational degrees of freedom, the matrix is populated by all 64 possible geometric products, by all possible 'two body' interactions [19] of the generators. Subsets shaded in blue belong to the even subalgebra, and yellow to the odd. The even subalgebra of STA (and of the Dirac algebra as well) is the Pauli algebra.

An earlier impedance analysis of the the photon-electron interaction [6], guided by the formalism of near-field QED [20, 21], suggests that even subalgebras correspond to eigenstates of the photon and electron, and odd to dynamics of the photon-electron interaction.

Energies associated with the modes have been calculated and are shown in figure 1. The broken symmetries associated with the representations at 3.7 KeV and 70MeV have been identified in earlier notes [7, 17]. The null modes are associated with scale invariant impedances, cannot communicate energy, only quantum phase. A familiar example is the quantum Hall impedance (another fundamental constant), denoted by big red dots in the figure.

Impedances of the interactions distinguished by blue and red dots, triangles, squares, and diamonds of figure 1 have been calculated, and are shown in figure 2 [22]. The impedance network shown there is background independent, gauge invariant, finite, confined, and by extension to the Planck particle contains gravity [6, 14]. Built from fundamental constants, it is precisely structured in powers of α, providing intuitive understanding of the ongoing part-per-billion successes of QED perturbative expansions in fitting and predicting experimental data.

The horizontal scale of the impedance network of figure 2 is in units of both length and energy. The length scale is defined at four fundamental lengths - classical, Compton, Bohr, and Rydberg. The energy scale is that of a photon of a given wavelength. For instance, the energy of a photon whose wavelength is the electron Compton wavelength is .511 Mev. The representations sit on the α-spaced coherence lines, at the nodes of the impedance network.

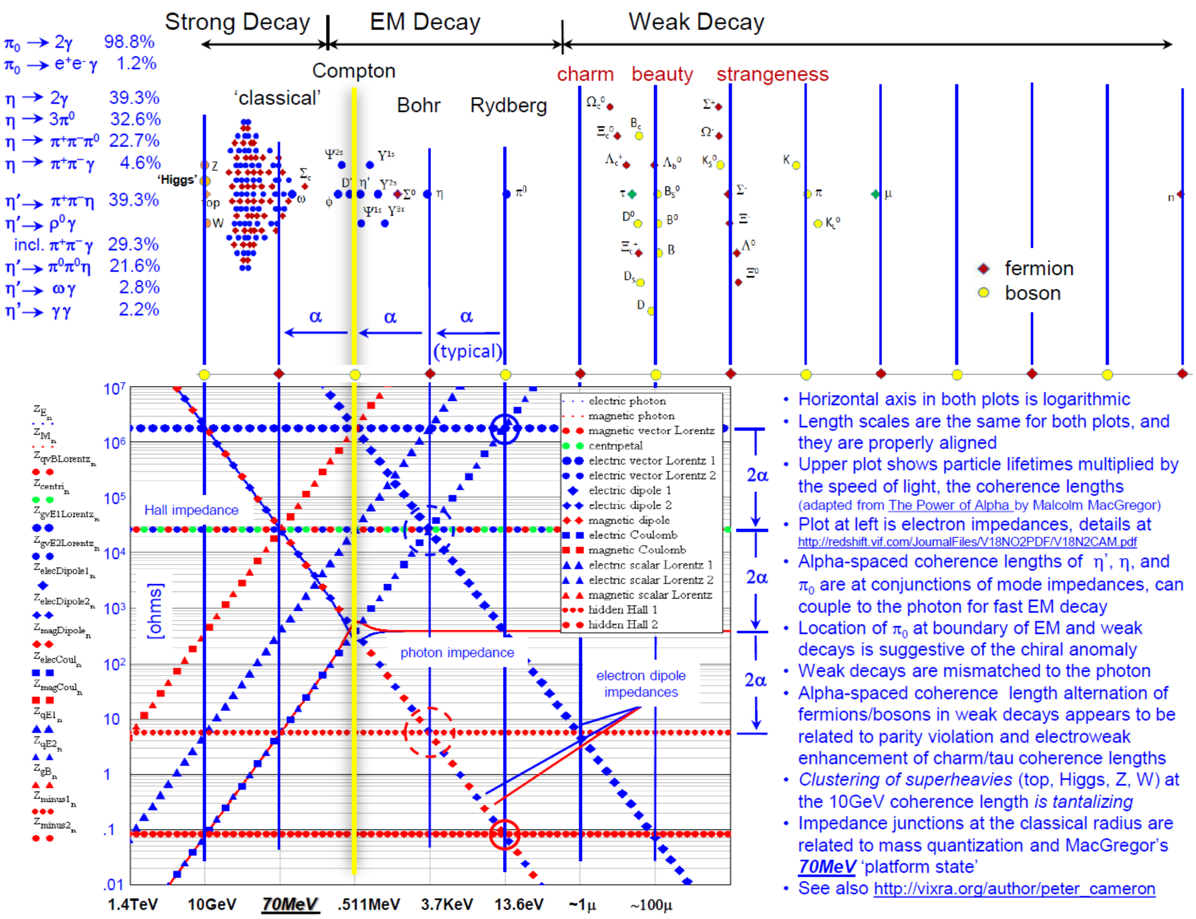

FIGURE 2. The 'One Slide' [22]

The network of figure 2 follows from a small subset of the group elements of figure 1. As figure 2 shows, additional representations exist for the superheavies (top, Higgs, Z, W,...), and for pizero, tau, the flavor families, muon, and neutron. It seems reasonable to suggest that at least some of these additional representations might follow from the remaining unplotted elements of figure 1, or from the orientational degrees of freedom not included in that figure, and would comprise a more complete electron gauge group. The stable proton is not shown on the plot of coherence lengths. And the neutrinos?

Group Requirements

It has been shown [23] that "every Lie algebra is isomorphic to some bivector algebra". The presence of the vector and pseudovector elements (shaded in yellow in figure 1) seems to immediately remove the possibility that the proposed gauge group is a Lie group. Given the earlier suggestion that odd subgroups comprise transitions and even subgroups eigenstates, it perhaps remains useful to consider whether the even subgroups might comprise a Lie group.

Doran and Lasenby [24] define a Lie group to be a "...manifold $\mathscr{M}$ together with a product $\phi(x,y)$..." such that "... the product has the correct group properties." Here the manifold $\mathscr{M}$ is the 4-dimensional vector manifold of STA, the product $\phi(x,y)$ is the geometric product of the group generators and/or elements, and the required properties are

- Identity - There exists an identity element e in $\mathscr{M}$ such that $\phi(x,e) = \phi(e,x) = x$ for every x in $\mathscr{M}$
- Inverse - For every element x in $\mathscr{M}$ there exists a unique element $\bar{x}$ such that $\phi(x,\bar{x}) = \phi(\bar{x},x) = e$
- Associativity - $\phi(\phi(x,y),z) = \phi(x,\phi(y,z))$ for every x,y, and z in $\mathscr{M}$
- Closure - The product $\phi(x,y)$ is a member of $\mathscr{M}$ for every x and y in $\mathscr{M}$

The first three in this list are true for all geometric algebras. However, closure as defined above is potentially problematic. The model presented here is of a self-interacting electron. One approach to this problem is to suggest that the product is realized in each representation by the coupled modes at the impedance conjunctions. Figure 1 is organized by geometric grade. The group as organized by representation is shown in Figure 3.

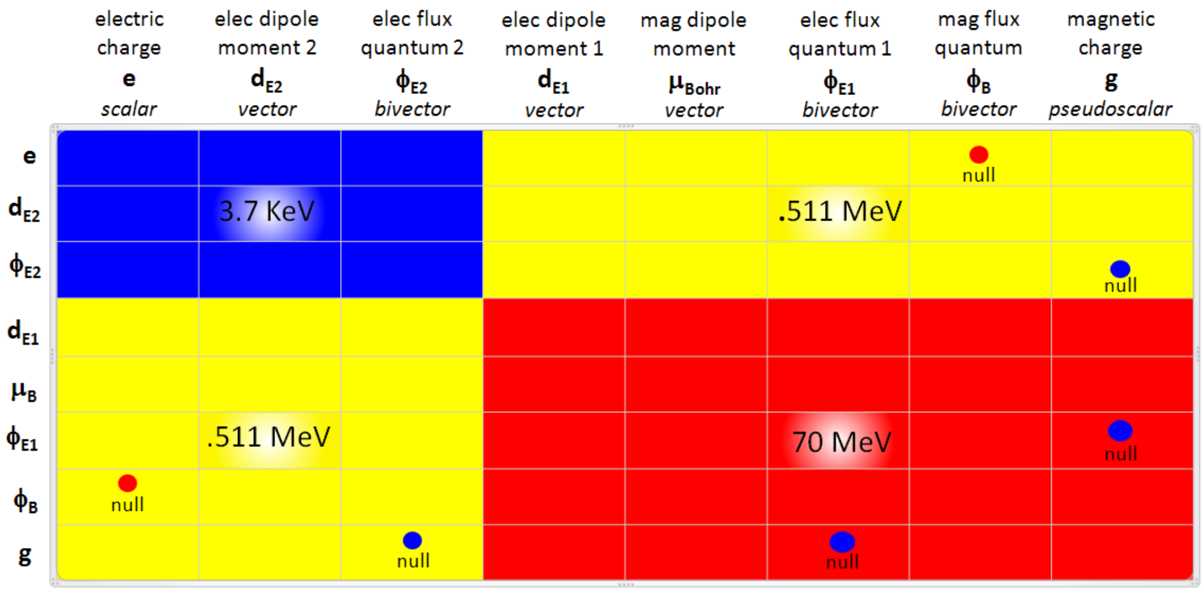

FIGURE 3. Proposed Gauge Group arranged by Representation

The null modes are variants of the vector Lorentz/quantum Hall impedance found in the impedance model. In mechanical units [25] rather than electrical, they correspond to centrifugal impedances.

They are scale invariant, cannot be shielded, and cannot communicate energy, only quantum phase (not a single measurement observable). This does not rule out the possibility that they might serve as mode couplers at the impedance conjunctions.

Another way to present the closure requirement is to say that every member of $\mathscr{M}$ can be written as a product $\phi(x,y)$ for at least one each of x and y in $\mathscr{M}$. This again suggests that the group requirement can be met by the physics of the interactions, by mode coupling of members of a given representation in linear superposition or between representations in non-linear effects, the noiseless parametric amplification conjectured to follow from the topological character of invariant impedances coming to mind in generation of particles heavier than the electron.

Taken for granted in the Lie group requirements [24] is that the group be continuous. The proposed group meets this requirement, being continuous in length/frequency/energy from something like the radius of the universe to the Planck length (and perhaps beyond to the singularity, totally decoupled by the infinite impedance mismatch to the dimensionless point) [26].

Exceptional groups have an additional requirement, a requirement usually reserved for discrete groups, that of reflection symmetry. Just if and where this symmetry might enter in the present approach is not yet at all obvious.

Conclusion

The ideas presented here have been guided not by any deep understanding of either Geometric Algebra or Group Theory. The present author claims no expertise in GA, and even less in GT. What has guided this work are early glimpses of the fundamental role of that which governs the flow of energy, glimpses of the fundamental role of quantized impedances in every aspect of the world of quantum phase coherence, of their role in gauge invariant theories.

Absent the expertise of one who has paid the seriously nontrivial dues needed to understand aspects both superficial and subtle, the material based in GA and GT and presented here is likely not without potentially embarassing gaffs. The hope remains that it will in some way prove useful, and the ignorance of the author may be noted in passing and forgotten (though not before correcting whatever misunderstandings may be remedied).

Transformation between impedance and scattering matrices is standard fare in electrical engineering. While the scattering matrix formalism is more familiar to the electrical engineer, in fact it originated not with the engineers but rather with the physicists, with Wheeler and company[27] in the 1930s. For present purposes the impedance approach has been more transparent. However in particle physics, and specifically while probing nucleon spin with polarized electrons [28], knowing the scattering matrix of the probe electron would be helpful both in motivating commencement of construction and in understanding the data [29, 30, 31].

And finally, coming back to the odd subalgebras of figure 1. In GA, multiplication of an odd subalgebra by γ_0 projects out the time/phase component, generating even subalgebras. The resulting group elements would be phase shifts, rather than that which generates phase shifts, namely the impedances. While the details are not clear, it might be that something like this would permit inclusion of the photon-electron interaction modes in the Lie algebra.

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"In the beginner's mind are many possibilities, in the expert's mind very few." [32]

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