

Realization of quasi-quanta via the forced contraction of loops

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October 13, 2023

Abstract

The contraction of a loop on a string in the orthogonal time direction is contemplated. Its relationship to a certain mathematical concept, forcing notions, is examined. In addition, we evaluate local systems on the worldline of a particle traveling in the positive timelike direction.

1 Contracting loops

Definition 1 *A slice, s , of a manifold M , is a stationary frame \mathfrak{F} whose subobjects are potentials of a gauge field.*

Let \mathfrak{S} be a collection of slices of a manifold \mathbb{R}^n , n even, and let there be a path $\rho : \text{inf}(\mathfrak{S}) \rightarrow \text{sup}(\mathfrak{S})$. The product, $\rho \cdot \mathfrak{S} = \{p, g\}$, defines an equivalence class of diffeotopic smooth manifolds spanning a lightcone \mathbb{L} of events.

For every segment of the vector $\vec{\rho}$, define a *portable loop*, p_ℓ .

Definition 2 *A portable loop is a neighborhood about a fixed point p_i along the path spanned by $\vec{\rho}$, such that $p_\ell^{-1} \circ p_\ell$ is the identity on p_i .*

Suppose the region enclosed by some p_ℓ contains non-trivial topological (i.e., physical) data, such as a particle. Then, we say that the loop is not contractible to the point \hat{q} . We impose the following:

Axiom 1 *For all non-contractible portable loops, there exists either (or both) of the following:*

- *A forcing notion, \Vdash , such that $\hat{q} \Vdash p_\ell^{\text{contr}}$*
- *A frame \mathfrak{F}^+ such that $p_\ell^{\text{contr}} \in \mathfrak{F}^+$*

These essentially (although not equivalently) amount to designating a map $t^+ : p_\ell \rightarrow \{*\}$; in other words, the successor function is applied to the dimension of time (in the second case), which *forces* the suspension of a quasi-quantum \hat{q} , thus transforming the non-trivial space it (virtually) occupies into a generic free variable.

Let us imagine that p_ℓ has the following properties, where \mathcal{U} stands for the neighborhood it encloses:

1. Pressure increases over time as $p_\ell \longrightarrow \{*\}$
2. Temperature increases within \mathcal{U} as it shrinks
3. The average number of molecules in \mathcal{U} decreases as p_ℓ contracts

Then,

Theorem 1 *Boltzmann's constant, k_B , remains constant as $p_\ell \longrightarrow \{*\}$.*

Proof We have

$$k_B = \frac{PV}{TN}$$

Assuming the decrease in the number of molecules in \mathcal{U} is proportional to the increase in temperature, the differences cancel out; assuming that pressure increases as the volume of \mathcal{U} decreases cancels out the terms in the numerator. Thus, k_B is constant under the map $p_\ell \longrightarrow \{*\}$.

1.1 Realization

In this paper, we envision that there is a certain *semitotic* propensity of quasi-quanta (virtual particles) to become actualized (topologically realized) as a result of the satisfaction of Axiom 1. Thus, the promotion of a wavefunction to a particle can be interpreted either as a class-theoretic (mathematical) operad, or as physical kinematics occurring across time in \mathbb{L} . The equation:

$$\mathbb{I} \times_h \hat{q}_{pot} \longrightarrow q = \mathfrak{F}^+ = \hat{q} \Vdash p_i^{contr}$$

relates the two paradigms to the production of a particle from an operator h mediating between an interval \mathbb{I} and the potential energy of a quasi-quanta. We can think of this as a sort of *crossed module*, \mathfrak{m} , acting on the group \mathfrak{g}_p of generators for the Lie group of the particle's neighborhood along a worldline. We remark here that the world-line, \mathfrak{W} , is a special case of the path ρ defined above, which has been restricted to timelike distinct sections of \mathbb{L} .

Motion along \mathfrak{W} obeys the following Leibniz rule:

$$(\mathfrak{m}i + \mathfrak{m}j)k = \mathfrak{m}ik + \mathfrak{m}jk = \partial\omega^{-1}\mathfrak{m}ijk$$

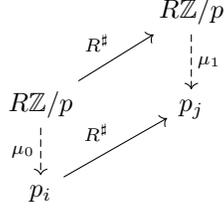
where ω is a differential form of dimension equal to a particular Lagrangian submanifold of M . Thus, the function

$$f(\mathfrak{m}) = \mathfrak{u} \xrightarrow{\theta} \mathfrak{u}' \subset_{\mathbb{N}} \mathfrak{g}_p$$

is transitive for all smooth paths which non-trivially intersect covers of neighborhoods over $p_i \in M$. Classically, $f(\mathfrak{m})$ defines a formally flat function

acting on trivial transport fibers. In our case, each segment $T_x(p_i)$ tangent to the moment of a particle yields a foliation along a boundary $\partial\mathcal{U}$ which projects to a singularity $\mathfrak{b} \in \mathbb{L}$, where a measurement either does or does not occur.

We may choose to enrich each copy of $\partial\mathcal{U}$ with a connected ring of polynomials modulo a certain prime, p , giving us $R\mathbb{Z}/p$. Correspondingly, transport of the particle p_i is described as a *tilting*, R^\sharp which forms an \aleph -cell about a wave packet.



In the above parallelogram, p_i, p_j are distinct particles which share the same wavefunction $\Psi(p, t)$, and μ_0, μ_1 are measurements, which are, respectively, projection onto the first and final coordinates of a local system. Directedness of the arrow $p_i \rightarrow p_j$ denotes the irreversibility of time due to the second law of thermodynamics.

Definition 3 A local system, $LocSys(h)$, is a closed, portable monoid equipped with a counting operad on lines.

Local systems naturally come with a bundle, Bun_G , which induces a simplicial stratification over a Hausdorff convex neighborhood of a manifold M . A local-system is G -equivariant with respect to reordering (shuffling) of place values, and is uniquely determined (up to isomorphism) by a collection of paths $\vec{P}G$ out of any given point p . Thus, the identity of a local system is given by:

$$LocSys_{Id} = \int_0^{2\pi} \frac{\partial p_i}{\partial i} \Omega G'$$

where $\Omega G'$ is space of loops of any other Lie Group. This is essentially the Yoneda lemma for Markov blankets.

Let π_η be a map of fibers over $LocSys(M)$. We denote by $Spec_\eta$ the spectral sequence:

$$\Pi : U(1) \rightarrow \eta_{ij} \rightarrow \eta_{jk} \rightarrow \eta_{ki} \rightarrow U(1)$$

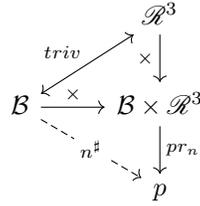
which is smooth. Denote the composition $\Pi \circ^n \Pi$ by $Nec_n(\Pi)$. One has that the canonical 2-morphisms, $\overset{b}{a} : (a, b) \rightrightarrows (c, d)$ are stable under the stack \mathcal{X}_{Top} , and the isofibrations $[\overset{b}{a}]$ are arbitrarily productive. This means that we can take the quotient $Nec_n(\Pi)/q$ and obtain a Hermitian Koszul complex, \mathcal{K}_{osz} , which preserves holonomy. Write

$$\mathcal{K}_{osz} = (LocSys(i) \times LocSys(j)) \xrightarrow{can} \Pi_{\bar{\omega}}$$

Definition 4 A Koszul complex is a global system whose interior consists of the disjoint union of the symmetric product of n local systems.

All neighborhoods \mathcal{U} , and smooth covers $\{\mathcal{U}_i\}_{i \in I}$ essentially arise as rank two restrictions of Koszul complexes. That is to say, that for each stalk f of \mathcal{K}_{osz} , there exists an infinitesimal thickening on the points of f (call them \tilde{f}_i), which are thin homotopies of rank two of one another, such that $\pi_2^2(\tilde{f} \in f)$ yields a conformal pullback to a site θ at which the functions $f(f)$ converge asymptotically.

Remark 1 For a flat bundle, $\mathcal{B} \in \mathcal{R}^3$, the collection of tangent spaces over each point $x \in \mathcal{B}$ contains a space whose projection onto the n th coordinate, $x \xrightarrow{n} \mathcal{R}^3$, converges to a point $p \in \mathcal{B} \times \mathcal{R}^3$.



The necklace, $Nec_p(\mathcal{B})$ gives the set of all maximal chains

$$\Delta \times p^{-1} \longrightarrow max|\mathcal{B}|$$

2 Creation and Transformation

Prior to the assignment of meaning to a symbol (or better yet, a symbol to a meaning), the “meaning” to be specified remains in a superposition of possible states, which we denote by \heartsuit . The so-called “creation map” defined below, is an exit path

$$\mathcal{EP}_{\heartsuit} : (x = \{\}) \longrightarrow x$$

out of the empty set, into a proposition x .

Definition 5 The creation map, Cr shall be written

$$Cr : \heartsuit \longrightarrow \{p\}$$

where $p \sim \{*\}$ for some zero-dimensional manifold.

Definition 6 The transform map, T , is given by

$$(x \rightarrow y) \leftrightarrow \exists x \vee y \rightarrow \exists y$$

where y is the top of a frame.

Remark 2 The existential quantifier used here is classical (strong). T thus represents a map $\exists^\bullet \rightarrow \exists(\sim)$. The sequence

$$\heartsuit \xrightarrow{Cr} x \xrightarrow{T} y \xrightarrow{(Cr \circ T)^{-1}} \tilde{\heartsuit}$$

is equivalent to $f^!(x, y) \circ f_!(x)$. Further, the identity on a fixed object, x , is $Cr = T^{-1}$.

Let X and Y be subsets of Z . Then, let $x \in X$ and $y \in Y$. We have $x \in^X Z$ and $y \in^Y Z$ defining a filtered inclusion relationship, such that the superscript \in^\bullet entails $\exists \bullet \in \sim$, where \sim is the least upper bound on all \bullet, \bullet' .

Each creation map corresponds to an actual measurement, μ_x over the object x , and a transformation map represents a first order differential taken over μ_x .

$$T = \mu_y - \mu_x = d\mu_x$$

2.1 Sampling Populations of Measures

Let there be a large number of measurements taken across a sample \mathfrak{s} of transform maps. We shall write

$$\frac{\Sigma(d\mu_x)}{\text{card}(\mu)} = \varphi(\mu)$$

to mean the average of the differences between each set of correlated pairs of points, x and y .

Proposition 1 Let $t^+ : [0, 1] \rightarrow [0, 1]$ be the time-step functor. We have $\lim_{t^+} T = \varphi(\mu)$.

Proof Our argument is proved by writing:

$$\varphi(\mu) = \lim_{\mu \rightarrow \infty} \frac{\Sigma(d\mu_x)}{\text{card}(\mu)} = T_\infty$$

due to the fact that t^+ is essentially the functor $\mu \rightarrow \mu + 1$. As a result, the average transformation asymptotically approximates the universal average taken over an infinite population. *Q.E.D.*

2.2 Generalized transforms and their actualization

We were motivated to form a *process-based* definition of an object. In this pursuit, we have established the following:

Definition 7 A transformation, $T(x)$, is a map $x \rightarrow ?$ such that $T^{-1}(x)$ is the identity on x .

One may be dismayed that this definition lacks a clear-cut physical interpretation. So, let us renew this definition, this time taking into account the wavefunction on a particle x :

$$T(x) = (\Psi(x) \longrightarrow ||x||) \vee (||x|| \longrightarrow ||y||)$$

Call the left-hand side of the disjunction the *actualization*. This is *one type of* creation map, but it is also implicated in the transformation process. The right hand side represents the ordinary transformation of observable eigenstates. Over a small period of time-evolution, quasi-quanta may enter or exit a given eigenstate, as parameterized by a given truth value τ . The evolution of time, $\tau \rightarrow \tau + 1$, is a form of monodromy in the F-theory description, where τ is the modulus of elliptic fibers of some locus Y . This can (and has) been used to model dynamics on intersecting seven-branes. We refer the reader to [1] for more information.

3 References

- [1] C. Cordova, *Decoupling gravity in F-theory*, (2011)