Quantizing Time and Space- From the Standing Wave to the Primary Gas Structure of a Particle – V

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Abstract

The author introduces the concept of a primary gas which is an abstract gas where the microstates are occupied successively in time unlike in the case of a real gas where the microstates are occupied simultaneously. He shows that a single plane wave associated with a standing wave formed by the confinement of a luminal wave could be treated as the microstate of the primary gas that represents a particle. This approach makes it possible to understand the dynamics of a particle in terms of the thermodynamics of the primary gas. In this approach, time and space turn out to be the intrinsic properties of the primary gas that represents a particle and the quantized nature of time and space emerges from it in a natural manner. It is shown that the action (with a negative sign) of a particle could be identified with the entropy of the primary gas and the principle of least action is nothing but the second law of thermodynamics. The author shows that the uncertainty relation of quantum mechanics can be derived directly from the equation for fluctuations and he explains the statistical basis of the virtual interactions.

1 Introduction

We saw that an electron could be represented by standing electromagnetic half wave called "staphon" which is formed by the confinement of a single circularly polarized electromagnetic wave called "photino". It is observed that such a standing wave not only acquires rest mass and the electric charge but also the half spin of a particle like electron and the plane wave representation becomes most appropriate when it acquires translational velocity [1],[2]. The staphon structure of a particle is seen to be compatible with the Dirac equation and it directly leads to the Pauli's exclusion principle[2],[3]. It was suggested that the confinement of the photino may be effected by its interactions with the Higgs waves created in the vacuum fluctuations[4]. The approach outlined above is based on the confinement of the photino and therefore the particles created could only be leptons. We know going by the Klein-Kaluza approach the electromagnetic field could be explained by invoking the existence of the fifth dimension which curves into itself. We may now assume that there could be composite waves (luminal waves) which have contributions from other dimensions also which could account for colors and flavors quarks on confinement. We shall now onwards use the term staphon in more general sense to represent the standing half wave formed by the confinement of such a composite wave.

It is now proposed that the states occupied by the staphon in its interactions with the vacuum fluctuations could be studied as a primary gas, which is an ideal gas where the microstates are occupied successively in time. The basic idea is to treat a particle as a thermodynamic system and to understand the dynamics of the particle in terms of the thermodynamic analogues. We know that a gas is one which is made up of millions of molecules moving randomly in all directions within a container having a definite volume. Statistically speaking, a gas is a system containing trillions of molecular states that exists at an instant with each molecule having a specific value of energy and momentum. We might as well imagine an abstract gas which is formed by trillions of molecular states occupied by a single molecule in its successive interactions with the walls of the container. We know that statistically speaking these two gases are equivalent. The only difference is that in the first case, all the states are occupied simultaneously, while in the second case they are occupied successively. However, this will not be important if we are dealing with a system in equilibrium which by definition is independent of time. We know that the thermodynamics of a real gas can be understood in terms of the statistical properties of the molecular states forming it. Based on the statistical equivalence of the two gases, it appears reasonable that the laws of thermodynamics could be extended to the abstract gas also. We shall call this abstract gas "the primary gas" and study the thermodynamics of this gas in more detail. Later we shall show that the constituent of the primary gas need not be a molecule, it could as well be a staphon.

The idea of a gas formed by a single molecule was first introduced by Leff a few decades back. He called it 'model gas' and it was constructed by the states occupied by a single particle with certain mass and kinetic energy confined in a one dimensional box [5]. His approach has been based on the kinetic theory and assumes the classical action function for the particle. In the entire analysis, it is presumed that the gas as a whole is stationary with regard to the observer. However, the approach we propose here involves the introduction of the additional thermodynamic coordinates like the translational velocity of the gas and the corresponding translational momentum.

2 Stochastic Equivalence

Let us take a real gas with N molecules contained in a vessel with conducting walls kept in a thermal bath. For the sake of generality, we shall use the term "microstates" to denote the state occupied by the molecules. We assume here that the molecular density of the gas is quite low and therefore for all practical purpose the molecules do not interact with each other; they interact only with the walls of the container. In short, we may treat it as an ideal gas. Let this gas be in thermal equilibrium with the macro-setting. We shall also assume that the ensemble of this gas contains R copies, where R could be taken as very large.

We may now imagine another identical container with only one molecule kept in it. It is possible to think of a very large number of states formed by this single molecule in its successive interactions with the container walls. Note that the average energy of the molecule will be determined by the temperature of the container walls as the system is in thermal equilibrium. If we take L number of such micro-states and divide it into 'R' groups, each group being comprised of N successive micro-states, where R and N are very large then, our assumption is that for each group having a certain profile of occupation of micro-states we can identify with an ensemble copy of the real gas having the same profile of occupation of the micro-states. Such a group of states occupied by a single molecule in successive interactions could be treated as an abstract gas. Here the sequence of the occupation of micro-states is not relevant, only the frequency profile of the occupation of the micro-states is important. In other words, if there are two abstract gases whose micro-states do not tally sequentially although the frequency of occupation the micro-states are identical, then the two abstract gas states may be treated as identical. In simple terms, what we are doing is analogous to comparing the outcomes of one die thrown thousand times with those of thousand dice thrown simultaneously. Obviously both results would be equivalent and the sequence of the outcomes in the case of a single die is unimportant in this process. Let us call this "the principle of the stochastic equivalence".

We know that the thermodynamic relations of a real gas can be derived purely on the basis of the probability of occupation of different gas states using the concepts of canonical ensemble and grand canonical ensemble. For example, for a given equilibrium macroscopic condition determined by, say, constant temperature, pressure and chemical potential, the probability of occupation will have a sharp maximum like a delta function for states with a particular value of energy, volume and number of molecules. The thermodynamic variables like energy, volume, number of molecules etc. of a real gas could be identified with such average values. The thermodynamics of the gas could be understood based on the ensemble approach by suitably defining entropy as a statistical quantity [6]. The stochastic equivalence of the ensemble of the primary gas with that of a real gas leads us to the conclusion that thermodynamics of a real gas and that of an primary gas are equivalent.

However, on certain aspects there are differences between a real gas and a primary gas. We shall examine these aspects in detail shortly. With this purpose in view, we shall construct a primary gas with its micro-states occupied by a single particle and derive the thermodynamic relations. Subsequently, we shall introduce translational velocity and momentum as additional thermodynamic variables and generalize the thermodynamic relations.

3 Thermodynamics of the Primary Gas

We assume here that the primary gas is formed by a single particle. Here the particle could be a single molecule or even an electron. The spread of the states occupied by the particle in time contributes to certain interesting properties. To avoid confusion regarding the identification of the particle state and that of the primary gas, we shall now onwards call the particle state as the micro-state of the primary gas. It should be noted that the number of micro-states in the primary gas is not constant, but increases as the time interval within which it is defined increases.

Let us now take a real gas whose number of molecules and volume are fixed. If we treat the system as a quantum mechanical one with quantized values for energy, then the notation for the energy of the gas will be ξ_k where 'k' denotes a complete set of quantum numbers. We now propose to derive the thermodynamic relations of the primary gas based on its equivalence to the real gas [7].

Let us consider L number of successive micro-states of the primary gas which are subdivided into R groups, each group being formed by N successive micro-states such that

$$N\,R \quad = \quad L \quad . \tag{1}$$

It should be noted that R represents the ensemble of the primary gas. L and R could be made as large as required. Let us denote by n_k the number of the ensemble copies for which the energy is ξ_k. Then, the ratio, n_k/R would give the probability for the gas to occupy the states with the specific values ξ_k. The logical foundations of the approach are based on the following. Let us consider a giant abstract gas formed by combining all of the R copies of the ensemble. Then the states with a given value ξ_k will play the same role as played by a box in the Boltzmann's method [8] and the number of copies n_k, the role of the number of molecules in the boxes.

The number of ways in which the copies of the ensemble can be distributed among the boxes is equal to

$$M \quad = \quad R!/\Pi_k\, n_k! \quad . \tag{2}$$

The copies of the ensemble will be assumed to be distinguishable as the primary gas state is a collection of micro-states, not an isolated micro-state. We shall later show that even if the states are taken as distinguishable, there will not be any change in the final result. We may assume that there exists a Gibbs micro-canonical distribution for the giant gas containing R copies. Therefore, the number of copies R and the total energy of all the copies ξ will be assumed to be fixed. These constraints may be expressed by the equations

$$\sum_k n_k \quad = \quad R \quad and \quad \sum_k \xi_k n_k \quad = \quad \widehat{\xi} \quad . \tag{3}$$

The approach we propose to follow is based on the assumption that the most probable state of the gas can be identified with that particular gas state which recurs most often in the ensemble. For this purpose, we should find out the maximum of M in (2). Keeping this in view, let us define a function σ given by

$$\sigma \quad = \quad K \ln M \quad , \tag{4}$$

where K is the Boltzmann's constant. We know that σ has a maximum when M has a maximum. So our effort boils down to finding the maximum of σ. Taking the conditions imposed by equations in (3) and using the Langrangean method of multipliers along with Sterling formula, we look for the maximum of σ in the expression

$$\phi \quad = \quad \sigma - \beta \widehat{\xi} + (\delta + 1) R \ , \tag{5}$$

where $\sigma \quad = \quad K \ln M \quad = \quad K \ R \ln R - K \sum n_k \ln n_k \, .$

Differentiating the n_k's as independent quantities and equating the first derivative of ϕ to zero, we obtain

$$n_k \quad = \quad e^{(\delta - \beta \xi_k)/K} \quad . \tag{6}$$

The Langrangean multiplier δ is obtained by substituting for n_k from (6) into (3) and is given by

$$e^{\delta/K} \quad = \quad R / \left(\sum e^{-\beta \xi_k / K} \right) \tag{7}$$

and $n_k / R \quad = \quad \left[Y(\beta) \right]^{-1} e^{-\beta \xi_k / K}, \quad where \quad Y(\beta) \quad = \quad \sum e^{-\beta \xi_k / K} \, .$ (8)

Since the number of the ensemble copies and as a consequence n_k can be made as large as desired, the application of the Stirling's formula is quite justified. The ratio $n_k/R = W_k$ is the probability that the system is in the state with energy ξ_k. If the energy level ξ_k is degenerate by a factor g_k, then (8) may be modified to give

$$W_k \quad = \quad n_k / R \quad = \quad \left[Y(\beta) \right]^{-1} g_k \ e^{-\beta \xi_k / K} , \tag{9}$$

where $Y(\beta) \quad = \quad \sum_k g_k \ e^{-\beta \xi_k / K} \quad .$ (9A)

Let us now examine the thermodynamic significance of *β*. Using (4), let us introduce a function S given by

$$S \quad = \quad R^{-1} K \ln M \quad = \quad \sigma / R \ . \tag{10}$$

S can be identified with entropy. The basis for identifying *σ/R* with entropy is firstly the additivity of the quantities *S* and *σ*. The second reason is that *S* and *σ* both reach their maximum values in the most probable state or the equilibrium state. The need to divide by *R* in (10) arises due to the fact that we want to determine the entropy of a real system which is represented by just one copy of the ensemble. Substituting the value of *σ* and using Sterling's formula to simplify, we get

$$S \quad = \quad - K \sum_k (n_k / R) \ln(n_k / R) \ \cdot \tag{10A}$$

This equation determines the entropy of an arbitrary state of a system, whether in equilibrium or otherwise. Substituting for n_k/R from (9) into (10A), we obtain

$$S \quad = \quad -K\sum_k (n_k/R)\{-\beta\xi_k/K - \ln Y(\beta)\}\ . \qquad (10B)$$

Note that (n_k/R) denotes the probability to occupy the kth energy state. Therefore, the term $\Sigma\,(n_k/R)\,\xi_k$ will give the average value of ξ_k and we obtain

$$S \quad = \quad \beta\bar{\xi} + K\ln Y(\beta)\ , \qquad (11)$$

where $\bar{\xi} = \hat{\xi}/R$ is the mean value of the energy of the system. It should be noted that the mean value $\bar{\xi}$ as per the statistical approach represents its value in the thermodynamic sense.

Let us now onwards, for the sake of notational convenience, replace $\bar{\xi}$ by ξ keeping in mind that it represents the average value, while ξ' will be used for the individual value in place of ξ_k. Accordingly (9) could be written as

$$W(\xi') \quad = \quad Y^{-1}\, g(\xi')\, e^{-\beta\xi'/K}, \qquad (12)$$

where

$$Y \quad = \quad \Sigma\ g(\xi')\, e^{-\beta\xi'/K}\ . \qquad (12A)$$

The mean energy and the entropy of the system are given by

$$\xi \quad = \quad -K\frac{\partial(\ln Y)}{\partial\beta} \quad \textit{and} \quad S \quad = \quad K\ln Y + \beta\xi\ \cdot \qquad (13)$$

By comparing (13) with a similar function obtained for an ideal gas, β can be easily identified with the inverse of temperature θ and $-K\ln Y$ with the ratio of the free energy F to the temperature of the system. These relations may be expressed as

$$\beta \quad = \quad 1/\theta \quad \textit{and} \quad K\ln Y \quad = \quad -F/\theta \quad = \quad S - \xi/\theta\ . \qquad (14)$$

The entropy may now be expressed in the familiar form

$$S \quad = \quad \xi/\theta - F/\theta\ . \qquad (14A)$$

We may now re-express (12) as

$$W(\xi') \quad = \quad g(\xi')\, e^{-(\xi'-F)/K\theta}, \qquad (15)$$

Reverting back to the quantum number k, the above equation may be written as

$$W_k \quad = \quad g_k\ e^{-(\xi_k - F)/K\theta}\ . \qquad (15A)$$

Here we should keep in mind that the free energy function, F is defined for a specific value of volume and number of micro-states. This is implicit in the steps we followed in the derivation. We may now re-express the second equation in (14) using (12A) as

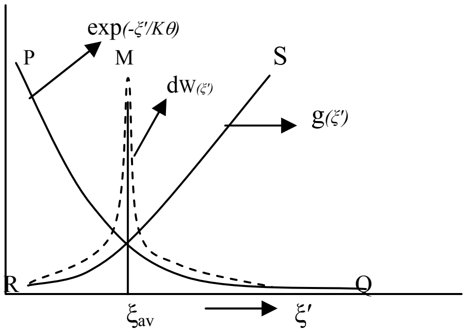
$$e^{-F/K\theta} \quad = \quad Y \quad = \quad \Sigma\ g_k\ e^{-\xi_k/K\theta}\ . \qquad (16)$$

Note that in (12), Y plays the role of the normalizing factor.

4 Broglieon gas as a Primary Gas

In the earlier papers we discussed about the structure of an elementary particle [1],[2],[3],[4]. There we studied the case of a photino which is confined to form a standing wave or a staphon. It was observed that the staphon is defined in the internal coordinates. But in the external coordinates what is observed is the plane wave. We also observed that the confined wave and the plane wave are always in phase. This means that by the time the confined wave travels one wave length between the mirrors, the plane wave would have traveled one wave length in the external coordinates. We may use the term broglino to identify a single plane wave. We know that any observable particle cannot be formed by a single wave. Note that a photon is represented by wave train. Likewise, we may assume that a large number of broglinos, say N broglinos, occupied successively forms the plane wave. Here N is left undefined. It could be taken as a large number. Let us call such a plane wave formed by N broglinos by the name broglieon (note that on the same logic, we should assume that N photinos constitute a photon). This broglieon state could play the role of the eigen state in quantum mechanics. This is because, observation is a macroscopic process and it would involve a large number of the microstates. We are assuming that the number of microstates involved is fixed and this group of micro-states taken together is treated as a primary gas. Since a staphon is formed by its interactions with the vacuum fluctuations, the energy and the momentum states occupied by it will be randomly distributed with a sharp maximum close to the average value. On this basis we may treat a broglieon state as a primary gas state. Now onwards we shall use the term primary gas exclusively for the broglieon gas.

The properties of a primary gas are similar to that of a real gas with which we are familiar. Let us take the state function represented by (12). One interesting property of $W(\xi')$ is that for given large values of N, as ξ' increases it shows a sharp maximum for the value $\xi' = \xi_{av}$, where ξ_{av} may be taken as the average value of the energy of the primary gas. The reason for the sharp maximum can be found in the fact that as ξ' increases, the exponential term in (12) decreases. However, the degeneracy of the state represented by $g(\xi')$ increases exponentially with the increase in ξ'. As a result, the product of the two functions exhibits a sharp maximum at the average value (figure.1). The sharpness of $W(\xi')$ increases as the number of microstates increases. When the number of microstates is in the range of 10^{21}, the function becomes so sharp that it could be taken as delta function. In such a situation practically all states of the gas would crowd around the average value. This is the reason why the thermodynamic values of energy, volume etc could be identified with the average values. On the basis of the probabilistic considerations, it is obvious that the same situation should hold good in the case of the primary gas representing a particle. Note that $g(\xi')$ represents the



PQ represents the Boltzmann factor which decreases as ξ' increases while RS represents degeneracy, $g(\xi')$ which increases exponentially as ξ' increases. The resultant curve RMQ has a sharp peak at M and has the form of a delta function.

Figure.1

degeneracy of the state or the number of ways the microstates could be occupied for a given value of the extensive energy ξ'.

In the case of the primary gas, the role of the molecular state will be taken up by the forward and reverse half photino states. In fact, we do not have to take up these two states separately, instead we may represent them by a single state whose energy and momentum are the average of these two states. Fortunately this is exactly the energy and momentum of the phase wave or the plane wave associated with the standing wave. Therefore, it will be more convenient to treat the broglino which stands for a single plane wave as the micro-state (equivalent to the molecular state).On this basis, the vessel containing the gas can be replaced by the set of two mirrors confining the photino. Therefore, in the present case the role of volume of the vessel will be played by the distance between the two mirrors. An important property of the broglieon gas is that its energy (or momentum as they are linearly related) determines the distance between the mirrors. Note that in the rest frame, the distance between the mirrors is $\frac{1}{2}\lambda_o$ while the energy of the photino is given by hc/λ_o. Therefore, once we fix the energy, the volume of the broglieon gas gets fixed automatically. In other words, for the broglieon gas volume is not an independent variable. Note that this holds good even when the system undergoes translational motion.

Let us now take up the case of the number of microstates contained in a broglieon gas. Needless to say, for a given duration, the number of microstates, N of the broglieon gas is not an independent variable. This is because the number of interactions undergone by the photino with the mirrors in one second is equal to its frequency which in turn is proportional to its energy. But we know that the time taken by the plane wave (phase wave) to complete one oscillation is exactly equal to the time taken by the confined photino one complete to and fro motion. Therefore, N will be inversely proportional to the frequency of the plane wave. In other words, the number of broglino states formed in the external coordinates will be exactly equal to the number of staphon states. This shows that N also is not an independent variable and is completely defined by the energy of the broglino state.

Here we should note the difference between a staphon and a broglino. A staphon is formed by the linear combination of a forward half photino and a reverse half photino. Remember that the staphon is defined in the internal coordinates. The broglino, on the other hand, is defined in the external coordinates and represents a single wave of the plane wave. While the period of the staphon and the broglino are the same, their wave lengths are different. Note that the staphon has a spread of $\frac{1}{2}\lambda_o/\gamma$, where λ_o is the wave length in its rest frame of reference whereas the wave length of the broglino is much larger [1][2] and is given by

$$\lambda = h/\text{þ} = h/(\gamma E_o \mathrm{v}/c^2) = (\mathrm{c}/\mathrm{v})\,\lambda_o/\gamma \; . \tag{17}$$

Needless to say, in the rest frame of reference λ will become infinity. However, the corresponding phase velocity given by c^2/v also would become infinity with the result that the frequency of the broglino would remain finite given by c/λ_o.

In the treatment we had followed for the primary gas, initially we had assumed that the volume and the number of states are fixed. However, in the case of the broglieon gas we now have a situation where the spatial spread which is equivalent to volume is no more an independent variable. It is just half of the wave length which is inversely proportional to the energy of the broglino. Likewise, it is obvious that the number of broglinos constituting the broglieon state also is completely determined by the frequency or the energy of the broglinos. In other words, the broglieon gas gets fully defined when we take only the energy of the micro-states into consideration. Therefore, the derivation of the state of the primary gas which was done in the preceding sections would entirely hold good for the broglieon gas also.

From the above discussion it is quite clear that N and V do not represent independent thermodynamic dimensions for a broglieon gas. This also shows that there is no method by which work could be extracted from the system of the broglieon gas. Here we should keep in mind that we are dealing with the broglieon gas in its rest frame and therefore it does not possesses translational velocity and translational momentum as its thermodynamic coordinates. We shall introduce them shortly. In the absence of any possibility of exchanging kinetic energy, it is obvious that we cannot make the staphon expand so as to extend its wave length and thereby make it do work. Therefore, we may treat *F* to be a constant. We may now introduce a modified entropy S′ where S′ = (S + F/θ). Accordingly we may express (14A) as

$$S' \quad = \quad \xi/\theta \quad = \quad NE/\theta \quad ,$$

Dropping the prime from S′, we may re-express the above equation as

$$S \quad = \quad \xi/\theta \quad = \quad NE/\theta \tag{18}$$

where E denotes the average energy of the micro-state. Therefore (15) may be expressed as

$$W(\xi') \quad = \quad g(\xi')\, e^{-\xi'/K\theta}. \tag{19}$$

5 Broglieon gas in motion at relativistic velocities

If we study a thermodynamic system like a gas having translational velocity in the relativistic range [9], then it is observed that its heat content or enthalpy given by (ξ+PV), would transform exactly the way the rest mass of a particle would transform. We may extent this property to the broglieon gas also without any logical inconsistency (see annexure). However, in the case of the broglieon gas, pressure and volume are not independent variables and therefore will not figure in the expression for enthalpy. In its place we may introduce variables representing its translational velocity and momentum. If we denote by G, the translational momentum of the gas as a whole, then we have

$$\xi \quad = \quad \gamma\,\xi_o \quad ; \quad \mathbf{G} \quad = \quad (\xi/c^2)\,\mathbf{v} \quad , \tag{20}$$

where $\gamma = (1\text{-}v^2/c^2)^{-\frac{1}{2}}$ and **v** is the translational velocity. It is obvious that ξ and **G** are related to each other exactly the way the energy and momentum of a particle are related to each other. In fact combining two equations in (20), we obtain

$$\xi^2 \quad = \quad \mathbf{G}^2 c^2 + \xi_o^2 \quad . \tag{20A}$$

This is the relativistic energy and translational momentum relation of the broglieon gas. If we take **v** to be along the x-axis, then by differentiating (20), one can readily show that

$$d\xi \quad = \quad \mathrm{v}_x\, dG_x \quad . \tag{21}$$

But from (18) we may write

$$\xi \quad = \quad NE \quad and \quad G_x \quad = \quad N\text{þ}_x \quad . \tag{22}$$

E may be called the intensive internal energy of the broglieon gas and þ its intensive translational momentum. Dropping the suffix from $G_{x,}$, þ_x and v_x while assuming they are directed along the x-axis, (19) can be written using (22) as

$$\text{þ} \quad = \quad \mathrm{Ev}/\mathrm{c}^2 \quad . \tag{23}$$

It is worthwhile to recall here that while the confinement of the photino takes place in the internal coordinates, the broglieon state which represents the particle is defined in the external coordinates. Therefore, in the external coordinates the microstates of the broglieon state will be represented by the broglinos. As already discussed, the broglino will be taken as the most basic entity for the primary (broglieon) gas just as the molecular state is taken as the most basic entity forming a real gas state. Although a broglino has the inner structure of a staphon (confined photino), this aspect plays no role in the nature of the broglieon state. Only the broglinos, which are the projections of the staphon states onto the external coordinates, are of importance. The need to get into the staphon structure will arise only to explain the fermionic nature of the particle. For all other purposes, the broglino structure will be adequate.

It should be noted that the introduction of the translational momentum, G is tantamount to the introduction of a new thermodynamic variable. Therefore, apart from (3), and (4) the following additional condition will have to be introduced where $\overline{G}$ is taken as fixed because the total number of copies in the ensemble is fixed. We shall for the sake simplicity take the direction of the translational motion to be strictly along the x-axis so that

$$\sum_k n_k G_k = \overline{G} \quad . \tag{24}$$

Using (24), equation (3) could be generalized to give

$$\phi = \sigma - \beta\overline{\xi} + \rho\,\overline{G} + (\delta + 1)\,R \quad , \tag{25}$$

where ρ is a constant which has to be determined. $\rho\bar{G}$ is naturally a scalar. Now following the steps taken earlier, equation (10B) would stand modified as

$$\begin{aligned} S &= -K\sum_k (n_k/R)\{-\beta\xi_k/K + \rho\,G_k - \ln Y_1(\beta,\rho)\}\ \cdot \\ &= \xi/\theta - \rho\,G - \phi/\theta\ , \end{aligned} \tag{26}$$

Here ξ and G represent the average values the internal energy and translational momentum while ϕ denotes the thermodynamic potential given by

$$Y_1(\beta,\rho) = e^{-\phi/\theta} = \sum g_k\, e^{-(\xi_{\mathbf{k}}/K\theta - \rho G_{\mathbf{k}}/K)} \tag{26A}$$

Note that if the energy states are lying close together, we may take them as continuous. Therefore, we may replace the summation by integration and also replace ξ_k and G_k by ξ' and G' respectively. We may now express the probability function corresponding to (12) as

$$W(\xi', G') = Y_1^{-1}\, g(\xi', G')\, e^{-(\xi'/K\theta - \rho G'/K)}, \tag{26B}$$

We know that in any reversible change we may take ϕ and S as constants. Therefore, taking differentials in (26), we obtain

$$d\xi/\theta = \rho\; dG \ , \tag{27}$$

Since $d\xi/dG = \mathrm{v}$, the translational velocity, we have $\rho = \mathrm{v}/\theta$. Accordingly we may re-express (26) as

$$S = \xi/\theta - \mathrm{v}\,G/\theta - \phi/\theta \tag{27A}$$

We may introduce a modified entropy function S′ given by

$$S_1 = S + \phi/\theta = \xi/\theta - \mathrm{v}G/\theta \tag{27B}$$

Note that S_1 also would remain an invariant in any reversible change. Keeping in mind that S_1 is the modified entropy function, we may drop the suffix from S_1 and express (27B) as

$$S \quad = \quad \xi/\theta - \mathrm{v}G/\theta \qquad (27C)$$

If the velocity and the translational momentum are along any general direction, then we have

$$S \quad = \quad \xi/\theta - \mathbf{v}.\mathbf{G}/\theta \;. \qquad (27D)$$

We may now express (26B) as

$$W\,(\xi') \quad = \quad g\,(\xi')\; e^{-(\xi' - \mathbf{v}.\,\mathbf{G}')/K\theta} \;. \qquad (28)$$

Here we should keep in mind that the translational momentum **G′** is taken along one direction only and this means ξ′ completely determines the value of **G′**.

It can be seen (see Annexure) that on a relativistic transformation, an ideal gas for all practical purpose would behave like a particle with rest energy. Needless to say, this applies to the case of the primary gas also. Since the number of microstates is a relativistic invariant, we may define E = ξ/N and þ = G/N with the energy and the translational momentum of a single microstate respectively. For the primary gas, an infinitesimal increase in the energy of the translational motion will be given by **v.dG**. In the case of particle mechanics, the equivalent function is **v.dþ.**

$$\mathbf{v}.d\mathbf{þ} \quad = \quad (\mathrm{d}\mathbf{r}/dt).d\text{þ} \quad = \quad \mathrm{d}\mathbf{r}.(d\mathbf{þ}/\mathrm{dt}) \quad = \quad \mathbf{F}.\mathrm{d}\mathbf{r} \;.$$

Now combining (27D) and (28) we may write

$$W\,(\xi') \quad = \quad g\,(\xi')\; e^{-[\xi' - \mathbf{v}.\mathbf{G}']/K\theta\}} \;. \qquad (28A)$$

(22) could now be written as

$$\mathbf{G} \quad = \quad N\mathbf{þ} \quad \text{and} \quad \mathbf{þ} \quad = \quad \mathrm{E}\,\mathbf{v}/c^2 \;. \qquad (29)$$

Equation (27D) could be expressed in a more general way as

$$S \quad = \quad N\breve{s} \quad = \quad NE/\theta - N\,\mathbf{v}.\mathbf{þ}/\theta \;. \qquad (30)$$

where $\breve{s}$ may be called the intrinsic entropy

We shall now try to express (28A) in terms of the intrinsic quantities like E′ and **þ′** instead of ξ′ and **G′**. We know that the probability density function *W(ξ′,G′)* has a sharp peak like a delta function for value ξ′ = ξ and **G′** = **G**, where ξ and **G** are the average values for the ensemble as a whole. Therefore, for all practical purpose the entire ensemble of the primary gas may be assumed to occupy states having the average values ξ and **G**. We shall now introduce an abstract gas formed by the entire ensemble of the gas which has Nn_e microstates. Here we have used n_e instead of n for a specific reason which will be clear later. For this abstract gas, the average intrinsic energy and intrinsic momentum of the microstate will be E ad **þ**. Remember that this abstract gas constituted by Nn_e microstates possesses the internal energy $\hat{\xi}$ and the translational momentum $\overline{\mathbf{G}}$. We shall ignore the problem regarding the indistinguishability of the micro-states for the present. Now following the Boltzmann's box method, It could be easily shown that the probability for occupying the micro-state with energy E′ is given by

$$W\,(E') \quad = \quad g\,(E')\; e^{-n_e N\,(E' - \mathbf{v}.\mathbf{þ}')/K\theta\}} \qquad (30A)$$

But we know that the number of microstates have a sharp maximum for energy and translational momentum which are close to the average values. Note that here the situation is similar to what is given in figure 1. In fact, equation (27) could be written as

$$(E' - E)/\theta \quad = \quad \mathbf{v}\ (\text{þ}' - \text{þ})\ /\theta \ ,$$

This can be re-expressed using (30) as

$$E' - \mathbf{v}.\text{þ}' \quad = \quad E - \mathbf{v}.\text{þ} \quad = \quad \breve{s} \ . \qquad (31)$$

Therefore, (30A) could be expressed using (31) as

$$W(E') \quad = \quad g(E')\, e^{-n_e N(E - \mathbf{v}.\textbf{þ})/K\theta\}} \qquad (31A)$$

In the rest frame of reference, the energy of a copy of the broglieon gas would be ξ_o and (31A) could be expressed as

$$W(E'_o) \quad = \quad g(E'_o)\, e^{-n_e N E_o/K\theta} \qquad (31B)$$

Since the energy of the individual microstates also would have a very sharp maximum for $E'_o = E_o$, we may replace E'_o by E_o for all practical purpose. Here *g(E′o)* which represents the degeneracy, increases as E'_o increases while the exponential factor decreases correspondingly with the result that $W(E'_o)$ gets a sharp maximum like a delta function at $E'_o = E_o$, where E_o is taken as the average value. Therefore, the probability to occupy any microstate with energy E'_o may be expressed as

$$W(E_o) \quad = \quad g(E_o)\, e^{-n_e N E_o/K\theta} \qquad (31C)$$

Using Sterling's formula it can be easily shown that for very large values of the microstates, the degeneracy

$$g(E_o) \quad \to \quad B\, e^{n_e N}$$

where B is a constant independent of n and N. Therefore, for all practical purposes we may take

$$g(E_o) \quad = \quad B\, e^{n_e N} \qquad (31D)$$

6 Quantizing Space and Time

We saw from the previous chapter that the intensive energy and the intensive momentum of a broglieon gas are related just the way the energy and the momentum of a particle are related. That is $\textbf{þ} = E\mathbf{v}/c^2$. Further we have $E = \gamma\ E_o$ where E_o is the value of E in the rest frame of reference. This leads us to the relativistic relation

$$E^2 \quad = \quad \text{þ}^2 c^2 + E_o^2 \quad . \qquad (32)$$

Let us now introduce two new thermodynamic quantities T_e and $\mathbf{R}_e$ defined by

$$T_e \quad = \quad h/K\theta \quad \text{and} \quad \mathbf{R}_e \quad = \quad \mathbf{v}h/K\theta \ . \qquad (33)$$

The factor h/K has been introduced into the equation so that T_e will have the dimension of time. It will be explained in section 10 why E = Kθ for the broglieon gas. Therefore, we have

$$T_e = h/E \quad . \tag{33A}$$

Let us now take (30) and substitute for $1/\theta$ and $\mathbf{v}/\theta$ from (33) and (33A) after multiplying it with n_e to obtain

$$S^{\#} = (K/h)[T_e n_e NE - n_e N\mathbf{R}_e . \text{þ}] \quad . \tag{34}$$

Here n_eNT_e and $n_eN\mathbf{R_e}$ have very interesting properties. n_eNT_e denotes the spread of the broglieon state in time while $n_eN\mathbf{R_e}$ represents the spatial distance covered during that period. Equation (34) pertains to n_e broglieon states. We may re-express the equation as

$$S^{\#} = (K/h)(Et_e - \text{þ}.\mathbf{r}_e) \quad , \tag{35}$$

where $$t_e = n_e NT_e \quad and \quad \mathbf{r_e} = n_e N\mathbf{R}_e \quad . \tag{35A}$$

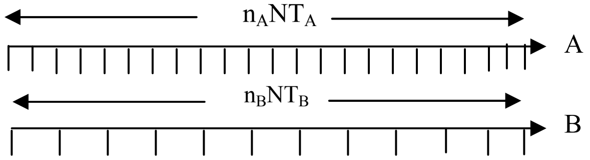
Here as n_eN increases, t_e also increases. In that sense t_e has the property of the progressive time. Likewise, $\mathbf{r_e}$ has the property of spatial coordinate.

The question which comes to our mind here is whether the time and the space coordinates denoted by t_e and $\mathbf{r_e}$ are specific to the broglieon gas under study or could it be taken to represent the laboratory (external) coordinate system. Since the equations in (33) which define T_e and $\mathbf{R_e}$ are specific to the broglieon gas, we have to assume that they pertain to a specific gas. But we have to consider the fact that if we take certain time interval, then the number of broglieons states, n_e contained in it will be inversely proportional to the energy of the broglieon gas. In (33A) we have equated the intrinsic time of a broglieon gas, T_e to the period of the broglino. It is obvious that the period of the broglino representing one particle will be different from that of another if they differ in their rest masses.

Let us now take the case of two broglieon gases A and B having rest energy E_A and E_B respectively, with $E_A > E_B$. Then, if T_A and T_B are respectively the intrinsic quantum of time (the period of the broglino) of A and B, it is obvious that

$$T_B > T_A \quad as \quad T_A = h/E_A, \quad T_B = h/E_B \quad . \tag{36}$$

Note that although T_B is greater than T_A, the number of broglieon states that exist in a time interval in the case of B is proportionately less than that of A. If we denote by n_A and n_B the number of broglieon states contained in a certain time interval by the broglieon gases A and B respectively, then presuming that n_A and n_B are very large numbers, we have



The interval between two consecutive vertical marking lines could be taken as the spread of the Broglieon state in time. Therefore, it is quite clear that for a given laboratory time interval $n_AT_A = n_BT_B$.

Figure.2

$$n_A N\, T_A = n_B N\, T_B = nN\, T \quad . \tag{37}$$

Here n and T pertain to some broglieon gas which can be taken as a norm. Note that T could be taken as small as required so that for all practical purpose, nNT could be taken as a continuous function and therefore it could be used to represent the external time coordinate. Substituting t for nNT, (37) could be written as

$$n_A N\, T_A \quad = \quad n_B N\, T_B \quad = \quad t \quad . \tag{37A}$$

It can be easily seen that this property given by (37A) could explain the perception of time as a universal background. Note that if **r** is the spatial coordinate of the broglieon gas taken as the norm, then **r** = **v**t.

We know that the extensive internal energy of n broglieon states, each having N microstates, may be represented by $\xi^{\#}$ where

$$\xi^{\#} \quad = \quad n\,\xi \quad = \quad nNE \quad . \tag{38}$$

If we now take two broglieon gases, A and B interacting with each other for a duration t, then their corresponding extensive energy could be taken as $\xi^{\#}_{A}$ and $\xi^{\#}_{B}$. Let the broglieon states of A and B involved in the interactions be n_A and n_B respectively. The corresponding number of microstates involved in the interactions will be Nn_A and Nn_B respectively. This would mean that, instead of taking the extensive internal energy of A and B, we might as well use the average energy of the microstates of A and B denoted respectively by $E_A = \xi^{\#}_{A}/Nn_A$ and $E_B = \xi^{\#}_{B}/Nn_B$.

Using (37A), we may now express the small variations in t and **r** as

$$\Delta t \quad = \quad \Delta n\; N\; T \qquad \mathit{and} \qquad \Delta \mathbf{r} \quad = \quad \Delta nN\,\mathbf{R} \quad . \tag{39}$$

where T and **R** pertains to the broglieon gas used as the norm for the laboratory coordinate system. We saw from the preceding discussion that we may take T and **R** as small as required so that t and **r** could be taken as continuous variables. Therefore, if Δn << n, then Δt can be replaced by dt and Δ**r** by d**r**. Let us now take (37A) and replacing the suffix A by e, we obtain

$$t_e \quad = \quad n_e NT_e \quad = \quad nNT \quad = \quad t \tag{40}$$

$$\mathbf{r_e} \quad = \quad n_e N\mathbf{v}T_e \quad = \quad nN\,\mathbf{v}T \quad = \quad \mathbf{r} \tag{40A}$$

Now substituting in (35) for $\mathbf{t_e}$ and $\mathbf{r_e}$ from (40) and (40A) we have

$$S^{\#} \quad = \quad (K/h)[\,Et\, - \,\text{\textbf{þ}}.\mathbf{r}\;] \quad . \tag{41}$$

We may also re-write (31A) using (33), (40) and (40A) as

$$W\,(E') \quad = \quad \mathrm{g}(\,\mathrm{E}'\,)\;\; \mathrm{e}^{-(\mathrm{Et}\;-\text{\textbf{þ}}.\mathbf{r}\;)/h} \quad . \tag{42}$$

Substituting for $S^{\#}$ from (41) into (42), we have

$$W\,(E') \quad = \quad \mathrm{g}(\,\mathrm{E}'\,)\;\; \mathrm{e}^{-S^{\#}/K} \quad . \tag{43}$$

Now we have to define the infinitesimal variations in entropy, energy and momentum. Here we should keep in mind that the broglieon state is the most basic level for observation. Therefore, any variation should be measured from one broglieon state to another. Let us take n number of broglieon states which are occupied successively. Let $S^{\#}$ be the extensive entropy of the group of broglieon states. Then we have

$$S^{\#} \quad = \quad nNS \qquad \mathit{and} \qquad \Delta S^{\#} \quad = \quad \Delta nN\;\, S \quad . \tag{44}$$

This means that as the laboratory time progresses through Δt, the extensive entropy increases by ΔnS. The variation defined by (44) is the extensive variation where only n varies. In a similar manner we may define differentials of ξ and G also.

7 Action Entropy Equivalence of the of the Broglieon gas

We saw from the earlier discussion that a particle can be represented by a broglieon gas. We shall now examine if the mechanics of a particle can be related to the thermodynamics of the broglieon gas. We saw that in a relativistic transformation, the intensive energy E and corresponding translational momentum þ of a broglieon gas changes exactly like the energy and linear momentum of a real particle. Let us now take the action function of a free particle given by

$$A \quad = \quad -(Et - \mathbf{þ.r}) \quad = \quad -\mathrm{E}_o t_o \;. \tag{45}$$

Here E_o is the energy of the particle and t_o its time coordinates in the rest frame of reference. It should be kept in mind that action is a relativistic invariant. Comparing (41) with (45) we obtain

$$S^{\#}/K \quad = \quad -A/h \qquad or \qquad dS^{\#}/K \quad = \quad -dA/h \;. \tag{46}$$

The invariance of entropy under a relativistic transformation holds good for both the extensive and the intensive aspects. The extensive aspect remains invariant because nN is an invariant in a relativistic transformation.

This equivalence of two of the relativistic invariants, action and entropy brought out by (46), is an exciting result. Being a thermodynamic system, entropy of the primary gas has a maximum in the equilibrium state. We know that entropy remains a constant in a relativistic transformation. From (46), we observe that when $S^{\#}$ is maximum, action A is a minimum. This means that the least action principle of the particle-mechanics turns out to be nothing but a restatement of the second law of thermodynamics applied to the broglieon gas. Let us now try to find out the thermodynamic equivalent of the Langrangean of the primary gas. We know that if we introduce to a system a small number of broglieon states having the same average characteristics, then it would result in increase in its heat content by a small value $dQ^{\#}$. This would result in change in the extensive aspect of entropy given by

$$dS^{\#} \quad = \quad dQ^{\#}/\theta \quad = \quad (K/h)\,T_e dn\; N\; q \quad = \quad (K/h)\,q\; dt \quad , \tag{47}$$

where q is the internal heat expressed in terms of a single microstate or one may call it the intensive internal heat. However, for a mechanical system

$$dA \quad = \quad L\,dt \quad , \tag{48}$$

where L is the Langrangean. Comparing (48) with (47) and (46) yields

$$q \quad = \quad -L \quad = \quad mc^2\sqrt{(1 - \mathrm{v}^2/c^2)} \;. \tag{49}$$

When v = 0, $q = mc^2$. Therefore, one may state that the rest mass of a particle is a measure of its internal heat. Note that the heat which we perceive in our daily experience is due to the random motion at the molecular level. However, the heat referred to here is created by random motion at the innermost or most level. Therefore it may be more appropriate to call it the **sub-quantum heat**.

In the light of the above discussion, we may examine the equation for the relativistic transformation of energy given by

$$E = E_o\Big/\sqrt{(1-\mathrm{v}^2/c^2)} = E_o\sqrt{(1-\mathrm{v}^2/c^2)} + (E_o\mathrm{v}^2/c^2)\Big/\sqrt{(1-\mathrm{v}^2/c^2)}$$

$$= mc^2\sqrt{(1-\mathrm{v}^2/c^2)} + \text{þv} \ . \qquad (50)$$

Using (49), the first term on the right hand side of the equation could be identified with the heat content while the second term could be identified with the free energy given by þv. Note that in the case of an ordinary gas, where the number of molecules is fixed, the free energy is given by PV, where P and V are pressure and volume respectively. But in the case of a primary gas, the equivalent function is þv. Therefore, (50) could be expressed as

$$E = q + \text{þv} \ . \qquad (50\text{A})$$

This represents the first law of thermodynamics. Actually the first law should be expressed using the variations as

$$dE = \Delta q + \Delta w \ . \qquad (50\text{B})$$

Here *w* denotes the work done on the system which is equal to þv. Note that Δq and Δw are improper differentials. This is because, a thermodynamic system can undergo variations in *q* and *w* along various paths or isothermals. Therefore, when Δq or Δw approaches zero, the paths followed by them are not uniquely defined. However, the linear combination of the two variations equals to the variation in the internal energy which is seen to be a proper differential and is therefore denoted by dE. In the case of the broglieon gas, we are dealing with a system whose entropy is conserved. Therefore, we are confining ourselves to the adiabatic (isentropic) changes only. We know that in the case the adiabatic changes, the variations in *q* and *w* are uniquely defined and therefore we may replace Δq and Δw by d*q* and dw respectively. Therefore, taking the constant of integration as zero, we may replace (50B) by (50A).

Let us now substitute for *q* in (50A) from (50) and take the differential to obtain

$$dE = d(mc^2/\gamma) + d(\text{þv}) = -\text{þ dv} + \text{þ dv} + \text{v dþ} = \text{v dþ} \ . \qquad (51)$$

Note that the variation in the internal heat is exactly compensated by one of the two terms representing the variation in the free energy leaving only vdþ as the resultant variation. This represents the variation in the kinetic energy (recall that $d[\frac{1}{2}m\mathrm{v}^2] = \mathrm{v}\ d$þ). This is the reason why we could ignore the primary gas structure of a particle and deal with it as a point particle. The idea that the mass of a particle represents the heat possessed by a particle due to its random motion at the sub-quantum level opens the window to a new breath-taking view of the internal structure of matter. In fact, this allows us to study the quantum world using the well understood principles of thermodynamics.

8 Statistical basis of the uncertainty principle

We shall now show that the Heisenberg's uncertainty principle can be derived from this idea of the interactions with vacuum fluctuations. We saw that the confinement of the photino could be attributed to the interactions with the vacuum fluctuations. It is logical to assume that the staphon occupies a range of energy states from E_1 to E_N on account of its interactions with the vacuum fluctuations. Since the number N need not be very large, this would mean that the energy of the broglieon states, each formed with N microstates, will be having values which are widely dispersed. If we take n such broglieon states, then they may occupy energy states from ξ_1 to ξ_n. If n is quite large, then the states occupied by the particle will crowd around the average value ξ.

Now applying the principle of fluctuations on these n broglieon states, we obtain

$$\Delta \xi / \xi = 1/\sqrt{n} \quad . \tag{52}$$

This is the equation for fluctuations [5]. If we take the case of a gas with N molecules, then, the energy of the gas as a whole will have fluctuation which is inversely proportional to the square root of the number of molecules. But we know that $\xi = Nh\nu = Nh/T_e$ where T_e is the period of the staphon. Therefore, we have

ie; $$\Delta \xi \, T_e = Nh / \sqrt{n} \quad . \tag{52A}$$

Let us now define the variation in the intrinsic energy as $\Delta E = \Delta\xi/N$. Let us also assume that n is a very small number which can be denoted by Δn. Now multiplying both sides of (56A) by Δn, we obtain

ie; $$\Delta E \, \Delta n \, T_e = \sqrt{\Delta n} \; h \quad . \tag{52B}$$

If T is the intrinsic quantum of time of the external coordinates, then from (40) we can always relate it to the intrinsic time of a particle by the relation

$$T_e = mNT \quad , \tag{52C}$$

where m is an integer. Here T is the intrinsic time of the broglieon gas which is taken as the norm for the external or laboratory coordinate system. NT would give us the time spread of the broglieon state which is taken as the norm. Since T could be made as small as required, equation (52C) implies that the intrinsic time of an elementary particle can be expressed as an integral multiple of the temporal spread broglieon gas which is taken as the norm. We may now express (52B) as

$$\Delta E \, \Delta n \, mNT = \sqrt{\Delta n} \; h \quad .$$

Taking $m \, \Delta n = \Delta n'$, the above equation may be written as

$$\Delta E \, \Delta n' N \, T = \sqrt{\Delta n} \; h \quad .$$

We know from (39) that "Δn'NT" can be taken as the durative time in the external coordinates and could be denoted by Δt. This gives us the relation

$$\Delta E \, \Delta t = \sqrt{\Delta n} \; h \qquad or \qquad \Delta E \, \Delta t \geq h \quad . \tag{53}$$

In a similar fashion, taking the case of the momentum and the spatial coordinates, we may derive the relation

$$\Delta þ \, \Delta x \geq \tfrac{1}{2} h \quad . \tag{53A}$$

The reason why $\frac{1}{2}$h appears on the right hand side is due to the fact that the photino forms a standing half wave, not a full wave.

9 Understanding the Nature of the Virtual Interactions

Now we shall examine the interactions which confine the photino forming the staphon. In classical physics, vacuum is taken as an inert medium or an arena on which the particles interact with each other. But in quantum field theory, vacuum is no more than an inert medium as it exists in a dynamical state due to the vacuum fluctuations. The uncertainty principle given below plays a very important role in the quantum field theory.

$$\Delta E \, \Delta t \quad \geq \quad h \quad and \quad \Delta \text{þ} \, \Delta \mathrm{x} \quad \geq \quad \tfrac{1}{2} h \ . \tag{54}$$

If the period Δt is long, then the fluctuation in energy becomes almost zero. We know from quantum field theory that in spite of the fluctuations, vacuum is to be taken as the ground state. Any fluctuation in energy which does not comply with the uncertainty relation given in (54) would remain a virtual one. In other words, if the vacuum fluctuations comply with the inequality $\Delta E \, \Delta t < h$, then such fluctuations would be virtual by nature.

Let us examine this issue in the light of the equation for fluctuations given in (52). Here we should keep in mind that when we fix the time duration Δt, it is equivalent to fixing the value of n. Therefore, when we take $\Delta E \, \Delta t > h$, we are taking $\Delta n > 1$. Similarly, when we take $\Delta E \, \Delta t = h$, we are basically taking $\Delta n = 1$. By the same reasoning, when we take $\Delta E \, \Delta t < 1$, we are assuming $\Delta n < 1$. Now we should try to understand what is the implication when we take $\Delta n < 1$. We know that a broglieon state, which is equivalent to the eigen state of quantum mechanics, is formed by N microstates. We can assume that the number of microstates, N_i keeps on increasing from 1, 2, 3 etc. till it reaches the value N when the broglieon state is formed or to put it differently, value of n becomes 1. Therefore, it is obvious that $n < 1$ pertains to the situation where $N_i < N$ or when the broglieon state or the eigen state is not yet fully defined. Let us now examine the fluctuations that could take place in the energy of the microstates. We know from the equation for fluctuations that

$$\Delta \xi_i \quad = \quad \xi_i \big/ \sqrt{N_i} \ , \quad or \quad \Delta E_i \quad = \quad E_i \big/ \sqrt{N_i} \ . \tag{55}$$

Here ξ_i represents the extensive energy of the system having N_i microstates while $E_i = \xi_i/N_i$ and $\Delta E_i = \Delta\xi_i/N_i$. It is quite obvious from (55) that smaller the time duration (note that N_i is directly proportional to the time interval) larger will be the fluctuations in the energy. This means that the so called virtual interactions undergone by a particle are caused by the usual statistical fluctuations. Therefore, even if vacuum is taken as the ground state, it will not preclude such virtual fluctuations. If we take the case of space and momentum, the requirement for the fluctuations to remain virtual will be $\Delta\text{þ}\Delta x < \tfrac{1}{2}h$.

One of the paradoxes of quantum mechanics has been the role played by the virtual interactions. While the interactions are termed virtual, their effects on the particles are real and observable like in the case of Lamb shift and Casimir effect. The interpretation of the uncertainty principle given above makes it clear that the so called virtual interactions with the vacuum are as real as any other interactions. The only reason why they appear virtual is that only a broglieon state formed by a group of N broglinos can be an observable. To understand the situation better let us examine an analogous case of a real gas. Let us assume that no sophisticated instruments are available to observe the molecules of a real gas directly, the instruments available can observe only macroscopic systems. In that case, as far as the observer is concerned, only the gas contained in a vessel would be a real entity while the molecules that form the gas would be virtual! Here, the virtual nature of the molecules is imposed by technology. However in the case of sub-quantum states, there seems to be a universal limit for observation. It seems that N microstates are required to form an observable state. But we do not know the exact value of N. We can only presume that it is not a small number.

Here we should understand how the uncertainty in the energy and the momentum of a particle are created. We saw from the last chapter that a particle could be identified with a primary gas formed by a group of microstates. In fact, the uncertainty in the energy and momentum of the particle could be traced to such a sub-structure. We may attribute the existence of these microstates to the interactions with vacuum. But if we assume that a single microstate in itself is not observable, instead a large group of such microstates only are observable, then, we have a very interesting situation. While the generation of the microstates – remember that a broglieon state is formed by a group of microstates - could be attributed to the vacuum fluctuations, the same vacuum acts as the ground state that can neither be a source nor a sink for energy. However, on the basis of the uncertainty principle, vacuum could still lend or borrow energy (and momentum) from particles so long as the period involved is $\Delta t < h/\Delta E$.

10 Conclusion

In the primary gas approach outlined above we have not elaborated much about the macro-setting. As discussed in the earlier paper [4], this role is played by the vacuum fluctuations in the Higgs field. There we saw that depending on the rest energy of the particle the energy of the virtual Higgs waves interacting with it would be equal to it. But now in the light of the new interpretation of the uncertainty principle given in section 8 and 9, we have to slightly modify the mechanism of the vacuum fluctuations which confines the photino. The interactions due to the vacuum fluctuations need not be restricted to one oscillation. It could last N_i oscillations so long as $N_i < N$. Here N emerges as the number of micro-states required to constitute a broglieon state that is equivalent to the eigen state. We do not know whether the value of N gets fixed arbitrarily in any observation or if it is a universal constant. In all probability N could be a universal constant and experiments designed appropriately may reveal its exact value in future.

Since the vacuum fluctuations play the role of the sub-quantum thermal bath, it is obvious that the internal energy of the particle will be equal to the energy of the vacuum fluctuations. Here we should keep in mind that the energy for one degree of freedom will be given by $\frac{1}{2}K\theta_o$. Since the Higgs wave, just like the electromagnetic wave, could be attributed two degrees of freedom by way of two planes of polarization, the total energy involved in the confinement will be $K\theta_o$. Therefore, the energy of the particle in the rest frame of reference will be given by $E_o = K\theta_o$.

The concept that an eigen state is represented by N broglino states would mean that a broglieon gas would be spread out in space over N wave lengths of the plane wave where N is not a small number. Therefore, it would appear that a particle cannot be localized under the primary gas approach. It will be shown in separate papers that this problem of the localization of the particle could be resolved by going deeper into the nature of time. As such We know from the equation given in (42) that the state function of the primary gas has the same form as that of a wave function except for the absence of the imaginary number "i" in the exponential term as a factor. In other words, we may say that in the wave representation of a particle, time is treated as an imaginary quantity while in the primary gas approach it is treated as a real quantity. This analysis appears to lead us to a new understanding of the nature of time and also to the very basis of quantum mechanics. However, before making such a claim, it is necessary to establish the thermodynamic basis of the basic postulates of quantum mechanics. This will be done in separate papers shortly.

Here it is interesting to note that de Broglie, towards the later part of his life, had come around to the view that a particle could be considered to have a hidden thermodynamic system and the mass of a particle could be taken as a measure of the hidden heat of the system while action represents its entropy[10]. However, his idea did not find acceptability due to certain serious shortcomings. For example de Broglie assumed that the particle could be treated like a real gas. However, the action function of a free particle at rest given by "$-E_o t_o$" increases as time progresses while the entropy of a real gas at rest is given by ξ_o/θ_o which does not have such a time dependence. This forced him to restrict the action function to one period of the phase wave artificially before equating it to the entropy. Besides, he was not clear as to how vacuum which acts like a sub-quantum thermal bath could sustain two particles with different rest masses continuously in the same region. We should keep in mind that he had not given up the concept of the point particle while attempting this intuitive jump. Therefore, although the concept of the action-entropy equivalence proposed by him was a laudable attempt, he could not put forward a consistent theory to back it up. That may explain why his attempt did not get the attention it actually deserved. All said and done, one has to salute this genius who was in the forefront in the quantum revolution during 1920s and is again in the forefront fifty years hence as the harbinger of what may be called the “sub-quantum revolution” which in all probability would take us to the “sub-quantum thermodynamics”.

References

1. V.A.Induchoodan Menon , vixra:0909.0035(2009), (quant- ph)
2. V.A.Induchoodan Menon, vixra: 0911.0017(2009), (quant-ph)
3. V.A.InduchoodanMenon , Vixra:1001.0008(2009), (quant-ph)
4. V.A.Induchoodan Menon, vixra:1004.0036(2010), (quant-ph)
5. Harvey S. Leff, Thermodynamic Insights from a One-particle Gas, Am.J.Phys. 63(10),1985, p.895-905
6. Yu.B. Rumer, M.Sh. Ryvkin, Thermodynamics, Statistical Physics, and Kinetics, Mir Publishers, Moskow,1980, p.309-322.
7. Yu.B. Rumer, M.Sh. Ryvkin, Thermodynamics, Statistical Physics, and Kinetics, Mir Publishers, Moskow,1980, p.310.
8. Arnold Sommerfield, Lectures on Theoretical Physics - Thermodynamics and Statistical Mechanics, Published by Levant Books, Kolkata, First Indian reprint (2005), p.213-20.
9. Richard C. Tolman, Relativity Thermodynamics and Cosmology, Published by Oxford At the clarendon Press, First edition (1934)
10. Louis de Broglie. Found. Of Ph*ys.,* Vol.1, 1970.(Thermodynamics of particle.)

Annexure

Treating an Ideal Gas as a Particle

We know from the relativistic thermodynamics that the pressure acting on a gas is a relativistic invariant just as its entropy is. If we denote pressure of the gas by P, its internal energy by ξ and its translational velocity by v, then we have that [1]

$$\xi + PV \quad = \quad \gamma(\xi_o + PV_o)\,. \qquad \text{(I)}$$

The corresponding relation for the momentum, **G** of the relativistic gas as a whole is given by

$$\mathbf{G} \quad = \quad \gamma(\xi_o + PV_o)\,\mathbf{v}/c^2\,. \qquad \text{(II)}$$

As pressure is a relativistic invariant, we may treat the relativistic transformation as an isobaric process. We know that in such a process, the enthalpy, denoted by H, is well defined. Since

$$dQ \quad = \quad d\xi + PdV \quad = \quad d(\xi + PV) \quad = \quad dH\,. \qquad \text{(III)}$$

where

$$H \quad = \quad (\xi + PV)\,. \qquad \text{(IIIA)}$$

Enthalpy represents the heat content of the gas. We now see that in a relativistic transformation, the enthalpy of the gas behaves exactly like the rest mass of a particle. Moreover, the translational momentum **G** also transforms like the momentum behaves like the momentum of a particle in a relativistic transformation. Combining (I) and (II), we obtain

$$H^2 - \mathbf{G}^2c^2 \quad = \quad H_o^2\,. \qquad \text{(IV)}$$

This is the equivalent of the relativistic energy-momentum relation for a particle. These similarities reinforce our conviction that a particle could be treated as a gas.

But when we extend these results to the case of the primary gas, we have to keep in mind that the primary gas does not have an independent dimension in pressure and volume. Therefore, (IIIA) will have to be modified to give

$$H \quad = \quad \xi\,. \qquad \text{(V)}$$

This means that the internal energy of a primary gas plays the role of the heat content. Accordingly, corresponding to (I) and (II) we have

$$\xi \quad = \quad \gamma\xi_o \quad \textit{and} \quad \mathbf{G} \quad = \quad \gamma\xi\,\mathbf{v}/c^2\,. \qquad \text{(VI)}$$

Since the number of microstates is an invariant in a relativistic transformation, we may re-express the two equations in (VI) in terms of the intrinsic values of energy and momentum as

$$E \quad = \quad \gamma E_o \quad \textit{and} \quad þ \quad = \quad \gamma\,\mathrm{Ev}/\mathrm{c}^2\,. \qquad \text{(VII)}$$

Reference

1. W.Pauli, Theory of Relativity, (Indian Edition), B.I.Publications, Bombay (1963), p.134-35.

Quantizing Time and Space- From the Standing Wave to the Primary Gas Structure of a Particle – V

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Abstract

The author introduces the concept of a primary gas which is an abstract gas where the microstates are occupied successively in time unlike in the case of a real gas where the microstates are occupied simultaneously. He shows that a single plane wave associated with a staphon formed by the confinement of an electromagnetic wave (photino) could be treated as a microstate of a primary gas that represents a particle. This approach makes it possible to understand the dynamics of the particle in terms of the thermodynamics of the primary gas and it offers a very simple way to quantize time and space. Time and space are shown to emerge from the intrinsic properties of the primary gas that represents particle. It is shown that the action (with a negative sign) of a particle could be identified with the entropy of the primary gas and the principle of least action could be nothing but the second law of thermodynamics. The author shows that the uncertainty relation of quantum mechanics can be derived directly from the equation for fluctuations and he explains the statistical basis of the virtual interactions.

1 Introduction

We saw that an electron could be represented by standing electromagnetic half wave called “staphon” which is formed by the confinement of a single circularly polarized electromagnetic wave called “photino”. It is observed that such a standing wave not only acquires rest mass and the electric charge but also the half spin of a particle like electron and the plane wave representation becomes most appropriate when it acquires translational velocity [1],[2]. The staphon structure of a particle is seen to be compatible with the Dirac equation and it directly leads to the Pauli’s exclusion principle[2],[3]. It was suggested that the confinement of the photino may be effected by the interactions with the Higgs waves created in the vacuum fluctuations[4]. The approach outlined above is based on the confinement of the photino and therefore the particles created could only be leptons. We know going by the Klein-Kaluza approach the electromagnetic field could be attributed to the existence of the vibrations in the fifth dimension which curves into itself. In that sense, the electromagnetic wave could be taken as an entity arising out of vibrations in the fifth dimension. We now assume that there could be composite waves which have vibrations in other dimensions also so that on confinement they could represent quarks having the properties like the color and the charm. We shall now onwards use the term staphon in more general sense to represent the standing half wave formed by the confinement of a composite wave.

It is now proposed that the states occupied by the staphon in its interactions with the vacuum fluctuations could be studied as a primary gas, which is an ideal gas where the microstates are occupied successively in time. The basic idea is to treat a particle as a thermodynamic system and to understand the dynamics of the particle in terms of the thermodynamic analogues. We know that a gas is one which is made up of millions of molecules moving randomly in all directions within a container having a definite volume. Statistically speaking, a gas is a system containing trillions of molecular states that exists at an instant with each molecule having a specific value of energy and momentum. We might as well imagine an abstract gas which is formed by trillions of molecular states occupied by a single molecule in its successive interactions with the walls of the container. We know that statistically speaking these two gases are equivalent. The only difference is that in the first case, all the states are occupied simultaneously, while in the second case they are occupied successively. However, this will not be important if we are dealing with a system in equilibrium which by definition is independent of time. We know that the thermodynamics of a real gas can be understood in terms of the statistical properties of the molecular states forming it. Based on the statistical equivalence of the two gases, it appears reasonable that the laws of thermodynamics could be extended to the abstract gas also. We shall call this abstract gas “the primary gas” and study the thermodynamics of this gas in more detail. Later we shall show that the constituent of the primary gas need not be a molecule, it could as well be a staphon.

The idea of a gas formed by a single molecule was first introduced by Leff a few years back. He called it ‘model gas' and it was constructed by the states occupied by a single particle with certain mass and kinetic energy confined in a one dimensional box [5]. His approach has been based on the kinetic theory and assumes the classical action function for the particle. In the entire analysis, it is presumed that the gas as a whole is stationary with regard to the observer. However, the approach we propose here involves the introduction of the additional thermodynamic coordinates like the translational velocity of the gas and the corresponding translational momentum.

2 Stochastic Equivalence

Let us take a real gas with N molecules contained in a vessel with conducting walls kept in a thermal bath. For the sake of generality, we shall use the term “microstates” to denote the state occupied by the molecules. We assume here that the molecular density of the gas is quite low and therefore for all practical purpose the molecules do not interact with each other; they interact only with the walls of the container. In short, we may treat it as an ideal gas. Let this gas be in thermal equilibrium with the macro-setting. We shall also assume that the ensemble of this gas contains R copies, where R could be taken as very large.

We may now imagine another identical container with only one molecule kept in it. It is possible to think of a very large number of states formed by this single molecule in its successive interactions with the container walls. Note that the average energy of the molecule will be determined by the temperature of the container walls as the system is in thermal equilibrium. If we take L number of such micro-states and divide it into 'R' groups, each group being comprised of N successive micro-states, where R and N are very large then, our assumption is that for each group having a certain profile of occupation of micro-states we can identify with an ensemble copy of the real gas having the same profile of occupation of the micro-states. Such a group of states occupied by a single molecule in successive interactions could be treated as an abstract gas. Here the sequence of the occupation of micro-states is not relevant, only the frequency profile of the occupation of the micro-states is important. In other words, if there are two abstract gases whose micro-states do not tally sequentially although the frequency of occupation the micro-states are identical, then the two abstract gas states may be treated as identical. In simple terms, what we are doing is analogous to comparing the outcomes of one die thrown thousand times with those of thousand dice thrown simultaneously. Obviously both results would be equivalent and the sequence of the outcomes in the case of a single die is unimportant in this process. Let us call this "the principle of the stochastic equivalence".

We know that the thermodynamic relations of a real gas can be derived purely on the basis of the probability of occupation of different gas states using the concepts of canonical ensemble and grand canonical ensemble. For example, for a given equilibrium macroscopic condition determined by, say, constant temperature, pressure and chemical potential, the probability of occupation will have a sharp maximum like a delta function for states with a particular value of energy, volume and number of molecules. The thermodynamic variables like energy, volume, number of molecules etc. of a real gas could be identified with such average values. The thermodynamics of the gas could be understood based on the ensemble approach by suitably defining entropy as a statistical quantity [6]**.** The stochastic equivalence of the ensemble of the primary gas with that of a real gas leads us to the conclusion that thermodynamics of a real gas and that of an primary gas are equivalent.

However, on certain aspects there are differences between a real gas and a primary gas. We shall examine these aspects in detail shortly. With this purpose in view, we shall construct a primary gas with its micro-states occupied by a single particle and derive the thermodynamic relations. Subsequently, we shall introduce translational velocity and momentum as additional thermodynamic variables and generalize the thermodynamic relations.

3 Thermodynamics of the Primary Gas

We assume here that the primary gas is formed by a single particle. Here the particle could be a single molecule or even an electron. The spread of the states occupied by the particle in time contributes to certain interesting properties. To avoid confusion regarding the identification of the particle state and that of the primary gas, we shall now onwards call the particle state as the micro-state of the primary gas. It should be noted that the number of microstates in the primary gas is not constant, but increases as the time interval within which it is defined increases.

Let us now take a real gas whose number of molecules and volume are fixed. If we treat the system as a quantum mechanical one with quantized values for energy, then the notation for the energy of the gas will be ξ_k where 'k' denotes a complete set of quantum numbers. We now propose to derive the thermodynamic relations of the primary gas based on its equivalence to the real gas [7].

Let us consider L number of successive micro-states of the primary gas which are subdivided into R groups, each group being formed by N successive micro-states such that

$$N\,R \quad = \quad L \quad . \tag{1}$$

It should be noted that *R* represents the ensemble of the primary gas. *L* and *R* could be made as large as required. Let us denote by n_k the number of the ensemble copies for which the energy is ξ_k. Then, the ratio, n_k/R would give the probability for the gas to occupy the states with the specific values ξ_k. The logical foundations of the approach are based on the following. Let us consider a giant abstract gas formed by combining all of the *R* copies of the ensemble. Then the states with a given value ξ_k will play the same role as played by a box in the Boltzmann's method [8] and the number of copies n_k, the role of the number of molecules in the boxes.

The number of ways in which the copies of the ensemble can be distributed among the boxes is equal to

$$M \quad = \quad R!/\Pi_k\, n_k! \quad . \tag{2}$$

The copies of the ensemble will be assumed to be distinguishable as the primary gas state is a collection of micro-states, not an isolated micro-state. We shall later show that even if the states are taken as distinguishable, there will not be any change in the final result. We may assume that there exists a Gibbs micro-canonical distribution for the giant gas containing R copies. Therefore, the number of copies R and the total energy of all the copies $\hat{\xi}$ will be assumed to be fixed. These constraints may be expressed by the equations

$$\sum_k n_k \quad = \quad R \quad and \quad \sum_k \xi_k n_k \quad = \quad \hat{\xi} \quad . \tag{3}$$

The approach we propose to follow is based on the assumption that the most probable state of the gas can be identified with that particular gas state which recurs most often in the ensemble. For this purpose, we should find out the maximum of M in (2). Keeping this in view, let us define a function σ given by

$$\sigma \quad = \quad K\,\ln M \quad , \tag{4}$$

where K is the Boltzmann's constant. We know that σ has a maximum when M has a maximum. So our effort boils down to finding the maximum of σ. Taking the conditions imposed by equations in (3) and using the Langrangean method of multipliers along with Sterling formula, we look for the maximum of σ in the expression

$$\phi \quad = \quad \sigma - \beta\hat{\xi} + (\delta + 1)R \quad , \tag{5}$$

where $\sigma \quad = \quad K\,\ln M \quad = \quad K\,R\ln R - K\sum n_k \ln n_k\,.$

Differentiating the n_k's as independent quantities and equating the first derivative of ϕ to zero, we obtain

$$n_k \quad = \quad e^{(\delta-\beta\xi_k)/K} \quad . \tag{6}$$

The Langrangean multiplier δ is obtained by substituting for n_k from (6) into (3) and is given by

$$e^{\delta/K} \quad = \quad R\Big/\Big(\sum e^{-\beta\,\xi_k/K}\Big) \tag{7}$$

and $$n_k\big/R \quad = \quad \big[Y(\beta)\big]^{-1}\, e^{-\beta\,\xi_k/K}\,, \quad where \quad Y(\beta) \quad = \quad \sum e^{-\beta\,\xi_k/K}\,. \tag{8}$$

Since the number of the ensemble copies and as a consequence n_k can be made as large as desired, the application of the Stirling's formula is quite justified. The ratio $n_k/R = W_k$ is the probability that the system is in the state with energy ξ_k. If the energy level ξ_k is degenerate by a factor g_k, then (8) may be modified to give

$$W_k \quad = \quad n_k\big/R \quad = \quad \big[Y(\beta)\big]^{-1} g_k \;\; e^{-\beta\,\xi_k/K}\,, \tag{9}$$

where $$Y(\beta) \quad = \quad \sum_k g_k \;\; e^{-\beta\,\xi_k/K} \quad . \tag{9A}$$

Let us now examine the thermodynamic significance of β. Using (4), let us introduce a function S given by

$$S \quad = \quad R^{-1}K \,\ln M \quad = \quad \sigma\big/R \;\; . \tag{10}$$

S can be identified with entropy. The basis for identifying σ/R with entropy is firstly the additivity of the quantities *S* and σ. The second reason is that *S* and σ both reach their maximum values in the most probable state or the equilibrium state. The need to divide by *R* in (10) arises due to the fact that we want to determine the entropy of a real system which is represented by just one copy of the ensemble. Substituting the value of σ and using Sterling's formula to simplify, we get

$$S \quad = \quad -\,K \sum_k \,(n_k\big/R)\,\ln(\,n_k\big/R) \;\; \cdot \tag{10A}$$

This equation determines the entropy of an arbitrary state of a system, whether in equilibrium or otherwise. Substituting for n_k/R from (9) into (10A), we obtain

$$S \quad = \quad -\,K \sum_k \,(n_k\big/R)\{-\,\beta\xi_k\big/K \,-\, \ln Y(\beta)\,\} \;\; \cdot$$

Note that (n_k/R) denotes the probability to occupy the k^th^ energy state. Therefore, the term Σ (n_k/R) ξ_k will give the average value of ξ_k and we obtain

$$S \quad = \quad \beta\overline{\xi} \,+\, K \,\ln Y(\beta) \;\; , \tag{11}$$

where $\overline{\xi} = \hat{\xi}/R$ is the mean value of the energy of the system. It should be noted that the mean value $\overline{\xi}$ as per the statistical approach represents its value in the thermodynamic sense.

Let us now onwards, for the sake of notational convenience, replace $\overline{\xi}$ by ξ keeping in mind that it represents the average value, while ξ' will be used for the individual value in place of ξ_k. Accordingly (9) could be written as

$$W(\xi') \quad = \quad Y^{-1}\; g(\xi')\, e^{-\beta\xi'/K}\,, \tag{12}$$

where $$Y \quad = \quad \sum \; g(\xi')\; e^{-\beta\xi'/K}\,. \tag{12A}$$

The mean energy and the entropy of the system are given by