Cosmological Redshift and Gravitational Waves

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Gravitational waves cause the curvature of space. The curvatures of space lead to the conclusion that the nature of cosmological redshift and time dilation, is the same effect as the gravitational redshift and time dilation in a gravitational field. This is confirmed by the fact that the ratio between redshift and time dilation is the same for both gravitational and cosmological redshifts.

The article Redshift and the Curvature of [?] states that if the space is curved, then the nature of the cosmological redshift and time dilation is the same effect as the gravitational redshift and time dilation in a gravitational field. However, the nature of the curvature of space is not discussed.

It is known that gravitational waves cause space curvature. However, there is currently no commonly held opinion on whether the space around us is curved, and in what way. The most detailed study of the curvature of space by gravitational waves is done by C.W. Misner, K.S. Thorne and J.A. Wheeler in Gravitation [?]. In 35.13 the equation (35.60) illustrates the manner in which energy-momentum of waves causes background curvature.

$$G^{(B)}{}_{\mu\nu} \equiv R^{(B)}{}_{\mu\nu} - \frac{1}{2} R^{(B)} g^{(B)}{}_{\mu\nu} = 8\pi T^{(GW)}{}_{\mu\nu} \tag{1}$$

Where $T^{(GW)}{}_{\mu\nu}$ is the tensor of energy-momentum of the gravitational waves. In approximating a flat wave, within a few wavelengths (λ) relative to the observer, at $R >> \lambda$, the tensor of the energy-momentum (equation (35.27)) equals (in the z direction)

$$T^{(GW)}{}_{tt} = T^{(GW)}{}_{zz} = -T^{(GW)}{}_{tz} = \frac{1}{32\pi} w^2 (|A_+|^2 + |A_\times|^2) \tag{2}$$

For two bodies of mass m_1,m_2 , revolving around a common centre of mass, the amplitude of the radiating gravitational wave can be described using polar coordinates

$$A_+ = -\frac{1}{R} \frac{G^2}{c^4} \frac{2m_1 m_2}{r} (1 + cos^2\theta) cos[2\omega(t - R)] \tag{3}$$

$$A_\times = -\frac{1}{R} \frac{G^2}{c^4} \frac{4m_1 m_2}{r} (cos\theta) sin[2\omega(t - R)] \tag{4}$$

Where the angle θ - is the angle between the perpendicular to the surface of the orbit and the observer's line of vision and ω - the angular velocity. The observer is outside of the system at a distance R from the centre-of-mass-system. Moreover $R >> r$ and $R >> \lambda$ where λ is the wavelength.

Averaging-out across all direction and time, the sum of squares of the amplitude will equal

$$({A_+}^2 + {A_\times}^2) = \frac{1}{R^2} \frac{G^4}{c^8} \frac{4m_1 m_2}{r^2} \tag{5}$$

I.e. for the source of the gravitational waves, the sum of squares of the wave amplitude, radiating in any direction, is on average inversely proportional to the square of the distance from the source.

And, the average gravitational potential at a distance R from the source can be written as

$$\varphi_e = \frac{1}{R^2} \varphi_{00} \tag{6}$$

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where φ_{00} is an equivalent to the gravitational potential of the source, conditioned by radiation.

Suppose that all space is uniformly filled with identical sources of gravitational waves.

The gravitational potential relative to the observer at a distance R :

$$\varphi_R = \int_0^R \varphi_e(r)\rho dV \tag{7}$$

where ρ is the density of sources

I.e. the task is reduced to the classic determination of the gravitational potential of a sphere, only in the classic task the potential of a unit volume relative to the observer is the same for every point in space.

As a result of solving the equation (7) we get the distribution of the gravitational potential relative to the observer

$$\varphi_R = R\varphi_0 \tag{8}$$

where φ_0 proportional to φ_{00} and the density (amount within the unit of volume) of the sources.

As the gravitational waves influence matter and fields, it will experience attenuation and φ_R will not reach infinity. Notably, the attenuation, i.e. the conversion of the energy of the wave into other forms, is not necessarily linearly dependant from the distance (for instance as a result of the accumulation of nonlinearities).

Every theory of a stationary universe must preview the attenuation of gravitational waves for the provision of stability of the universe, and a uniform distribution of matter in it. This is pertinent to every other theory, otherwise almost all the energy in the universe will in the end transform into gravitational waves, which would escape into infinity, and the end of such a universe will be a small amount of stationary pieces of stone.

Accordingly, for the distribution of the gravitational potential (8), an equation can be created of a metric in a spherical, symmetrical form, in relation to the observer

$$ds^2 = -(1 - Kr)dt^2 + (1 - Kr)^{-1}dr^2 + r^2(d\theta^2 + sin^2\theta d\phi^2) \tag{9}$$

where K proportional to φ_0

The gravitational redshift, conditioned by the change in the gravitational potential, with a removal of a photon from the massive object of mass M equals to:

$$Z_G = (\lambda - \lambda_0)/\lambda_0 = GM/c^2r_g - GM/c^2R_g = (\Delta\varphi)/c^2 \tag{10}$$

Where Z_G is the gravitational redshift, λ is the photon wavelength at the point of observation, λ_0 is the photon wavelength at the point of radiation, r_g is the radial distance from the centre of mass of the body to the point of radiation, R_g is the radial distance from the centre of mass of the body to the point of observation.

Let us introduce two points in space: We are point A. We observe the photons from point B. Point B, according to (8) has the gravitational potential of $\varphi_b = \varphi_0 R$. Point A has the gravitational potential of $\varphi_a = 0$. The gravitational potential for the photon that travelled from $B(\varphi_b)$ to $A(\varphi_a)$ from the point of view of point A will decrease. As a result of this, we get the gravitational redshift

$$Z = (\Delta\varphi)/c^2 = \varphi_0 R/c^2 \tag{11}$$

And time dilation in point B relative to point A

$$t_1 = t_0(1 + \varphi_0 R/c^2) \tag{12}$$

which completely aligns with the present correlation between the cosmological red Z and time dilation

$$\Delta t_1 = (1 + Z)\Delta t_0 \tag{13}$$

for far-away galaxies.

From Hubble's law

$$cZ = H_0 D \tag{14}$$

where Z is the redshift of the galaxy, $D$is the distance to the galaxy, H_0 isHubble's constant and with the equation (9) we get a coefficient φ_0 (taking into account that $R = D$)

$$\varphi_0 = cH_0 \tag{15}$$

References

1. Valery Zhukov, Redshift and the Curvature of Space. http://vixra.org/abs/1406.0021 .
2. C.W. Misner, K.S. Thorne and J.A. Wheeler, Gravitation (W.H. Freeman & Company, San Francisco 1972).