The mathematical expression and approximate numerical value of the counterexample of the Riemann hypothesis

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abstract

In the process of searching for counterexamples of the Riemann hypothesis using a computer, I accidentally discovered the possibility of counterexamples in a region. After delving into the derivation of mathematical formulas, I found that a perfect mathematical expression can be used to describe them. The position of the counterexample is right next to the area where s=1.

For Riemann zeta functions

$$\zeta(s) \equiv \begin{cases} \sum_{k=1}^{\infty} \frac{1}{k^s} & \Re(s) > 1 \\ \frac{1}{1-2^{1-s}} \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^s} & 0 \le \Re(s) \le 1 \\ 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s) \zeta(1-s) & \Re(s) < 0 \end{cases}$$

After some deduction, it can be concluded that

$$\xi(s) = \frac{\eta(s)}{1-2^{1-s}}$$

$$\eta(s) = \eta(r+it) = \sum_{n=1}^{\infty} \frac{(-1)^n \cos(-t \ln n)}{n^r} + i \sum_{n=1}^{\infty} \frac{(-1)^n \sin(-t \ln n)}{n^r}$$

We define

$$f(r,t) = \sum_{n=1}^{\infty} \frac{(-1)^n \cos(-t \ln n)}{n^r}$$

$$g(r,t) = \sum_{n=1}^{\infty} \frac{(-1)^n \sin(-t \ln n)}{n^r}$$

$$H(r,t) = f(r,t) f(r,t) + g(r,t) \; g(r,t)$$

It can be inferred that

$$H(r,t)=\sum_{n=1}^{\infty}\sum_{m=1}^{\infty}\frac{(-1)^{n+m}\cos\left(t(\ln n-\ln m)\right)}{(nm)^{r}}$$

When r=0.997 and t=9.007, there is

1

$$\sum_{n=1}^{100}\sum_{m=1}^{100}\frac{(-1)^{n+m}\cos\left(t(\ln n-\ln m)\right)}{(nm)^{r}}$$

$= 0.00330933761355$

2

$r=0.997$

−10 … 10

3

$t=9.007$

−10 … 10

4

$$\sum_{n=1}^{100}\frac{(-1)^{n}\cos(-t\ln n)}{n^{r}}$$

$= -0.00706827807124$

5

$$\sum_{n=1}^{100}\frac{(-1)^{n}\sin(-t\ln n)}{n^{r}}$$

$= 0.057090954263$

Means

η（0.997+9.007i）=0

Bring in

$$\xi(s)=\frac{\eta(s)}{1-2^{1-s}}$$

obtain

$$\xi(s) = \frac{\left(1-2^{1-r}\cos(t\ln 2)\right)\sum_{n=1}^{\infty}\frac{(-1)^n\cos(-t\ln n)}{n^r} + 2^{1-r}\sin(t\ln 2)\sum_{n=1}^{\infty}\frac{(-1)^n\sin(-t\ln n)}{n^r}}{\left(1-2^{1-r}\cos(t\ln 2)\right)^2+\left(2^{1-r}\sin(t\ln 2)\right)^2}$$

$$+ \frac{\left(1-2^{1-r}\cos(t\ln 2)\right)\sum_{n=1}^{\infty}\frac{(-1)^n\sin(t\ln n)}{n^r} - 2^{1-r}\sin(t\ln 2)\sum_{n=1}^{\infty}\frac{(-1)^n\cos(-t\ln n)}{n^r}}{\left(1-2^{1-r}\cos(t\ln 2)\right)^2+\left(2^{1-r}\sin(t\ln 2)\right)^2} i$$

ξ（0.997+9.007i）=-1.41730580011−0.130738376876i

This is because both the numerator and denominator are almost zero at the same time More generally, we have

η（1+2πni/ln2）=0(N is an integer and n！ = 0)

In the next step of work, for ξ (r+ti)=0, I will provide the expression for r and t.If the accuracy requirement is not high, it can be considered that r=1，t=2πn/ln2

References

1. **viXra:2005.0284** The Riemann Hypothesis Proof **Authors:** Isaac Mor