The Weird World of Nonstandard Analysis (NSA), the GD and the GID (General Intelligent Design) Models

Robert A. Herrmann*

20 NOV 2014. Revised 24 DEC 2016

Abstract: The very unusual symbolic aspects of Nonstandard Analysis are discussed. Most formal constructions are not presented. The definitions, interrelations and symbolic manipulations of six distinct NSA symbolic languages are presented. Examples are given for these interrelations and symbolic manipulations. These symbolic forms are interpreted theologically relative to the characterizing of Divine attributes and relative to the intelligent design of a General Grand Unification Model produced physical universe. The predicted theological conclusions are summarized by the numbered statements in Sections 5 and 6 of this article. This article is intended to present the necessary material that should make these statements fully comprehensible.

1. Introduction.

Mathematics and its applications used throughout almost all of present day society is highly distinct from a rather obscure subject termed **Nonstandard Analysis (NSA)**. Although NSA was considered when first proposed from 1961 - 1966 to be a remarkable new area of mathematical discourse, expecially since it solves the "problem or Newton and Leibniz" and was applied to many mathematical and physical science areas, it is, except for a few controversial topics, slowly pasting from the scene. This is understandable since the subject is a product of formal logic and abstract model theory, two subjects that are not part of ones mainstream mathematical eduction and informal mathematical discourse.

Further, throughout the years of its development, it was felt necessary to adhere to the rather rigid standards required of these two subjects. Thus, the explanatory linguistic terms that were assiduously used in the past were not altered from what they meant. Due to the subject matter to which NSA is applied, there arose a clash in terminology that has, over the years, clouded the meanings of the terms employed. Material is often written only for the highly experienced who do, indeed, "know what the author means" when much is not specifically stated. The informal language, like the one being presently employed, has a great deal in common with the language

*Professor of Mathematics (Ret.), The United States Naval Academy, Annapolis, MD, U. S. A. *E-mail* drrahgid@hotmail.com

employed in formal logic and model theory. This tends to lead to misunderstandings. In definitions, further explanations and proofs of theorems, adjectives are dropped from terms, modifiers that tell one what a symbol actually signifies since the reader is suppose to "know what they signify."

Although it is not specifically stated, much of NSA is but abstract mathematics. That is, it is symbol manipulation that follows either explicitly stated or implied but accepted rules. These rules are often not stated but are learned via practice. Informal mathematics is most often written in terms of a "first-order predicate language with constants." Other types of entities such as functions are also included. If the language refers to set-theory, then technically such additional language entities are not necessary.

A predicate is usually thought of as a simple declarative sentence that states a relation between noun type words, where the names of the noun can vary. For example, a two place predicate "- is going to - " or the one place predicate "- is a horse" states the relation part, where the missing noun can vary. The fact that it can vary is expressed by "variables." So, one has, "x is going to y " or simple "x is a horse." The language uses a list of terms and forms, the two standard or equivalent **quantifiers**, "for each," "there exists," as well as the words "and," "or," "not," and the forms "If -, then - " and "- if and only if -". The term first-order means that the variable is a predicate variable and it is not a predicate itself. That is, the predicates are not quantified. This normal informal language of mathematical discourse can be written in a more formal pure symbolic form.

A **formal language** uses constant symbols like $a,\ b,\ B,\ E$, special symbols such as $\forall,\ \exists,\ \wedge,\ \vee,\ \neg,\ \to,\ \leftrightarrow$, for two quantifiers, specific words and two forms, respectively. Then it uses distinctly different variables symbols like $x,\ y,\ z$. Here are a few lines from a claimed "informal" proof. The $\in$ and $=$ are symbols for two place predicates.

"Let (A, E) be an infinite graph. . . . Let B be a hyperfinite subset of $^*A$ . . . such that $\forall a \in A,\ {}^*a \in B$. Then $B \in {}^*\mathcal{F}$ . . . such that $\forall x,\ y \in B$, if $(x, y) \in {}^*E$, then . . . . In particular, $\forall a,\ b \in A$, if $(a, b) \in E$, then $({}^*a,\ {}^*b) \in {}^*E$, so $g({}^*a) \neq g({}^*b)$." (Loeb and Wolf, ((2000, p. 49)).

Notice the different constants taken from the first part of the alphabet and the use of the formal symbol $\forall$. Often when a mathematician writes a statement very informally, such as in a classroom, this symbol might be used as a mere abbreviation for the phrase "for each" as well as $\ni$ for the phrase "such that," but due to their more formal usage in NSA the practice should be otherwise avoided. The constants are modified with the * notation but there are also two variables $x,\ y$ used. As will be discussed later, the part of the proof using the variables is not necessary and the phrase "In particular" is also not necessary. In this case, the formal $\forall$ is also employed. This statement is a “translation” from a formal-logic expression used to express behavior of the objects in a model, a nonstandard one. Further, the objects are given names such as the word “hyperfinite” which is important. One learns specifically described rules for writing a formal expression from left-to-right and a formal expression associated with this proof partially looks like

$$\forall z \in \mathcal{F}(\exists w \ldots (\forall x \forall y \in z)[(x, y) \in E \to w(x) \neq w(y)].$$

(Note: I have made an alteration in the original to correspond to what is discussed later in this article.)

I would not have used the variables in two context, the formal and the informal. Indeed, it is unnecessary to do so. As mentioned, I would not have used the formal $\forall$ for the phrase in the informal statement but rather its informal meaning “for each.” Further, for the phrase “in particular” and the statement $\forall a,\ b \in A$, I would have written this, as I do throughout my published papers and books, as “Let $a,\ b \in A$.” It is standard practice that one word is understood when one informally writes “Let $a,\ b \in A$.” The complete phrase is “Let arbitrary $a,\ b \in A$.” The same holds when “Let” is replaced by “Given.” For NSA, these are arbitrary members of the set of language constants. After these phrases, any additional restrictions placed on these arbitrary members of a set are stated.

In order to obtain its results, abstract algebra is a subject that employs rules for symbol manipulation. These rules are applied to the same set of symbols. The same concept can be applied to NSA. However, there are different sets of symbols employed and, for the modern treatment, one very special form of manipulation. Further, one does not just manipulate symbols with no particular goal in mind, but, rather, there is a particular reason for doing so.

2. The Beginnings of the Symbolic World of NSA.

The first major introduction to NSA occurred in 1966 with the publication of Robinson’s book on the subject (Robinson, 1966). In order to not rely merely upon the customary axiom systems for specific mathematical theories, Robinson employed the notion of the “higher ordered structure” and a modification to a very general formal approach known as a “theory of types” (Robinson 1966, p. 19). These concepts are not part of the mainstream of mathematical discourse and its applications and few individuals have a background in formal type theory. In order to simplify this approach, a second method was introduced called “pseudo-set-theory.” Your author wrote his dissertation in pseudo-set-theory and published a few papers using this approach. Then, in 1969, a third approach was introduced that employs a standard form of set-theory and the formal logical discourse is that of the more familiar type taught in first courses in Mathematical Logic - the first-order predicate calculus with, at the least, enough constants.

Since most practical mathematics is treated axiomatically in such a manner that a slightly modified form of modern set-theory is sufficient, only two special predicate symbols are employed. They are the "element-hood" binary predicate $\in$ and the customary equality symbol $=$, with its symbolic properties. The $=$ is interpreted as the equality of set-theory and a form of logical identity for members of a foundational set. It is this third form, the superstructure approach, that almost always comprises the subject as of today. While this approach has greatly simplified the basic requirements, it has made one aspect somewhat more complex. Six distinct symbolic "languages" are necessary in order to properly discuss the details of NSA.

Relative to the usual set-theory axioms, there is an additional one. In order to apply NSA to numerously many informal mathematical and physical science subjects, there is selected a type of "starting" object upon which the models are built. This is called the set of "individuals" or "urelements" and, for the set-theory, the general term **atoms** is employed. This is a rather special nonempty set. The objects in it are not considered as sets and, hence, the reason for their special name. For a specific set of individuals H, if $r \in H$, then one cannot write $a \in r$ within this theory. Notice that from the language viewpoint all this means is that if a is a constant or variable then predicate $\in$ does not apply to a. Thus, you have a rule that states that a symbolic form cannot be used. Outside of this one special axiom, all the other axioms of ZF set-theory are employed and usually the axiom of choice, C, is added.

When it comes to set-theory, one uses the **structure** idea and "constants" as a "type of model." That is, nonempty sets are but composed of symbols. And what one does is to manipulate the symbols in accordance with the axioms. Of course, additional symbols are defined and they have their manipulation properties. Thus, when one writes that $a \in A \cap B$ one can merely "think of this" as meaning that the symbol a is intuitively contained in the set A and in the set B. The word "and" takes on the common meaning. Then one writes, via symbol manipulation, that $a \in A$ and $a \in B$. Hence, in an informal proof, one would write, "If $a \in A \cap B$, then $a \in A$ and $a \in B$."

When one introduces models into a first course in Mathematical Logic only rather concrete models are used. For example, you have a binary predicate denoted by $P(-,-)$. Then a formal axiom $\forall x \forall y \forall z (P(x,y) \wedge P(y,z) \wedge P(x,z))$. The P is considered as a **dedicated** constant. There is an axiom in formal first-order predicate calculus with constants that states that we can substitute into this formal statement, for the

x, y, z, any constants from the list of constants. Thus, it is logically legal to write $P(a,b) \wedge P(b,c) \wedge P(a,c)$. Abstract model theory, as presented in Herrmann (2006), can be based upon this replacement. You can also use an equivalent process for $\exists z$. The mathematical foundation is an informal nonempty set called the **domain** and this set has constants that name its elements. This P symbol corresponds to a binary relation and with its domain yields a **structure**.

Constants from the formal language are in one-to-one correspondence with the constants "names" for domain members. In Herrmann (2006), these new constant names are denoted by the language constants with an additional prime attached. However, in the literature, this approach is also used but with a slight difference.

Then, for a simple model, this is translated into simple set-theoretic terms. All one needs is one set over which the x, y, z are to vary. But, how, in general, are the domain constants denoted? For application to this form of logic, in Herrmann (2006), $P(a,b) \wedge P(b,c) \wedge P(a,c)$ is translated into $P'(a',b') \wedge P'(b',c') \wedge P'(a',c')$, a statement about members of D. The slight difference is that, elsewhere, the same constants are used in the structure and in the formal language. The $\forall$ is translated to mean that the statement holds in the structure as you vary named constants over the entire domain D. The $\wedge$ has the common meaning of "and." The structure is a **model** for the translated formal statement if it **informally holds** within the structure. The best way to understand this is by example since, in this case, to hold informally is a product of human thought.

In Herrmann (2006), the formal language constants for actual objects in the set-theory are not used since they are considered as something rather more "pure" in character. But on the other hand, this is all but types of symbol manipulation. As an example, consider the informal set of constant symbols $D = \{a'\}$. So, D contains but one constant. This set is good enough to model the formal statement if one selects the correct set of ordered pairs for the predicate. Let $P' = \{(a',a')\}$. This gives a structure $\langle D, P' \rangle$. And, it is a fact that $(a',a') \in P'$ and $(a',a') \in P'$ and $(a',a') \in P'$. (By-the-way, this is obtained via human observation which is a basic method used in order to prove a mathematical statement.)

However, is it possible that this is a model for the formal statement? Yes! The reason for this is that in the formal statement there is no statement that "$(x \neq y) \wedge (x \neq z) \wedge (y \neq z)$. The formal statement is written in this fashion only to accommodate, in another structure, possibility different values for the variables. This one simple structure shows that the statement, an hypothesis say, is consistent. Thus, in this case, $\langle D, P' \rangle$ is a model for the statement. This also indicates that when there is a finite combination of $\wedge$ and the appropriate number of $\forall$ and nothing more, then there is a structure for which such a statement holds.

Is there a structure that models the statement $\forall x \forall y(P(x,y) \wedge (\neg P(x,y))$, where $\neg$ is interpreted as "not"? Let domain D be a nonempty set as they all must be. One can argue as follows: suppose that there is a nonempty set P' of ordered pairs that correspond to P. Let $(a',b') \in P'$. (We do not assume that a' and b' are necessary distinct.) Then, it is a contradiction to state that $(a',b') \in P'$ and $(a',b') \notin P'$, (i.e. $\neg((a',b') \in P')$). (Note: I did not write that "observationally" this follows, although that is the only thing done to arrive at the conclusion.) This also holds if P$'$ is an empty relation. Thus, since no such set of ordered pairs exists, there is no possible P' and the statement cannot hold in any structure.

As a final example, consider the formal statement $\forall x \exists y(P(x,x) \to P(x,y))$. Let $D' = \{a'\}$. Notice that the "there exists y" means that we must have a nonempty set of ordered pairs to associate with P. Let $P' = \{(a',a')\}$. This is actually sufficient since there is no requirement that $\neg(x = y)$. And, it is true that for each x, in particular a', there exists a member of D, in particular a', such that $(a',a') \in P'$. Then the following informal statement, "If $(a',a') \in P'$, then $(a',a') \in P'$" holds. Hence, $\langle D', P' \rangle$ models this formal expression.

What is the must significant aspect of the first and third examples? It is the fact that, in each case, since the statement has a structure as a model, then any expression, with the appropriate number of quantifiers $\forall$, $\exists$, that is logically deduced from the one hypothesis using the rules for first-order predicate deduction will hold, be true, in the structure.

Although this model theory material does seem rather straightforward, it can become rather more complex when the structure used is generated by another mathematical theory with its own peculiar set of symbols. This is exactly what happens for NSA.

3. The Actual Set-Theory Used.

Thus far, the set-theory used has been rather informally presented. The actual set-theory uses two classes of variables the "sets," and the non-sets, the urelements, as well as two dedicated constants $\emptyset$, and A that name two special sets and two dedicated constant binary predicate symbols $\in$ and $=$. The variables for non-sets are all related to A in that when one writes $\exists w(w \in A)$ this signifies that in this form "w is a urelement variable and not a set variable. Thus, unless specific variables are assigned to the urelements, then, in any formal statement involving urelement variables, they must be identified as being $\in$ related to A. For set-theory, the predicates are not denoted in the usual way, but by $x \in y$ and $x = y$.

The actual set-theory does not require any additional formal constants and the properties of $\emptyset$ and $=$ are defined only in terms of variable symbols. For a basic definition, Jech, (1971, p. 122) restricts some variables. This is not necessary for NSA. In NSA, it is required that the formal statement variables be restricted by the formal statement itself. The quantifiers are required to be "bounded." For example, $\forall x(x \in X_n \to \cdots$. If $X_n \neq A$, then x is a set variable. If $X_n = A$, then x is a urelement. It can also be either if $x \in X_1 = X_0 \cup \mathcal{P}(X_0)$, where $\mathcal{P}(\ )$ is the set of all subsets of X_0. If $x \in X_0$, then it behaves like a urelement. If $x \in \mathcal{P}(X_0))$, then x behaves like a set. For sets, the $=$ is set equality. For urelements, $=$ is logical identity.

A standard superstructure is set-theoretically generated. For example, as indicated above, in one formulation for a particular "level," the general form is $X_{n+1} = X_n \cup \mathcal{P}(X_n)$. Each distinctly different set necessarily has one and only one distinct language constant name. In mathematical proofs and discussions, the ordinary language used to discuss formal languages is called the **meta-language**. In proofs or elsewhere, what may appear to be two different sets as denoted by two different meta-language informal symbols may in reality be the same set. This is established if they are shown to be equal. In this case, one of the symbols can be used as its formal name. If used, there is the $=$ notion relative to urelements. This should be considered as a type of logical identity, where informally, for clarity, two or more meta-langrage symbols have been used where but one may actually be the constant name.

A major difference is introduced into NSA by also allowing constant "names" to replace the variables under an allowed logic axiom for each member of the standard superstructure and using other types of names for members of the other superstructures. Since for the axioms of this type of set-theory only two non-predicate constants are employed, then this implies that for these structures one can consider the sets and urelements as modeled by mere sets of constants. These include the constants of the formal language itself.

4. The Actual Symbolic World of NSA.

NSA is an elaborate addition to ordinary mathematical discourse. But, just a few specific members of the basic structure employed in this mathematical theory are used for the GID-model. Consider an infinite set of individuals X_0, the atoms for the set-theory. Often this is intended to be a well known informal entity like the natural numbers $\mathbb{N}$, integers $\mathbf{Z}$, rational numbers $\mathbf{Q}$, the real numbers $\mathbb{R}$ or merely an infinite set. Such a selected set is called the **ground set**. From this, the present approach is to build, using the set-theory axioms, a collection of sets called a **superstructure** that includes almost all of the relations and the usually defined objects associated with this "ground" set that are employed throughout a particular axiomatically controlled mathematical theory. For example, number theory, various algebraic theories, real number theory (analysis), the calculus, complex number theory, general or algebraic topology and many more. These do not include the set-theory used to construct the superstructures.

It is not important that the technical aspects of this construction be given here. There is a one-to-one correspondence between each formal language constant and members of this superstructure. They are said to "name" the members of the superstructure. But, essentially what one has is actually but a specially constructed collection of sets of constants. For NSA, this is called the **standard superstructure** and each entity is a **standard entity**. The language constants form the **standard language** that names each constituent of this structure.

> When a formal statement is written using only the predicates $\in$ and $=$, variables and standard constants, then the most important special NSA rule for symbol manipulation is a back and forth process. One can change the expression into one where all of the standard constants are altered and conversely. For NSA, a formal statement, or its informal counterpart, holds if and only if the statement one obtains by altering the constants and only the constants holds for entities in a specially constructed **nonstandard structure**. This is called the *-transform (transfer) processes. This structure is contained within another superstructure, the Y in the model $\mathcal{Y} = \langle Y, \in, = \rangle$, which is called in Herrmann (1978 - 1993), the *Grundlegend Structure* or G-structure.

Notice that if we only used sets of constants and their modifications and nothing more, then the behavior of the constants holds if and only if the behavior of the modified constants holds. By construction, the specific standard superstructure is a model for almost all aspects of the theory being considered. Certain purely set-theoretic aspects, relative to the structure, are not modeled, however. The usual Roman-type alphabet symbols are primarily used as constants as well as all of the special symbols that have been developed for a particular theory.

Then the next steps in the construction of a nonstandard model for the theory being considered are rather complex and uses methods that are not part of an individual's usual educational experiences with mathematics. This includes graduate education in subjects other than mathematics and even within graduate courses given for advanced degrees in mathematics. The construction leads to a set of entities called the **internal entities**. *The language problem is that the new entities employed in the construction are, technically, can be extremely different from those entities originally named by the standard language constants.* Further, $\lambda \in Y$ is internal if and only if there is a set

${}^*X \in Y$, such that $\lambda \in {}^*X$.

> As mentioned, there is a one-to-one correspondence, the * correspondence, between the standard constants and the extended standard constants. *The special NSA rule, *-transform, states that any formal statement that holds in the standard structure also holds in $\mathcal{Y}$ upon replacing the standard constants with the corresponding * altered constants AND conversely.* For <u>finitely</u> expressed set-theoretic statements, this yields the rules for symbol manipulation. Relative to expressing theory behavior via the formal statements, there are also new language symbols used to name other members of Y that behave in the exact same manner as the standard entities.
>
> The * altered standard constant symbols are called the **extended standard**. These and new language symbols form the **internal language symbols.** If we were unable to analyze and compare expressions using the standard and internal symbols, then we would not recognize that, in comparison, there can be considerable differences in behavior being expressed. It is the comparative differences that, for NSA, yields the "nonstandard" behavior.

Thus, standard language symbols are in one-to-one correspondence with a subset of these internal language symbols. Symbolically, for each standard language symbol a, these new names are denoted by ${}^*a$. This is even so for the standard symbols like $\int$ and sin and cos, where these are *-transformed to ${}^*\!\int$, ${}^*\sin$, ${}^*\cos$ symbols. The * is informally called the star or hyper operator. But, there are a vast "number" of other internal entities distinct from the named extended standard ones, where some are named by symbols such as taken from the Greek alphabet and usually one states that they are for internal entities.

The type of construction employed has another property. Suppose that the ground set includes the natural numbers $\mathbb{N}$, or at least, a set that behaves like it. Then in ${}^*\mathbb{N}$ is an internal λ such that for any $n \in \mathbb{N}$, ${}^*n < \lambda$. And, clearly, $\lambda \notin \mathbb{N}$ for if it were we would have the contradiction that $\lambda < \lambda$ since by the *-transform process there are no members of ${}^*\mathbb{N}$ with this property. (Actually, I have used a certain convention in order to write it this way since the $<$ is actually an extension of the natural number relation $<$ and it satisfies all the modeled natural number properties if restricted to the extended standard symbols obtained from $\mathbb{N}$. Some authors would have written this as ${}^*<$.)

The rules for symbol manipulation include those that allow one to go from the standard symbols to the * symbols. Via *-transform, the rules are generally obtained as follows: take any standard statement that holds in the standard structure. Write it using only standard symbols without any variables. Express it only in a finite real or conceptually finite form. Then put a * on each of the constant language symbols. Symbols such as $\{,\ \}, (,\ )$ and the like are not standard constant symbols. For example, if one knows that $\{a, b, c, d\} \subset B$, then we know that ${}^*\{a, b, c, d\}$ means $\{{}^*a, {}^*b, {}^*c, {}^*d\} \subset {}^*B$. Why have I not writing ${}^*\subset$? The $\subset$ relation is defined by the $\in$ in the standard structure, and the construction of Y is of a very special type so that the $\in$ is the same throughout the entire superstructure. When *-transform is applied, the $\subset$ only refers to "internal sets." As mentioned, properly written formal standard statements always need to include a portion that specially states that the variables are members of a set in the standard superstructure. Sometimes, this requirement is missing but is to be "understood." The superstructure Y includes all the internal entities as well as many others not yet discussed. For an infinite B, there are subsets of ${}^*B$ that are "external" objects and some of these are identified below.

Then we have the sets that are carved out of the standard structure by "set-builder" notation. The same holds in that case also. For example, define $B = \{x \mid (x \in \mathbb{N}) \land (x > 6)\}$. Then ${}^*B = \{x \mid (x \in {}^*\mathbb{N}) \land (x > 6)\}$. It seems I may have just broken one of the rules. The totally correct set-builder definition for ${}^*B$ should have been ${}^*B = \{x \mid (x \in {}^*\mathbb{N}) \land (x \ {}^*\!> {}^*6)\}$. The reason for this is that $>$ is a standard name for a binary relation on the natural numbers $\mathbb{N}$ and 6 is a standard name for one of them. In the literature, this way of defining ${}^*B$ is retained. This alteration in what one should expect follows what mathematicians term as "conventions" (special rules).

The set $\mathbb{N}$ is a subset of the atoms A in our set-theory. So, let $a \in A$. Then the *-transform is ${}^*a \in {}^*A$ and conversely. By definition, the expression $b \in a$ has no meaning using the language of the set-theory. Hence, a set of symbols strings like ${}^*b \in {}^*a$ has no meaning in $\mathcal{Y}$, in this case. The superstructure Y is constructed within our set-theory from the set ${}^*A$. So for this superstructure, the ${}^*A$ behaves as we had hoped like a set of superstructure atoms. This is one reason that, by convention, most individuals who have worked in NSA simply use the older standard symbol for members of ${}^*A$, it being understood that they are not actual members of A. This seems to make A a subset of ${}^*A$ which, technically, it is not. This will be more fully explained below.

A convention is also used for various $<$ type names. This relation is defined for members of $\mathbb{N}$ in this case. So if we restrict the ${}^*\!<$ to them, then, by the previous convention, another convention drops the * notation. But, this does not explain why it is also dropped even if one or more of the objects to which ${}^*\!<$ applies are internal nonstandard objects with their internal names. This is yet another convention. Mathematically, under the previous convention, when restricted to the symbols used for members of what is now $\mathbb{N}$, $<$ is used rather than $^*<$. So, why not specialize this just a little bit more and not use $^*<$ for any members of $^*\mathbb{N}$ and then simply consider the $<$ as it is used for the now $\mathbb{N}$ looking members of $^*\mathbb{N}$ as but a restriction of $<$ relative to how the symbol is employed for $^*\mathbb{N}$. The $^*<$ relation has the same first-order properties on internal objects as $<$ has on standard objects. Note that each $^*n = n$, (the notational convention) is internal since $^*n \in {^*\mathbb{N}}$.

Is there a reasonable explanation why we might have added these conventions to the already large storehouse of new symbols and rules for how to manipulate them? What are the natural numbers? Although we investigate them by manipulating mere collections of symbols are we actually investigating a unique set of entities termed the natural numbers? In fact, natural numbers properties correspond to "anything" that satisfies a specific set of theory statements. Relative to collections of symbols, they are any collection of symbols that satisfies the axioms. If the axioms are stated in the form of variables, with maybe one or two dedicated constant, then the symbols form a model for these axioms. Are there collections of such symbols used to name members of the standard superstructure in the structure $\mathcal{Y}$ that do satisfy the standard structure natural number theory statements? The answer is yes and, in the literature, they may or may not be identified. They are just the collections of all of the extended standard language symbols, relative to $\mathbb{N}$.

The nonstandard structure is determined by means of the **ultrapower** construction using a Theorem such as 8.2 in Hurd and Loeb (1985). When used for comparison purposes these sets are defined as follows: let B be a member of the standard superstructure. Then for each $a \in B$, $[\overline{a}]$ is the equivalence class that contains the constant a sequence. Define ${^\sigma B} = \{[\overline{a}] \mid a \in B\}$. (Note: This definition for $^\sigma$ is not the one that appears in many of my writings. There I often define ${^\sigma B} = \{{^*a} \mid a \in B\}$.) If you do this for all of the atoms and sets in the standard superstructure and only consider these sets and of objects which are also members of Y, then the entire collection behaves like the corresponding named objects without the $^\sigma$ attached. *That is, they form a model within superstructure Y for statements that hold in the standard superstructure.* So this gives an indication as to why on the level of the atoms and relations relative to them this special convention is employed. It allows one to write a comparative statement such as $\mathbb{N} \subset {^*\mathbb{N}}$, $\mathbb{N} \neq {^*\mathbb{N}}$, where, technically, this means that ${^\sigma\mathbb{N}} \subset {^*\mathbb{N}}$, ${^\sigma\mathbb{N}} \neq {^*\mathbb{N}}$.

The structure $\mathcal{Y}$ has a special property which is identified by its name. The structure is a **polysaturated enlargement**. (This only refers to the degree of "saturation.") If B is any infinite set in Y, then ${^\sigma B}$ is an "external" object member of Y and an external subset of ${^*B}$ and ${^\sigma B} \neq {^*B}$. The conventions used allow for one to write for the (external) set of atoms $A \subset {^*A}$. I tend to follow these conventions.

On the other hand, to make comparisons within "higher-levels" within the superstructure Y, the $^\sigma$ notion is used. This does not invalidate the comparison relative to the original superstructure since one can merely consider this is a change in the names employed. Some authors simply construct their theorems and proofs so that such comparisons are not needed. Consider the following three statements.

(1) For each $y \in \mathbb{N}$, $\exists x((x \in \mathbb{N}) \wedge (x > y))$. (2) For each $y \in {^\sigma}\mathbb{N}$, $\exists x((x \in {^\sigma}\mathbb{N}) \wedge (x \ {^\sigma}\!\!> y))$. (3) For each $y \in {^*}\mathbb{N}$, $\exists x((x \in {^*}\mathbb{N}) \wedge (x \ {^*}\!\!> y))$ (without the convention). Each of these statements holds in their respective structures. It is statement (3) that, due to the construction of Y, leads to the further conclusion that there is a $\lambda \in {^*}\mathbb{N}$ such that for each $n \in {^\sigma}\mathbb{N}$, $\lambda \ {^*}\!\!> n$ or simply $\lambda > n$.

Thus, the major difference between NSA and other mathematical approaches is in its use of different sets of symbols that use the customary rules for their manipulation and new rules not seen before. There are two other sets of symbols not yet mentioned. After you remove all the internal language symbols there remains many more members of Y that one needs to name. They are named by the **external language symbols** or the **the symbols for the external entities.** Thus, there are internal objects in Y and all others are external objects. The Robinson set of infinitesimals and the set of infinite numbers are external objects in Y. But, how do we investigate the interplay between distinctly different members of Y? As mentioned, in mathematical logic, the ordinary language we use to discuss formal languages is the meta-language. This is what is done in this case. The meta-language one sees in NSA proofs, discussions, and definitions includes the other languages and all else that is needed.

The meta-language used to construct the superstructures and structures used for NSA is not used in this article. It is unnecessary if one trusts the mathematicians who have developed NSA. This is an example as to why such material does not appear here. "Set $\Pi^0_{\mathcal{U}} V(X) := \bigcup_{n=0}^{\infty} \Pi_{\mathcal{U}}[V_n(X) - V_{n-1}(X)]$. . . . When $[a],\ [b] \in \Pi^0_{\mathcal{U}}$, we write $[a] \in_{\mathcal{U}} [b]$ if $a_i \in b_i$ a.e."

Such an elaborate addition to mathematical discourse seems rather purposeless unless it gives results that yield significant results not obtainable by less than rigorous means. The most significant, if they exist, would be in areas of application to other disciplines. Although NSA solves the Newton-Leibniz problem, as well as others, and clarifies and improves comprehension as mathematics is applied to various physical areas, apparently these results and applications are not considered significant enough so as to replace the older well established methods. The General Grand Unification Model, its GID-model and the GD-model interpretations should be exceptions to this.

5. The Grundlegend-Deductive (GD)-model.

For non-mathematical applications, the term "model" is generalized to include a one-to-one correspondence between the mathematical symbols as they appear in a mathematical theory and meaningful terms taken from another discipline. Properly defined discipline items, represented by formal and informally defined sets, are also in one-to-one correspondence with mathematical symbols.

The first theological application of NSA is the Grundlegend-Deductive (GD)-model. This model is a comparative model for God's Old and than New Testament Biblically stated attributes. This was followed, in 1983, by a creationary interpretation for the NSA produced General Grand Unification Model (GGU-model). This solves the General Grand Unification Problem that was considered as unsolvable by members of the mathematics and physics departments of Princeton University. Applying NSA to theology yields the first mathematical models for various theological concepts and establishes there rationality.

The assignment of the abstract symbols to other symbols or terms within mathematical theories or non-mathematical disciplines is called, in general, an **interpretation**. The term "vector" as used in the theory "linear algebra" can take on many different interpretations. Vectors can be line segments with identified end points in geometry. They can be assigned the physical notions of a direction as coupled with a numerical value. They can yield significant physical measures in quantum physics. They can even take on terms from economics.

Hence, once one has the mathematical language used to produce the results contained within NSA, then unless one wishes to remain within its technical boundaries, one interpretes it in mathematical subjects such as real or complex analysis, or within other disciplines considered as exterior to mere symbolic manipulation.

The subject of Mathematical Logic is an application of metamathematics to the mathematical languages themselves, which are expressed formally. Thus, if one applies NSA to aspects of this subject there is a considerable clash in terminology. NSA has its languages and we are using that language to investigate other "languages." This can be more confusing if the concepts of **universal logic** are investigated by NSA since universal logic investigates the notion of languages <u>in general</u> not just those that use written alphabets or formal expressions. I term this as a **general language** and it includes, at least, all forms we use to communicate ideas in visual form. *Thus, when the NSA symbols are interpreted as entities related to a general language, different terms need to be used or understood and the interpreted symbols themselves should carry their interpreted meanings.* Further, the symbols used for the interpretations may be different in formal technical papers than employed for more informal presentations. This often occurs due to the lack of fonts.

> In physical modeling, it is often the case that the mathematical theory exists prior to an application. Then one begins with a specific discipline and assigns symbols from a mathematical theory to the discipline terms.

The application of NSA to a general language begins with an **informal model** using informal set-theory for items in very basic **word theory**. The symbols used for this interpretation are Roman fonts. Then this informal model is put into one-to-one correspondence with a more formally constructed mathematical theory. The corresponding symbols are written in bold font. Certain common mathematical entities such as the natural and rational numbers and customary symbolic representations for these retain their mathematics italics form. Further contextually, the same symbols are often used for the informal natural numbers, the rational numbers and the like as used for the formal standard structure. Thus $\mathbb{N}$ denotes the natural numbers in both contexts and, for example, $\mathrm{f}(i)$, as $i \in \mathbb{N}$, are the informal values of a sequence f.

Given a general language alphabet ALP, one constructs from ALP, Markov (1954) styled **words** by intuitively writing a finite list of such alphabet items, with repetition, from right-to-left. The alphabet ALP is for a general language. This yields the informal set of words, today technically denoted by W', - an **informal general language**. Notationally, W' is most often written as L. A single alphabet symbol is a word. Usually, however, words are considered as composed of finitely many words combined by the "juxtaposition" operator. Mathematically this operator yields an abstract algebraic structure termed a "monoid." I have a copy of the book of Matthew as transcribe in the Greek of 3'rd century. It contains neither punctuation, nor spaces between the words, the sentences, the paragraphs. It is composed of one "continuous" collection of old-styled capital Greek alphabet letters printed on 77 pages.

The informal language L was originally encoded as subsets of the standard natural numbers $\mathbb{N}$, which is a subset of the set of atoms. The entire NSA model prior to interpretation was technically a pure NSA theory of numbers. Recently, this has been modified to incorporate, if one wishes to do so, a Robinson approach, where W' is consider as a subset of the set of atoms. The ground set for the standard superstructure is, at least, the set $\mathrm{W}' \cup \mathbf{Q}$, a subset of the set of atoms, where $\mathbf{Q}$ is considered as the set of rational numbers. The properties that identify $\mathbf{Q}$ as such a set are contained in rather immediate superstructure levels. The $\mathbf{Q}$ is used to identify GGU-model events.

So as to incorporate the fact that various combinations of words yield the exact same word, members of W' are encoded in such a manner to preserve this fact and embedded into the standard superstructure. Under this embedding, the "language" is denoted by $\underline{\mathcal{W}}'$ or **L**. Any symbolized object within word theory, K, that is so embedded is symbolized by a bold **K** and the same terminology used in the informal theory is used for this standard embedding. Thus, for a word w ∈ L, $\mathbf{w} \in \mathbf{L}$ denotes the corresponding word in the corresponding embedded general language.

Consider the term "intelligent." Such a word can be modified by the word "very," or a similar word, to indicate a strength of intelligence when compared to others. If an entity is considered as intelligent, then a second entity, in comparison, can be considered as "very intelligent," then a third one can be considered as "very, very intelligent." In these forms a small logical process is must often applied. "Very, very intelligent" implies "very intelligent," which implies "intelligent." The pattern is similar to how we compare numbers such as $\{0, 1, 2\}$, $2 > 1 > 0$, and $2 > 0$, if one uses these numbers to count the number of "very" strings placed to the left of the attribute. This form of reasoning is termed as **adjective reasoning** and is an actual restriction of our most basic model for deductive thought, propositional deduction. Other constructions, such as "much, much more", are also employed that have the same intuitive meaning.

For the GD-model, one selects other nouns that characterize such comparative human attributes that can be so modified by the word "very" or equivalent. Then all of these finitely long words and an operator that mimics this reasoning process are embedded into the standard superstructure. The standard language that represents each of these constructed words is a subset of **L**. The entire embedded collection of all such words is denoted by **BP**, where **b** is the basic attribute, and the **better than** or **stronger than** ordering applies to those words that have the same attribute. This ordering, denoted by $\leq_B$, is defined by the ordering of the natural numbers, where the number is the number of "very" strings attached to the left of an attribute.

In order to informally interprete Theorem 4.4.1 in Herrmann (1978-1993), a term is employed that is intuitively considered to indicate that there are "infinitely many" "very" strings attached to an attribute **b**. The term used is **ultraword**. For a particular $\mathbf{b} \in \mathbf{BP}$, there is an ultraword $c \in {}^*\mathbf{L}$ such that for each $\mathbf{a} \in \mathbf{BP}$, where $\mathbf{b} \leq_B \mathbf{a}$, the ultraword c has the property that ${}^*\mathbf{a} = \mathbf{a} \, {}^*\!\leq_B c$. Notice that c is written in a mathematics italic font. It is an "hyperfinite" infinitely long word representable by an infinite set $[f]$ that is an internal object, and it is not representable by an extended standard symbol. This follows from the fact that $c \in {}^*C_b$ and each object in C_b is a finite set. Hence, if ${}^*\mathbf{a} \in {}^*C_b$ and $c = {}^*\mathbf{a}$, then $c = {}^*\mathbf{a} = \mathbf{a}$ is a finite set; a contradiction

Hence, informally (intuitively), from how increasingly stronger attributes are described by adding "very" strings to an attribute, an ultraword description is predicted that describes an attribute that is infinitely stronger than any such attribute used to characterize any biological entity. If one considers such an "infinite" notation as indicating the size of a set, than it has been shown that the size of this infinite is greater than the size of any such infinite measure for any member of the standard superstructure. The mathematical measure for it is "outside" of the superstructure, so to speak. Such predicted hyper-attributes, higher-attributes, yield a partial description for God's corresponding Biblical attributes. The "Omni" characterizing attributes are also rationally predicted. Recently, (Herrmann (2014), it has been shown that this notion of the "infinite" is actually better described as "larger yet" and is actually unlimited and mathematically immeasurable. Additional Divine characteristics, such as distinct tri-category characteristics, are predicted after equation (2) in Herrmann (2013a).

> (1) In summary, it is predicted that God's attributes are infinitely more powerful, stronger than or greater than comparable human attributes. Further, the "Omni" attributes and distinct tri-category characteristics are rationally predicted.

6. The General Intelligent Design (GID) Model.

In Herrmann (2013), GGU-model processes are used to predict five universe generating schemes. These schemes are termed as the **secular model**. For the single-complexity universe, there are four schemes and each displays signatures for intelligent design by an higher-intelligence. The "strongest" display is exhibited by schemes (S) and (S'). As pointed out at the end of Section 7, in Herrmann (2013), these mathematically represented schemes yield analogue behavioral models for processes relative to intelligent design by an higher-intelligence.

It is predicted that within a non-physical substratum world there exists entities called **ultra-propertons** that have properties but they should not actually be visualized as some sort of non-physical object. An **info-field** is a specific and unique combination of ultra-propertons. A single process applied to an info-field, the realization process, yields a physical reality. Various aspects of the secular model are translatable using the language of intelligent design.

The major GID-model intelligent design statements are translations taken from Herrmann (2013b, 2002). The following statements are translations of the material that appears in Herrmann (2014a, 2013, 2013b, 2002) for the strongest display of intelligent design for a single complexity universe. They employ the previous "intuitive" notion of an infinitely "long" ultraword based upon comprehensible finitely long members of **L**. These are coupled with an infinite form of higher-intelligence deduction that is predicted from the simplest linguistically modeled form of human deduction as used within the physical science community and throughout our daily lives. This form is as follows: suppose that P and Q are meaningful phrases. Then P, Q $\in$ L and the sentence If P, then Q. $\in$ L. The form of deduction is: given "P" and "if P, then Q." then "Q" is deduced.

In the following, some of the symbol names for certain entities are employed.

(1) Within our universe, every physical entity and the behavior of every physical combination of these entities are intelligently designed by a higher-intelligence.

(2) The designs are produced by a predicted infinite form of measurable intelligence and they satisfy the known and verified physical laws.

(3) Within our universe, every physical entity and the behavior of every physical combination of these entities is indirect evidence for the existence of a Biblically described creator.

(4) There exists a predicted higher-intelligence designed ultraword, $W \in {}^*\mathbf{L}$, that contains completely detailed descriptions, ${}^*\mathbf{f}(i,j) \in {}^*\mathbf{L}$, for each specific (i,j) identified universe-wide slice of a universe (a universe-wide frozen-frame) at each (i,j) moment in its development. This collection of descriptions is called a *developmental paradigm.

(5) Associated with W, there exists a second higher-intelligence designed ultraword, $W_I \in {}^*\mathbf{L}$, that contains specific instructions as to how to reproduce each designed ${}^*\mathbf{f}(i,j)$ via distinct info-fields $IF(i,j)$. The W_I is called an *instruction paradigm. (Note: The * is usually translated by the term "hyper.") The designed ultrawords display an higher-form of rational behavior.

(6) It is predicted, from the simplest form of modeled human deduction $\mathcal{A}$, that there exists a higher-form of deduction ${}^*\mathcal{A}$ such that, when ${}^*\mathcal{A}$ is applied to either W or W_I, each ${}^*\mathbf{f}(i,j)$ or $IF(i,j)$ is *deduced in the correct (i,j) order, respectively.

(7) Each component of each of the five secular model schemes is intelligently designed by a higher-intelligence.

(8) For each moment (i,j) in the development of a physical universe, application of the realization process, $\mathtt{St}$, to each $IF(i,j)$ yields the physical-systems contained within a specific (i,j)-universe-wide frozen-frame. The $\mathtt{St}$ process has an intelligent agency signature.

(9) The ultrawords correspond to higher-intelligence thoughts that are transformed into a physical reality.

(10) For a participator universe, where participators such as us can seemly alter a development, there are infinitely many such ultrawords that allow for such altered universes to be produced in the correct step-by-step order (Herrmann, (2014a).

For the weakest scheme for the development of a universe, (PWM), only the (generalized) sequences of identified info-fields $IF(i,j)$ exist. The intelligent design is displayed for every physical-system within each specific universe-wide frozen-frame as it is produced, via the realization process, by the refined method used within the GGU-model to design each info-field. That is, each physical-system within a specific universe-wide frozen-frame is intelligently designed by a higher-intelligence and the design is displayed as each info-field is realized.

References

Herrmann, R. A., (2014). The strengths of superstructure modeled divine attributes and an ultra-infinite bound, http://vixra.org/abs/1407.0052

Herrmann, R. A., (2014a). The participator model, http://vixra.org/abs/1410.0181

Herrmann, R. A., (2013). Nonstandard ultra-logic-systems applied to the GGU-model, http://vixra.org/abs/1308.0125

Herrmann, R. A., (2013a). Modeling God's attributes and the Biblical God is a scientifically rational concept, http://vixra.org/abs/1312.0203

Herrmann, R. A., (2013b). GGU-model ultra-logic-systems applied to developmental paradigms, http://vixra.org/abs/1309.0004

Herrmann, R. A., (2006). Logic for Everyone, http://arxiv.org/abs/math.GM/0601709

Herrmann, R. A., (2002), Science Declares Our Universes IS Intelligently Designed, Xulon Press, Fairfax, VA and other addresses.

Herrmann, R. A., (1978-1993), The Theory of Ultralogics,
Part I, http://arxiv.org/abs/math/9903081
Part II, http://arxiv.org/abs/math/9903082 and book 3 in
The newest version is found at http://vixra.org/abs/1606.0160 Book I of III,
http://vixra.org/abs/1606.0159 Book II of III,
http://vixra.org/abs/1606.0158 Book III of III

Hurd, A. E. and Loeb, P. A., (1985), *An Introduction to Nonstandard Real Analysis,* Academic Press, New York

Jech, T., (1971). Lectures in Set Theory with Particular Emphasis on the Method of Forcing, # 217 in Lecture Notes in Mathematics, Springer-Verlag, New York.

Loeb, P. A. and M. Wolff. (2000). Nonstandard Analysis for the Working Mathematician, Kluwer Academic Publishers, Dordrecht.

Markov, A. A., (1954). The Theory of Algorithms, Tr. Mat. Inst. Steklov, XLII. (Translated: Office of Technical Services, U. S. Department of Commerce, 1962.)

Robinson, A., (1966). Non-Standard Analysis, North-Holland Publishing Co, Amsterdam.