

Deduction of orbital velocities in disk galaxies.

or: “Dark Matter”: a myth?

by using Gravitomagnetism.

T. De Mees - thierrydemees@pandora.be

Summary

In my paper “*A coherent dual vector field theory for gravitation*” is explained how simply the Gravitation Theory of Newton can be extended by transposing the Maxwell Electromagnetism into Gravitation. There exists indeed a second field, which can be called: co-gravitation-, Gyrotation- (which I prefer), gravito-magnetic field and so on. In this paper, I will call this global theory the Maxwell Analogy for Gravitation (MAG) “Gyro-Gravitation”.

One of the many consequences of this Gyro-Gravitation Theory that I have written down, is that Dark Matter does not exist. At least far not in the quantities that someones expect, but rather in marginalized quantities. Many researchers suppose that disk galaxies cannot subsist without missing mass that, apparently, is invisible, and which has to be taken into account in the classic Newton-Kepler model to better explain the disk galaxies' shapes.

An remarkable point is that Gyro-gravitation Theory is not only very close to GRT, but more important, easy to calculate with, and coherent with Electromagnetism. It is no coincidence that nobody found the same result with GRT, not because GRT would obtain some other result, but because it is almost impossible to calculate with it.

A demonstration is again given in this paper, where I deduce the general equations for the orbital velocities of stars in disk galaxies, based on the assumption of a simple mass distribution of the initial spherical galaxy.

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1. Pro Memore : Symbols, basic equations and philosophy.

1.1 Maxwell Analogy Equations in short – The two fields.

The formulas (1.1) to (1.5) form a coherent set of equations, similar to the Maxwell equations. The electrical charge q is substituted by the mass m , the magnetic field \mathbf{B} by the *Gyrotation* $\mathbf{\Omega}$, and the respective constants as well are substituted (the gravitation acceleration is written as \mathbf{g} and the universal gravitation constant as $\mathbf{G} = (4\pi \zeta)^{-1}$). We use sign \Leftarrow instead of $=$ because the right hand of the equation induces the left hand. This sign \Leftarrow will be used when we want to insist on the induction property in the equation. \mathbf{F} is the induced force, \mathbf{v} the velocity of mass m with density ρ . The operator \times symbolizes the cross product of vectors. Vectors are written in bold.

$$\mathbf{F} \Leftarrow m (\mathbf{g} + \mathbf{v} \times \mathbf{\Omega}) \quad (1.1)$$

$$\nabla \mathbf{g} \Leftarrow \rho / \zeta \quad (1.2)$$

$$c^2 \nabla \times \mathbf{\Omega} \Leftarrow \mathbf{j} / \zeta + \partial \mathbf{g} / \partial t \quad (1.3)$$

where \mathbf{j} is the flow of mass through a surface. The term $\partial \mathbf{g} / \partial t$ is added for the same reasons as Maxwell did: the compliance of the formula (1.3) with the equation :

$$\text{div } \mathbf{j} \Leftarrow - \partial \rho / \partial t$$

It is also expected

$$\text{div } \mathbf{\Omega} \equiv \nabla \cdot \mathbf{\Omega} = 0 \quad (1.4)$$

and

$$\nabla \times \mathbf{g} \Leftarrow - \partial \mathbf{\Omega} / \partial t \quad (1.5)$$

All applications of the electromagnetism can from then on be applied on the *gyrogravitation* with caution. Also it is possible to speak of gyrogravitation waves.

1.2 The definition of absolute local velocity – The velocities are not relativistic.

When it comes to a competition between GRT and MAG, attention should be paid to two very important differences.

The first one is that the actual MAG that I use is not really relativistic (although one could speak of semi-relativistic; I prefer to speak of Dopplerian). It works like the Newton and the Kepler theories, and like non-relativistic Electromagnetism.

Newton and Kepler did not see that the second field existed, caused by the second term in $\mathbf{G} m m' (1 + v^2/c^2)/r^2$. This expression is namely the simplest form for the Gyro-gravitation forces, and it is applicable between two identical moving masses in one dimension of place (see “*A coherent dual vector field theory for gravitation*”, last chapter).

This second term, which is very small and which is -by the way- often wrongly seen as an expression related to relativistic phenomena (I would rather say: transversal Doppler-effects), was not observed at that era. The relation to Doppler-effects will not further be discussed in this paper.

The extension of the theory for very fast velocities in non-steady systems has been settled by *Oleg Jefimenko* in several of his books, and is very analogical to what is called “relativistic electromagnetism”, where the field retardation -due to the finite velocity of gravitation- has been taken into account.

The consequence of this first difference is that in the framework of MAG, we should only study the kinds of steady systems, wherein the retardation of the fields, due to their finite velocity, is not of any crucial importance.

The second difference is that absolute velocity really exists. Not “absolute” with regard of the “centre” of our Universe, but “locally absolute” in the observed system wherein the forces interact within a given time-period. This means that the solar system can be studied as a closed system for “short” time periods of several years. However, I found that Mercury's perihelion advance is induced by the sun's motion in the Milky Way (see “*Did*”).

Einstein cheat ?”). Also the solar system, together with its motion in the Milky Way, can be seen as a closed system too.

When the system of our Milky Way is considered, there is no need to also consider the cluster wherein our Milky Way is just a tiny part of, etc.

Without much more explanations, you feel already what I mean by “local absolute velocity”.

One of the facets is indeed the place- and time-magnitude of what is to be observed or to be calculated. Yes, that magnitude can be 'the quantity of elapsed time' for that particular system as well. The gyrotation part of Mercury's perihelion advance is only visible after many years compared with the very visible gravitational orbital motions of the system.

The correct way to settle it, is to understand that each gravitation field of any particle can be seen as the *local absolute velocity* zero in relation to all the other particles. Not the observer can be at an absolute local velocity of zero, unless he is a dynamic player in the system with a significant mass. Each motion of one body will generate the gyrotation field onto any other body of the system and vice-versa. This means that in a moving two-body-system (without any other body in the universe), we have to consider the gravitation centre of the bodies as the zero velocity of the system, just as we used to for Newtonian systems, in high school. And every rotational motion of each particle plays a role in the gyrotation calculation of the system.

2. Why do some scientists claim the existence of “dark matter”?

2.1 The orbital velocity of stars in a disk galaxy – The velocities are constant.

One of the mysteries of the cosmos is the discovery that in disk galaxies, the velocity of the stars of the disk is almost constant. The Milky way characteristics are shown in Fig. 1 (from Burton 1976 Ann. Rev. 14, 275, shown from the ADS).

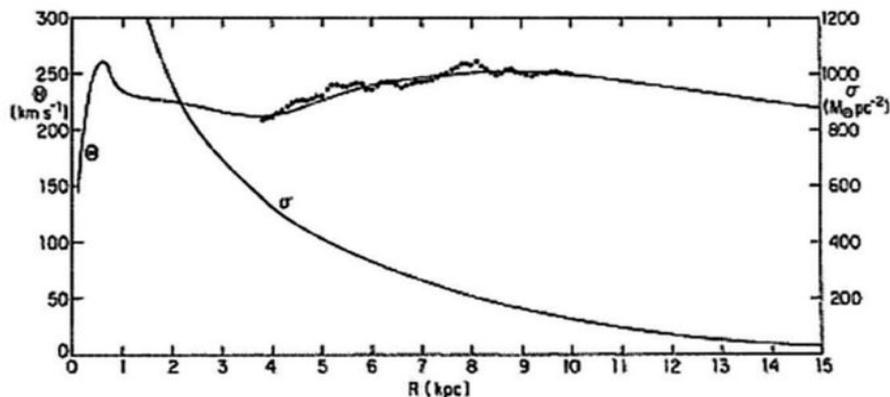


Figure 2 Variation with distance from the galactic center of the linear velocity of differential rotation, $\Theta(R)$, according to Simonson & Mader (1973) at $R < 5$ kpc and according to Schmidt (1965) at $R > 5$ kpc, and of the corresponding total galactic mass surface density, $\sigma(R)$, according to Innanen (1973). The dots show the rotational velocities found from H I observations of the subcentral-point region by Shane & Bieger-Smith (1966).

Fig. 2.1.

The linear velocity of the stars is given by the curve $\Theta(R)$ and is fairly constant from the distance of 1 kpc from the centre on. The curve $\sigma(R)$ represents the observed mass surface density. This curve is smooth and resemble a hyperbolic function. Much discussion exist on the correctness of curve $\sigma(R)$ because of the very high luminosity of accretion disks nearby black holes, which give a high apparent mass that is not in correct relation with their real mass content.

In Fig. 2. some other velocities are shown of several other disk galaxies (from Rubin, Ford, and Thonnard 1978 ApJL 225, L107, reproduced courtesy of the AAS). In general, we can say that the velocity of the stars is fairly constant, beginning at a distance of 2 or 3 kpc.

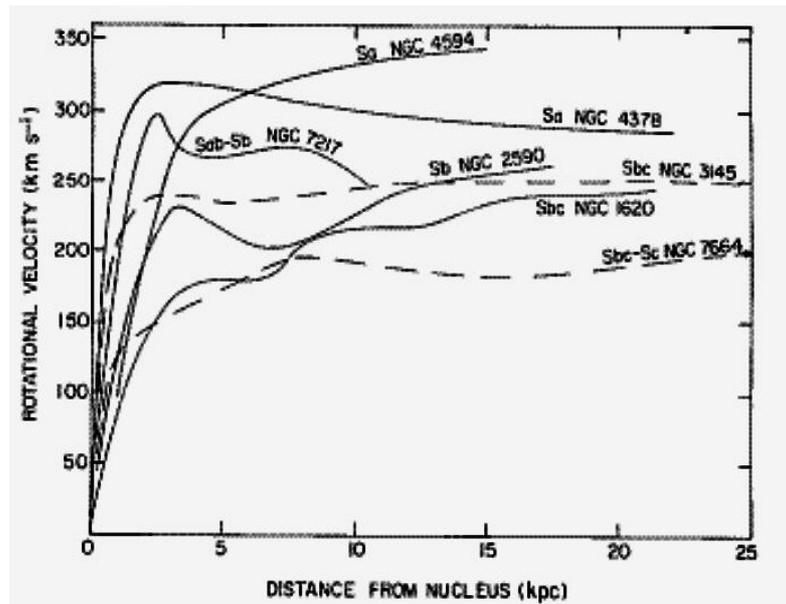


Fig. 2.2.

Rotational velocities of stars in several disk galaxies. Most of them have a similar graphic: a fast, almost linear increase near the nucleus, a small collapse of the velocity before 5 kpc, and a stabilization in the disk at (nearly) one single velocity.

The centre of the bulge has no specific (average) velocity, which result in a zero velocity on the figure. The first part of the disk outside the bulge, at nearly 2,5 kpc has often gotten a some higher velocity. And over 4 kpc, the velocity is almost linear, sometimes sinusoidal. Often, this linearity is almost constant or stays in a short range of values.

2.2 What did Kepler claim ? – The velocities decrease with the distance.

In a planetary system as the solar system, the planets follow a quite simple rule. The square of the orbit velocity of the planet is inversely proportional to its distance from the sun. This law has been written down by Kepler.

$$v^2 = G M / r \quad (2.1)$$

For low velocities, this law is correct and can be applied in this paper as such, even if the correct equation for higher velocities is somewhat different, as I explained in “*On the orbital velocities nearby rotary stars and black holes*”, in chapter 3, equation (3.10).

By increasing distances from the sun, planets will rapidly decrease its orbit velocity. And this law is nothing more than a geometrical one.

There is no *a priori* reason that the same law wouldn't be true for stars in a galaxy. But reality is different ! Equation (2.1) is extremely different from what is observed in galaxies.

The purpose of this paper is to find out why this is so.

2.3 Is there a way to get the Kepler law working? – The easy hypothesis: Missing Mass

There is a logical problem, and it should be solved logically. Thus, in order to get disk galaxies complying with Kepler's Law, what could be different that we cannot see? Galaxies and stars in general are observed, and classified by its distance to us, their weight, their motion in relation to us and so on. For long time, we only had light as sole measuring instrument to define all these properties. Since a few decades, this has been extended by waves of other frequencies than just light: X-rays.

But still, the method is very uncertain if masses are not bright, but cold.

At the other hand, the Kepler Law and Newton's laws only got two variables: mass and distance. The universal gravitation constant could be variable too, but until now, no evidence has been found for this.

because the radial component of the gyrotation force is small as well. That component will only slightly change the apparent mass of the planet, compared with its velocity and its orbit radius. The relationship between these parameters is given in my paper “*On the orbital velocities nearby rotary stars and black holes*”, chapter 3, equation (3.10), admitting that the orbit radius remains quasi constant.

When the planet was originally orbiting in retrograde direction, the gyrotation force will push the planet away from the sun's equator. Since the orbit's radius will only change very slightly during this orbital swivelling, the swivelling will continue until the entire orbit becomes prograde, and further converge to the sun's equator.

3.2 Equations for the accelerations nearby spinning stars.

In former papers, we found the equations for the accelerations upon an orbiting object about a spinning star, due to the gravitation and gyrotation fields. The orbit here is not forming a plane that is going through the star's origin, but an orbit that is parallel to the star's equator. The reason for that choice will follow further on.

$$\mathbf{a}_{x, \text{tot}} = -\frac{3G m \omega \omega' R^2 \sin^2 \alpha}{5 r^2 c^2} - \frac{G m \cos \alpha}{r^2} \quad (3.1)$$

$$\mathbf{a}_{y, \text{tot}} = -\frac{3G m \omega \omega' R^2 \sin \alpha \cos^2 \alpha}{5 r^2 c^2} - \frac{G m \sin \alpha}{r^2} \quad (3.2)$$

These can be written in the more adequate formulation in relation to the radial and the tangential components of the gyrotational part :

$$\mathbf{a}_r = -\frac{G m R^2 \omega \omega' \cos^2 \alpha}{5 r^2 c^2} \quad (3.3)$$

$$\mathbf{a}_t = \frac{G m \omega \omega' R^2}{5 r^2 c^2} \sin 2\alpha \quad (3.4)$$

R is the star's radius, m the star's mass and ω the spinning velocity of the star; α is the angle between the star's equator and the considered point \mathbf{p} , ω' the orbit angular velocity of the point \mathbf{p} (the parallel-orbiting object) and r the distance from point \mathbf{p} to the star's centre; c is the light's speed and G the universal gravitation constant.

4. From a spheric galaxy to a disk galaxy with constant stars' velocity.

4.1 The global stars' velocity in disk galaxies.

Relationship between the spherical and the disk galaxy.

We have to consider some other facts before we go for an analysis of the stars' velocities in the disk galaxy: we need a reconstruction of the original spherical galaxy. And we analyse the disk part of the disk galaxy as well.

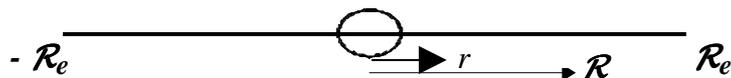


fig. 4.1

The schematic view of a disk galaxy with radius R_e . The bulge is nearly a sphere or an ellipsoid. The bulge area, the disk and the fuzzy ends are studied separately. R is the considered place, r is the variable place (for integration).

In fig. 4.1, we show the schematics of a disk galaxy, with the fuzzy ends of the disk $-\mathcal{R}_e$ and \mathcal{R}_e , and with the fuzzy bulge. The considered place \mathbf{p} is at a distance \mathcal{R} from the galaxy's centre. The variable \mathbf{r} is used for integration purposes.

When we call the spherical galaxy “1” and the disk galaxy “2” the following infinitesimal volumes are :

$$\mathbf{d}V_2 = 2\pi r h \mathbf{d}r \quad \text{and} \quad \mathbf{d}V_1 = 4\pi r^2 \mathbf{d}r$$

Since for every concentric location \mathbf{r} with the respective volumes of cases “1” and “2” we can say that $\mathbf{d}M_1 = \mathbf{d}M_2$ (because only the densities and the volumes got changed), it follows that $\rho_1 \mathbf{d}V_1 = \rho_2 \mathbf{d}V_2$

or :

$$\rho_1 = \frac{\rho_2 h}{2r} \quad (4.1)$$

The spherical density distribution is given by $\rho_1(\mathbf{r}) = \frac{3M_1(\mathbf{r})}{4\pi r^3}$ by definition,

or :

$$\mathbf{d}M_1(\mathbf{r}) = 4\pi r^2 \rho_1(\mathbf{r}) \mathbf{d}r .$$

While the expression for the disk galaxy's mass is : $\mathbf{d}M_2 = 2\pi r \rho_2(\mathbf{r}) h(\mathbf{r}) \mathbf{d}r$.

In order to fix the ideas, we go further and we simplify as follows.

Idealizing and simplifying the gravitational part.

The value of $M_1(\mathbf{r})$ can be found by assuming that the density distribution of the original spherical galaxy responds to a simple formula. We could sensibly simplify our analysis by assuming that for every concentric part of the spherical galaxy is valid that :

$$\frac{\mathbf{d}M_1(\mathbf{r})}{\mathbf{d}r} = \text{constant} = \frac{M_0}{R_0} \quad (4.2)$$

wherein M_0 and R_0 are the total mass and the radius of the bulge. This choice is only made in order to get simpler results. Besides, such a relationship is not totally unexpected: when we look at a spherical galaxy as a succession of spherical layers that have the same thickness, from the bulge to the “end” of the galaxy, we can expect that the masses could possibly be equal for each layer. The volume of each layer increases dramatically while the mass for each layer stays the same. At the “end” of the galaxy, the density decreases dramatically as well.

Combining (4.1) and (4.2), we get for the disk galaxy :

$$\rho_2(\mathbf{r}) = \frac{M_0}{2\pi r R_0 h(\mathbf{r})} \quad (4.3)$$

Now, we also know that for the disk galaxy: $\mathbf{d}M_2 = 2\pi r \rho_2(\mathbf{r}) h(\mathbf{r}) \mathbf{d}r$, so that when combining with (4.3) :

$$\frac{\mathbf{d}M_2(\mathbf{r})}{\mathbf{d}r} = \frac{M_0}{R_0} \quad \text{or} \quad M_2(\mathbf{r}) = \frac{M_0}{R_0} r \quad (4.4)$$

Filling in this equation in the Kepler equation gives : $\frac{v_g^2}{r} = \frac{GM_2(\mathbf{r})}{r^2}$

This means that in this special, simplified case, we get for the overall stars' velocity the simple equation:

$$v_g = \sqrt{\frac{GM_0}{R_0}} \quad (4.5)$$

showing that the overall velocity of the stars in the disk of a disk galaxy is constant and equal to the Kepler velocity at the boundary of the bulge.

Remark however that we just manipulated formula's mathematically without respecting the full physical meaning during the deduction. Firstly, in (4.4) we considered only the mass from the galaxy's centre to the place r and not the mass further away from the galaxy's centre. Secondly, we considered the mass to be concentrated into a point mass at the galaxy's centre.

Although the observed velocities stay in a restricted range, close to the velocity defined in (4.5), the reality shows slightly different local velocities. The origin of these differences interested me, and will be unveiled hereafter.

5. Origin of the variations in the stars' velocities.

5.1 The galaxy's bulge area.

5.1.1 Gyrotation acceleration of stars inside the bulge.

Let us start thinking of a spherical galaxy, whereof the centre is rotating, say, one or more massive black holes. These black holes are fast spinning, and many stars near the center of the spherical galaxy are spinning as well.

When we look at a disk galaxy, we observe that the central bulge is not a sphere like the sun, full of matter, but that the bulge is a system by itself.

The summation of the gyrotation field of all the fast spinning stars of the bulge creates a global, fuzzily spread gyrotation field, which is difficult to analyze as long as the distribution of the spinning stars is unknown.

Since it is even more difficult to know the local gyrotation acceleration *inside* the bulge without knowing the locations of the individual black holes, it seems that the spread of gyrotation would be rather *a priori*- random-based.

But even if there are several spinning black holes rotating in different directions through the bulge, the global gyrotation field of the bulge apparently allowed the formation of the disk galaxy. The disk of the galaxy finds its origin in a global gyrotation field vector, which is perpendicular to the disk.

5.1.2 The fuzzy gyrotation field of the bulge.

Let us think of the fuzzy gyrotation field of the bulge again.

Theoretically, we get, based on (3.4) and with a good approximation, the tangential gyrotation acceleration :

$$\mathbf{a}_t = \frac{G \omega'}{5c^2} \sum_{i=1}^n \frac{m_i \omega_i R_i^2}{D_i^2} \sin 2\alpha_i \quad (5.1)$$

where ω_i symbolizes that the n fast spinning stars can be situated anywhere in the bulge. In fig 5.1 , the meaning of the symbols is visually shown. The values D_i and α_i are variables in time.

The locations and the parameters of the fast spinning stars and black holes are not known. Some statistics could be used here, but this is not the aim of the present paper.

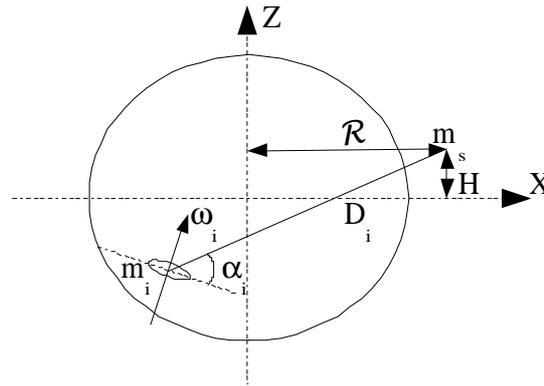


fig. 5.1

The bulge of the disk galaxy. A mass m at a vertical height H and a horizontal distance \mathcal{R} from the centre is influenced by the gyrotation of black hole i . The surroundings of the bulge are fuzzy, caused by a random distribution of n black holes which result in unwell defined vectors of the gyrotation fields.

The local thickness of the bulge and its surroundings is symmetric for the z -axis and is determined by (5.1). The summation-part in equation (5.1) indeed represents a spread of gyrotation sources that has a standard deviation and results in a Gaussian probability curve around the x - y -plane, but also an axi-symmetric one about the z -axis. Even if the individual black holes are distributed randomly and asymmetrically, we may assume that the x - y - and the z -distribution are Gaussian. This means that also in the z -direction, a number of stars inside and outside the bulge could have been trapped by some black holes whose rotation axis lays parallel to the x - y -plane.

The radial component of the gyrotation acceleration, as given in (3.3), is valid here as well, but its influence with regard to the stars' velocities is not significant compared to the gravitation part.

Concerning the influence of gyrotation and gravitation for the stars' velocities in the bulge, I expect that the effective gyrotation acceleration in the bulge is low, because in (5.1), the number of fast spinning black holes will probably be several thousands of times less than the total number of stars in the bulge. Moreover, the orientation of the fields of each black hole's gyrotation field will be randomized, so that the sum of all such fields will be very limited. It follows that the gravitational acceleration is dominant inside and nearby the bulge.

5.1.3 Gravitational acceleration in the bulge.

Let us do now the easiest part of the work: the gravitation acceleration of the bulge. When the motion of the stars is not taken into account, we speak of pure gravitation. The Newton's law for the gravitation acceleration inside homogeneous full spheres gives, at a radius \mathcal{R} :

$$\mathbf{a}_{g,\mathcal{R}_0}(\mathcal{R}) = -\frac{GM_0}{R_0^3}\mathcal{R} \quad (5.2)$$

With the little information we have got about the bulge, this is the best possible equation. The minus sign shows an attraction.

5.1.4 Stars' velocities in the bulge.

If only the gravitational part of the accelerations is significant for the orbital velocities, the star's orbital velocity at a radius \mathcal{R} is defined by :

$$\boxed{v_{g,0}(\mathcal{R}) = \sqrt{\frac{GM_0}{R_0^3}\mathcal{R}}} \quad (\text{for } 0 < \mathcal{R} < R_0) \quad (5.3)$$

As observed, the velocity is linear with the radius inside the bulge (Zone 0).

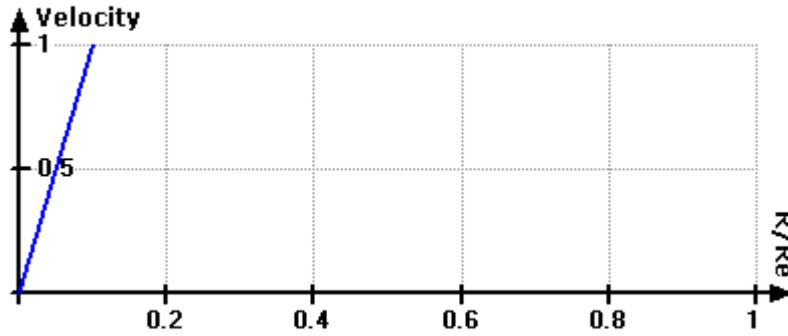


fig. 5.2

The orbital velocity in the bulge is linear and reaches its maximum at the bulge's boundary.

In fig. 5.2 we see the graphic of the velocities for such a bulge, arbitrary supposed here to be 10% of the diameter of the total disk.

5.2 The zone near the bulge.

5.2.1 More localized gyrotation activity.

The shape of the disk galaxy's section nearby the bulge is resembling a Gauss probability distribution. In the horizontal direction (x -component), the 'random' distribution of spinning black holes in the bulge and the overall orbital motion of the stars in the bulge contribute in a more accentuated overall gyrotation vector that is perpendicular to the galaxy's disk. This means that the z -component of the gyrotation is far more dominant than the x - y -component.

The gyrotation forces constrain the orbits to swivel down, the more away they are from the bulge. This shape will influence the gravitational mass to be taken in account in that area, resulting in different orbital velocities.

5.2.2 The gravitational formulation.

The shape of the disk galaxy near the bulge is flattening the more we go away from its bulge.

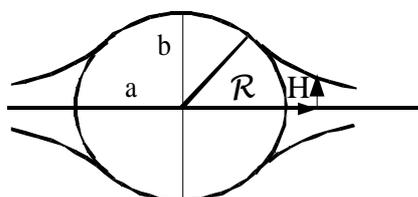
For stars laying in the disk's plane at a radius $(\mathcal{R}^2 + H^2)^{1/2}$ from the galaxy's centre (see fig.5.3) , the orbit velocity will be defined by the mass contained within that radius. For that part of the equation we can argue that the relatively wide spread of the stars in this area allows us to use the Kepler equation near the bulge.

For any star in the galaxy, the bulge's area can be seen as a point mass with mass M_0 . The corresponding orbit acceleration is given by:

$$a_{(\text{bulge})x} = \frac{GM_0 \mathcal{R}}{(H^2 + \mathcal{R}^2)^{3/2}} \quad (5.4)$$

$$a_{(\text{bulge})y} = \frac{GM_0 H}{(H^2 + \mathcal{R}^2)^{3/2}} \quad (5.5)$$

But also the mass outside of that radius will influence that orbit velocity. That part of the equation will better be described by a mass-distribution of a disk.



Using (4.3) , we find :

$$\mathbf{d} \mathbf{a}_{\mathcal{R}(r,\alpha)D^*} = \frac{GM_0}{2\pi R_0} \frac{(H^2 + (\mathcal{R} - r \cos \alpha)^2) \mathbf{d}r \mathbf{d}\alpha}{(H^2 + \mathcal{R}^2 + r^2 - 2\mathcal{R}r \cos \alpha)^{3/2}} \quad (5.11)$$

In order to find the horizontal and the vertical component of the acceleration, a projection with angle γ is needed. Due to symmetry, I disregard the y -component in the plane of the disk.

which result in a multiplication of $\mathbf{d} \mathbf{a}_{\mathcal{R}(r,\alpha)D^*}$ with $\cos \gamma$ for $\mathbf{d} \mathbf{a}_{\mathcal{R}(r,\alpha)x}$ and with $\sin \gamma$ for $\mathbf{d} \mathbf{a}_{\mathcal{R}(r,\alpha)z}$:

Therefore, notice that :

$$\tan \gamma = \frac{H}{\mathcal{R} - r \cos \alpha} \quad (5.12)$$

Using (5.10.b) for the angle γ , and (5.12) , the following components are found:

$$\mathbf{d} \mathbf{a}_{\mathcal{R}(r,\alpha)x} = \frac{GM_0}{2\pi R_0} \frac{(\mathcal{R} - r \cos \alpha) \mathbf{d}r \mathbf{d}\alpha}{(H^2 + \mathcal{R}^2 + r^2 - 2\mathcal{R}r \cos \alpha)^{3/2}} \quad (5.13)$$

and

$$\mathbf{d} \mathbf{a}_{\mathcal{R}(r,\alpha)z} = \frac{GM_0}{2\pi R_0} \frac{H \mathbf{d}r \mathbf{d}\alpha}{(H^2 + \mathcal{R}^2 + r^2 - 2\mathcal{R}r \cos \alpha)^{3/2}} \quad (5.14)$$

Equation (5.14) is different from zero if $H \neq 0$. From (5.13) and (5.14) follow that the orientation γ of the infinitesimal vector $\mathbf{d} \mathbf{a}$ is given by (5.12).

The integration of both (5.13) and (5.14) has to be taken between the following limits (the same limits are valid for the x - and the z -component).

$$\mathbf{a}_{\mathcal{R}(r,\alpha)x} = \int_{R_0}^{\mathcal{R}_e} \int_0^{2\pi} \mathbf{d} \mathbf{a}_{\mathcal{R}(r,\alpha)x} \mathbf{d}\alpha \mathbf{d}r \quad \text{and} \quad \mathbf{a}_{\mathcal{R}(r,\alpha)z} = \int_{R_0}^{\mathcal{R}_e} \int_0^{2\pi} \mathbf{d} \mathbf{a}_{\mathcal{R}(r,\alpha)z} \mathbf{d}\alpha \mathbf{d}r \quad (5.15) (5.16)$$

Remember that for the bulge part, we have got another equation. Of course, the integrals (5.15) and (5.16) are meant to be non-trivial. The integral from 0 to 2π corresponds to twice the integral from 0 to π .

5.3.2 Finding the gravitational equations in the disk.

In the first place, we will integrate the x -component. Remember that the parameters \mathcal{R} and H must be taken constant during the integration. H is not supposed to describe the profile of the galaxy.

Integrating first for r , we find :

$$\mathbf{a}_{\mathcal{R}(\alpha)x} = \frac{GM_0}{2\pi R_0} \int_0^{2\pi} \left[\frac{\mathcal{R} \mathcal{R}_e \sin^2 \alpha + H^2 \cos \alpha}{(\mathcal{R}^2 \sin^2 \alpha + H^2) \sqrt{H^2 + \mathcal{R}^2 + \mathcal{R}_e^2 - 2\mathcal{R} \mathcal{R}_e \cos \alpha}} - \frac{\mathcal{R} R_0 \sin^2 \alpha + H^2 \cos \alpha}{(\mathcal{R}^2 \sin^2 \alpha + H^2) \sqrt{H^2 + \mathcal{R}^2 + R_0^2 - 2\mathcal{R} R_0 \cos \alpha}} \right] \mathbf{d}\alpha \quad (5.17)$$

This integral has been taken between R_0 and \mathcal{R}_e .

Also the z -component can easily be integrated for r , which gives the following result:

$$\mathbf{a}_{\mathcal{R}(\alpha)z} = \frac{GM_0}{2\pi R_0} \int_0^{2\pi} \left[\frac{H(\mathcal{R}_e - \mathcal{R} \cos \alpha)}{(\mathcal{R}^2 \sin^2 \alpha + H^2) \sqrt{H^2 + \mathcal{R}^2 + \mathcal{R}_e^2 - 2\mathcal{R}\mathcal{R}_e \cos \alpha}} - \frac{H(\mathbf{R}_0 - \mathcal{R} \cos \alpha)}{(\mathcal{R}^2 \sin^2 \alpha + H^2) \sqrt{H^2 + \mathcal{R}^2 + \mathbf{R}_0^2 - 2\mathcal{R}\mathbf{R}_0 \cos \alpha}} \right] d\alpha \quad (5.18)$$

This integral has been taken between R_0 and \mathcal{R}_e as well.

Since the integration of (5.17) and (5.18) to α is complicated, I could integrate it numerically from 0 to 2π . However, I consider that stars at a certain distance H will orbit in a plane under a certain angle with the disk, but I don't expect a significant difference of velocity compared with stars which lay in the disk's plan.

Thus, $H = 0$ is a valid option in order to get a first idea of the orbital velocities of the stars. This makes (5.17) considerably simpler.

$$\mathbf{a}_{\mathcal{R}(\alpha)x|H=0} = \frac{GM_0}{2\pi R_0} \int_0^{2\pi} \left[\frac{\mathcal{R}_e}{\mathcal{R} \sqrt{\mathcal{R}^2 + \mathcal{R}_e^2 - 2\mathcal{R}\mathcal{R}_e \cos \alpha}} - \frac{\mathbf{R}_0}{\mathcal{R} \sqrt{\mathcal{R}^2 + \mathbf{R}_0^2 - 2\mathcal{R}\mathbf{R}_0 \cos \alpha}} \right] d\alpha \quad (5.19)$$

By putting aside the factor $GM_0/(2\pi R_0)$, we look at the remaining part between the brackets and integrate it.

Therefore, remark that the integral from 0 to 2π corresponds to twice the integral from 0 to π .

$$\mathbf{a}_{\mathcal{R}, \text{disk}|H=0} = \frac{2GM_0}{\pi R_0} \left\{ \frac{\mathcal{R}_e}{\mathcal{R}(\mathcal{R}_e - \mathcal{R})} \left[\mathbf{F} \left(\frac{-4\mathcal{R}_e \mathcal{R}}{(\mathcal{R}_e - \mathcal{R})^2}, \frac{\pi}{2} \right) \right] - \frac{\mathbf{R}_0}{\mathcal{R}(\mathbf{R}_0 - \mathcal{R})} \left[\mathbf{F} \left(\frac{-4\mathbf{R}_0 \mathcal{R}}{(\mathbf{R}_0 - \mathcal{R})^2}, \frac{\pi}{2} \right) \right] \right\} \quad (5.20)$$

wherein $\mathbf{F}(x, \pi/2)$ is the Complete Elliptic Integral of the First Kind.

The equation (5.20) combined with (5.4) wherein we set $H = 0$ form the overall equation for the orbital acceleration of the stars of the disk galaxy, simplified for stars in the disk's plane, and according the mass distribution of equation (4.2).

$$\mathbf{a}_{\mathcal{R}, \text{tot}|H=0} = \frac{GM_0}{\mathcal{R}^2} + \frac{2GM_0}{\pi R_0} \left\{ \frac{\mathcal{R}_e}{\mathcal{R}(\mathcal{R}_e - \mathcal{R})} \left[\mathbf{F} \left(\frac{-4\mathcal{R}_e \mathcal{R}}{(\mathcal{R}_e - \mathcal{R})^2}, \frac{\pi}{2} \right) \right] - \frac{\mathbf{R}_0}{\mathcal{R}(\mathbf{R}_0 - \mathcal{R})} \left[\mathbf{F} \left(\frac{-4\mathbf{R}_0 \mathcal{R}}{(\mathbf{R}_0 - \mathcal{R})^2}, \frac{\pi}{2} \right) \right] \right\} \quad (5.21)$$

In the next section, I will deduce the orbital velocities for stars in the disk galaxy and find the corresponding graph.

5.4 The global orbital velocities' equation of disk galaxies.

The equation for the orbital velocities of the stars in the disk galaxy follows out of $v^2 = a \mathcal{R}$.

$$\mathbf{v}_{\mathcal{R}, \text{tot}|H=0} = \sqrt{\frac{GM_0}{\mathcal{R}} + \frac{2GM_0}{\pi R_0} \left\{ \frac{\mathcal{R}_e}{(\mathcal{R}_e - \mathcal{R})} \left[\mathbf{F} \left(\frac{-4\mathcal{R}_e \mathcal{R}}{(\mathcal{R}_e - \mathcal{R})^2}, \frac{\pi}{2} \right) \right] - \frac{\mathbf{R}_0}{(\mathbf{R}_0 - \mathcal{R})} \left[\mathbf{F} \left(\frac{-4\mathbf{R}_0 \mathcal{R}}{(\mathbf{R}_0 - \mathcal{R})^2}, \frac{\pi}{2} \right) \right] \right\}} \quad (5.22)$$

This equation (5.22) gives the orbital velocity equation in the disk's plane for $R_0 < \mathcal{R} < \mathcal{R}_e$. Remark that these velocities are only initial velocities, just after the orbit swivelling.

5.4.1 Interpreting the gravitational equations.

The velocities' table is easier to deduce numerically from (5.19) than using equation (5.22) , by avoiding the Elliptic Integral. By choosing the values $R_0 = 1$ and $R_e = 10$, and by varying \mathcal{R} between 1 and 10 , the general profile of the disk galaxy's orbital velocities will appear clearly enough. I leave to the reader to experiment with other mass distributions and with more detailed data by using (5.17) and (5.18).

$$\frac{dM_2(r)}{dr} = \frac{M_0}{R_0} \Rightarrow$$

\mathcal{R}	1	1,2	2	3	4	5	6	7	8	9	10
v	1	0,83	1,54	1,75	1,84	1,92	2	2,07	2,17	2,34	2,78

tab.5.1

Comparing the figures in tab.5.1 suggests that the galaxies NGC 4594 , NGC 2590 and NGC 1620 (see fig.2.2) respond quite well to the mass distribution of equation (4.2). Other mass distributions will result in other velocity distributions.

We are then able to link mass distributions to velocities and check the theory's validity.

6. Conclusion : are large amounts of “dark matter” necessary to describe disk galaxies ?

With the calculations in this paper, we demonstrated that the gyrotational swivelling of the orbits of elliptical or spherical galaxies permitted to find a consequent velocity deduction for the stars. The found velocities for a mass distribution of $dM_2(r)/dr = M_0/R_0$ gave encouraging results. They describe the stars' velocities of a certain number of disk galaxies without the need of dark matter. The order in r of the last equation's right hand is zero. This kind of disk galaxies I will call galaxies of order zero.

The physical basics of the MAG theory, with swivelling orbits about spinning black holes in the bulge, seems to lead to at least one kind of disk galaxies: galaxies of order zero.

The used mathematical model seems to be totally consistent with galaxies of order zero as well. But other orders of disk galaxies have still to be analysed.

7. References and interesting lecture.

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