A Probabilistic[1] Proof of Goldbach's Conjecture[2]

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Probabilistic Proof of Goldbach's Conjecture is presented. Philosophical, political, and historical narratives tangentially related to Goldbach's Conjecture are provided.

Prologue

Hello everyone, thank you for your kind and generous readership //:-D This is a research paper, but I will keep it as entertaining as possible. Please enjoy-

I. Introduction to Goldbach's Conjecture

1. Historical Background

Goldbach's conjecture is a hypothesis:

[1] Professor Erdos once said, I recall, and I'm paraphrasing, "probabilistic approach to number theory may be fruitful." And we agree //:-) See https://en.wikipedia.org/wiki/Paul_Erd%C5%91s . When this author started to write this paper, he did not have a complete proof yet, as he discovered errors in his original design of the proof. But as he was still writing this paper, he fixed those errors on a daily basis for over a month. He jokingly used to title this paper, 'Probabilistic (if not problematic) Proof of Goldbach's Conjecture'. Once he finally found a complete proof on 3/19/23, the day he discovered and partially proved the 'POPS(N) function convergence theorem,' he took out the parenthetical phrase //:-)

[2] This paper is dedicated to the People in the world who support this author's 2024 US Presidential campaign: his social media and internet Friends (in DailyMotion, YouTube, Twitter, Facebook, Instagram, SSRN, and VIXRA and other websites), his past and current in-person Friends, and his Family in Korea. Started being written on 3/14/2023. He's a secular-religious, politically independent, and a private academic. The author is running for the US President in 2024 as an independent thinker.

[3] A lawyer by trade, a scientist by hobby, a humanologist by mission, a U.S. Army veteran by record, a former computer programmer, a prior PhD candidate in computational biology (withdrawn after 2 years without a degree), a former actor/writer/director/indie-filmmaker/background-music-composer. Born in the USA, 1978. Grew up in Seoul, South Korea as a child and a teenager. Returned to America as a college student. Still growing up in America as a person //!-)

Every even number can be expressed as a summation of two prime numbers.

For instance,

$2 = 1 + 1$

$4 = 3 + 1$

$6 = 3 + 3 = 5 + 1$

$8 = 5 + 3 = 7 + 1$

...

Christian Goldbach was an attorney by trade, a mathematician by hobby, just like this author is.[4] He, an expatriate in Russia from Germany, befriended a professional mathematician named Leonhard Euler, an expatriate in Russia from Switzerland. Two lonely men in a foreign land, whose native languages are German, became naturally good friends as they also shared the interest in mathematics.

The amateur mathematician Goldbach discovered Goldbach's conjecture and the professional mathematician Euler refined that conjecture though neither of them proved it.[5] Since 1742 of the two mathematical friends' initial discussion, 281 years have passed. For nearly three centuries, both amateur and professional mathematicians from all around the world tried to prove the conjecture but no one so far achieved that goal.

Goldbach's conjecture has been a back burner hobby project for both amateur and professional mathematicians in the world for that long a time,[6] even inspiring fiction writers.[7] There have been many discussions[8] and attempts at its proof. Many mathematicians, including

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[4] See https://en.wikipedia.org/wiki/Christian_Goldbach ; https://en.wikipedia.org/wiki/Independent_scientist ; https://en.wikipedia.org/wiki/List_of_amateur_mathematicians .

[5] See https://en.wikipedia.org/wiki/Goldbach%27s_conjecture .

[6] See https://en.wikipedia.org/wiki/Henry_Pogorzelski ; https://mathoverflow.net/questions/38324/did-pogorzelski-claim-to-have-a-proof-of-goldbachs-conjecture ; https://www.degruyter.com/document/doi/10.1515/crll.1977.290.77/html ; https://www.deepdyve.com/search?query=goldbach+conjecture .

[7] See https://en.wikipedia.org/wiki/Uncle_Petros_and_Goldbach%27s_Conjecture .

[8] See https://mathoverflow.net/questions/27755/knuths-intuition-that-goldbach-might-be-unprovable ; https://math.stackexchange.com/questions/545607/proving-goldbachs-conjecture-hypothetically-probabilistic-argument ; https://math.stackexchange.com/questions/3327489/has-goldbachs-conjecture-been-proven ; https://htt219390965.wordpress.com/2020/08/23/goldbachs-conjecture/ ; https://math.stackexchange.com/questions/547858/why-goldbachs-conjecture-is-difficult-to-prove ; http://www.vedicganita.org/Goldbach.aspx ; https://www.vedicmaths.org/2000-newsletter-index/newsflash-goldbach-s-conjecture ; https://www.quora.com/How-do-I-prove-Goldbachs-conjecture . See also page 4 of http://www.ams.org/notices/200203/fea-knuth.pdf .

this author, thought that they proved it but later they or others realized some flaws or logical errors in the proofs.[9] A weaker version of Goldbach's conjecture was proven by a professional mathematician.[10]

This author has been working on the proof of the conjecture on and off for about three years. Some episodes in the author's humanology series back in the days in YouTube and nowadays in DailyMotion contain the author's past works on the proof attempts.[11]

This time around, the author made an interesting improvement and wants to present the latest proof attempt to the world. It may contain some errors and flaws but it is worthwhile to share with the world so that perhaps the future generation of mathematicians may improve upon the author's work.

2. Mathematical Background

A prime number is a number that cannot be divided.[12] The opposite is a composite number,[13] which can be expressed as a product of prime numbers.[14] In a sense, prime numbers are like purely refined metals like gold[15] (prime numbers are rare and needs to be looked for with efforts) and composite numbers are like natural ores that have several minerals mixed together. Mathematicians proved what's called prime number theorem,[16] which states that there are less and less prime numbers as numbers get big. But Euclid's theorem proved by Euclid also states that there are infinitely many primes.[17] It's just that prime become rarer and more sparse among big numbers compared to small numbers. That is, the average distance between two adjacent primes increases.

Centuries ago, mathematicians used to regard '1' as a prime. Later on, mathematicians agreed to exclude '1' from the set of primes, because in the multiplicative world, '1' is always a prime factor of any number, rendering it trivial in prime factorization. But, in additive world,

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[9] See https://www.academia.edu/35560601/A_simple_proof_of_Goldbachs_conjecture ; https://math.stackexchange.com/questions/3735655/about-cohens-proof-for-goldbachs-conjecture ; https://vixra.org/pdf/1702.0150v1.pdf ; https://hal.science/hal-01900627/document ; https://www.researchgate.net/publication/323377615_Goldbach_conjecture_proof ; https://goldbachconjectureproofs.com/Goldbach%20ConjSbm2.pdf ; https://vixra.org/pdf/1308.0032v1.pdf . Whether correct or not, the attempted proofs are both educational and inspirational and the author enjoyed studying them, but the author came up with the ideas in this paper on his own, independently.

[10] See https://arxiv.org/pdf/1305.2897v4.pdf ; https://en.wikipedia.org/wiki/Goldbach%27s_weak_conjecture .

[11] See https://www.youtube.com/watch?v=0b_DiVbfjoo ; https://www.dailymotion.com/video/x8j2sl2 .

[12] See https://en.wikipedia.org/wiki/Prime_number .

[13] See https://en.wikipedia.org/wiki/Composite_number .

[14] See https://en.wikipedia.org/wiki/Integer_factorization .

[15] See https://en.wikipedia.org/wiki/Gold .

[16] See https://en.wikipedia.org/wiki/Prime_number_theorem .

[17] See https://en.wikipedia.org/wiki/Euclid%27s_theorem .

where Goldbach's conjecture belongs to, we should reverse that mathematical convention and include '1' back to the prime number set.

Also, '2' has always been included in the prime set, as in multiplicative context, 2 is a very important prime factor, as its absence or presence in the prime factorizations categorizes all the numbers into the odd set and the even set. But, in additive context of Goldbach's conjecture, we should exclude '2' in prime set for two reasons. First, '2' is the only even prime number and all other primes are odd numbers. Second, 4 is the only even number that can be expressed as the summation of two even primes, and all other even numbers can only be expressed as the summation of two odd primes. We are interested in general cases and '2' gets in the way of generalization, so we will remove it from consideration. We will only deal with odd prime pairs that sum up to an even number.

3. An Algorithm to find Goldbach Pairs

A Goldbach pair is a pair of odd primes that sum up to an even number. Goldbach pair set of an even number, 'N', is a set of Goldbach pairs that sum up to 'N'. An algorithm to find Goldbach pairs of an even number is as follows. First, split the number in the middle and express it as an addition of two equal numbers. Second, if the two numbers are both even numbers, pass over '1' from the right number to the left number. Third, pass over '2' from the right number to the left number. Fourth, continue that process of passing over '2' from the right to the left until the right number becomes '1'. Now you have a set of all odd pairs that sum up to 'N'. From that set of odd pairs, remove pairs that contain composites. Then you have left with the set of Goldbach pairs, the Goldbach pair set. For example,

44 *= 22 + 22*

= 23 + 21

= 25 + 19

= 27 + 17

= 29 + 15

= 31 + 13 ← *Goldbach's Pair*

= 33 + 11

= 35 + 9

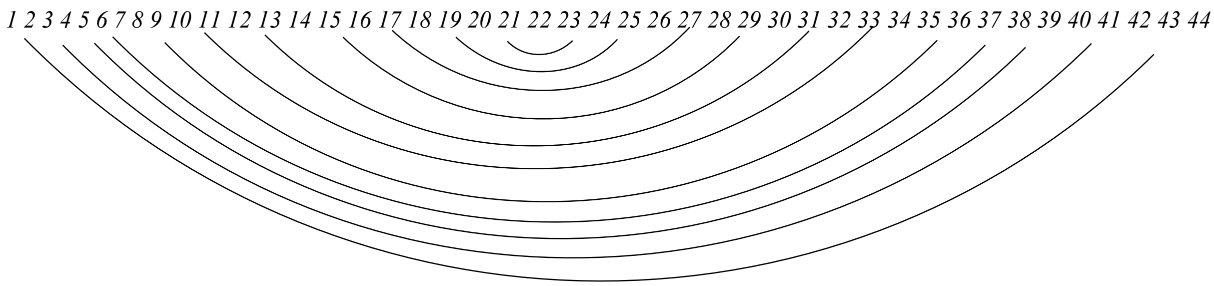
= 37 + 7 ← *Goldbach Pair*

= 39 + 5

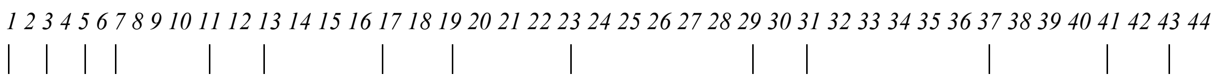
$= 41 + 3$ ← *Gold Pair*

$= 43 + 1$ ← *Gold*

One interesting fact about the odd pairs is that they're symmetric pairs, meaning that the two numbers of a pair are equidistant from the center and from the left and right ends:



Also note the distribution of prime numbers getting sparser as numbers get larger:



Basically, the intuition we can get from the two pictures above is as follows. We are pairing a small number and a large number. This is the outer pair. The left small number is very likely to be a prime but the right big number is less likely to be a prime, so the big probability of left number being a prime compensates for the small probability of the right big number being a prime.

For an inner pair, left number is not to small, so their probability of being a prime is moderately big. The right number is not too big, so their probability of being a prime is moderately small. Again, they compensate each other like yin and yang, in a very symmetric and duodualistic way.[18]

Also, notice that, as the even number N gets bigger and bigger, it has more number of odd pairs under it, increasing the probability of the odd pair being consisting of two odd primes. With computers,[19] mathematicians verified that Goldbach's conjecture is true up to about 10^{18}. So we are not really worried about small N in the proof of Goldbach's conjecture. We wonder, when N is very big, whether it is possible that N has no odd prime pairs summing up to N. If we prove that is possible, then that would be a disproof of Goldbach's conjecture. If we prove that

[18] See the author's past humanology papers at https://papers.ssrn.com/sol3/cf_dev/AbsByAuth.cfm?per_id=4395089 and https://vixra.org/author/huhnkie_lee .

[19] See https://en.wikipedia.org/wiki/Goldbach%27s_conjecture#Verified_results

N necessarily has at least one odd prime pair summing up to N, that would be a proof of Goldbach's conjecture.

4. Prime Counting Function[20] and Prime Number Theorem[21]

Prime counting function, π(N), is the number of primes between 1 and N, inclusive. For our purpose of Goldbach's conjecture proof, we only consider where N is even, and all the pairs of odd numbers that sum to N.

Prime number theorem states:

$$\lim_{N \to \infty} \pi(N) = N/\ln N$$

Which means, if N is big enough,

$$\pi(N) \approx N/\ln N$$

Let's look at the right side of the equation above.[22] If we use L'Hopital's rule,[23] as it is infinity over infinity type, the right side becomes N. So,

$$\lim_{N \to \infty} \pi(N) = \lim_{N \to \infty} N = \infty$$

Which also confirms Euclid's theorem that states there are infinitely many primes.

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[20] See https://en.wikipedia.org/wiki/Prime-counting_function .

[21] See https://en.wikipedia.org/wiki/Prime_number_theorem .

[22] Also notice that the left side of equation is an integer (the number of primes between 1 and N), while right side of equation is a fraction (logarithm is typically an irrational number). It is not exact equality but an asymptotic approximation, so we are relaxing the mathematical equality using limits and calculus.

[23] See https://www.mathsisfun.com/calculus/l-hopitals-rule.html ; https://en.wikipedia.org/wiki/L%27H%C3%B4pital%27s_rule . Basically the rule says the limit of a fraction is the same as the limit of another fraction both of whose numerator and denominator are differentiated. N' = 1, and (ln N)' = 1/N. 1/(1/N) = N. So lim (pi(N)) = lim (N/ln N) = lim (1/(1/N)) = lim N = infinity.

II. A Proof of Goldbach's Conjecture

1. Sketch and Overview of the Proof

The concept of the proof is surprisingly simple and it requires only non-math-major college level knowledge of mathematics.

Basically, we look at all the symmetric odd pairs that sum to N. There are N/4 such odd pairs, because there are N/2 odd numbers and we're pairing them, which results in another division by 2.

And the odd pair has a small left number and a large right number.[24] We want to calculate probability of the left number being a prime, and we calculate the probability of the right number being a prime. Since there is no intersectionality or dependency between the two simultaneous events (left number being a prime and right number being a prime at the same time), we can bypass the Bayesian issue[25] and safely use the product rule.[26]

Basically, we are calculating the probability of the left odd number being a prime, and the probability of the right odd number being a prime, both at the same time, by multiplying the two probabilities. After that, we add up all such possibilities of all the odd pairs, using the addition rule.[27]

If we can prove that final probability (i.e., the probability that there is at least one odd prime pair summing to N) is more than zero, that would be a Goldbach's conjecture proof.

And because we are wondering the asymptotic behaviorism of that probability, we will let N goes to infinity and make use of L'Hopital's rule and other techniques in calculus in order to simplify the equations and terms.

2. Probability of an Odd Number being a Prime under N ("POP(N)" as its acronym)

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[24] Of course the left and right odd numbers can be equal numbers, like 10 = 5 + 5.

[25] See https://en.wikipedia.org/wiki/Bayesian_probability ; https://en.wikipedia.org/wiki/Bayes%27_theorem .

[26] See https://byjus.com/question-answer/what-is-product-rule-in-probability/ ; https://en.wikipedia.org/wiki/Independence_(probability_theory) . Actually, later the author found out there is some intersectionality issue, but as we will see later in this paper, it becomes a non-issue. The author, as of writing these days, has been constantly experiencing trial and error correction routines, so far, like five times in the past week alone //:-)

[27] See https://corporatefinanceinstitute.com/resources/data-science/addition-rule-for-probabilities/ ; https://corporatefinanceinstitute.com/resources/data-science/addition-rule-for-probabilities/ https://proofwiki.org/wiki/Addition_Law_of_Probability ; https://en.wikipedia.org/wiki/Probability . Of course, technically, we need to do the subtraction with intersection part, but later, it turns out that we don't need to worry about that scenario.

Let us look at odd numbers between 1 and N. Again, we assume that N is an even number. There are N/2 odd numbers. Again, we only consider odd primes. There are $\pi(N)$ odd primes between 1 and N. Well, for simplicity, we'll use equality symbol in lieu of approximation symbol. So, the Probability of a random Odd number under N being a Prime is:

$$POP(N) = \pi(N) / (N/2) = 2\, \pi(N) / N$$

Next, let us pick a two natural numbers under N. Let's call them x and y. x is less than y. And let us make a set of all odd numbers between x and y, and denote that set as [x, y]. The number of primes under y is $\pi(y)$. The number of primes under x is $\pi(x)$. Then the number of primes between x and y is:

$$\pi(y) - \pi(x)$$

The number of odd numbers between x and y is:

$$(y - x) / 2$$

Then, the Probability of an odd number between x and y being a prime is:

$$\begin{aligned} POP([x, y]) &= (\pi(y) - \pi(x)) / ((y - x) / 2) \\ &= 2(\pi(y) - \pi(x)) / (y - x) \\ &= 2(y / \ln y - x / \ln x) / (y - x) \\ &= \frac{2(y \ln x - x \ln y)}{(y-x) \ln x \ln y} = \frac{21\ \frac{x^y}{y^x}}{(y-x)\, ln\, x\, ln} \end{aligned}$$

In particular, when y = x + 2,

$$POP([x, x+2]) = \frac{(x+2)\ln x - x\ln(x+2)}{\ln x \ln(x+2)}$$

Without losing generality,[28] let's assume x is an even number. Then the formula above represents the asymptotic probability of any odd number to be a prime number, which is 'x+1', between 1 and N.

Then, the probability of the odd number 'x+1' being a prime would be:

$$POP(x+1) = POP([x, x+2]) = \frac{(x+2)\ln x - x\ln(x+2))}{\ln x \quad (x+2)}$$

If we let the odd number to be z,

$$z = x + 1$$

$$POP(z) = \frac{(z+1)\ln(z-1) - (z-1)\ln(z+1)}{\ln(z-1)\ln(z+1)}$$

$$= \frac{\ln\frac{(z-1)^{(z+1)}}{(z+1)^{(z-1)}}}{ln(z-1)\, ln(z+1)}$$

Next, let us consider the following. From prime number theorem, we know that prime numbers get rare and rare. It means that the density of prime gets smaller. Intuitively then, the probability of a random odd number being a prime would get smaller too. Let us prove it.

Let 'r' be a positive variable. Then 'z+r' represents a number larger than z. We want to prove that POP(z+r) is smaller than POP(z). To do so, we need to prove the following hypothesis:

[28] See https://en.wikipedia.org/wiki/Without_loss_of_generality .

$$\lim_{r\to\infty}\left(\frac{POP(z)}{POP(z+r)}\right) > 1$$

Let us plug in the numbers and letters to the left side of the inequality above:

$$\frac{POP(z)}{POP(z+r)} = \frac{(z+1)\,ln(z-1) - (z-1)\,ln(z+1)}{ln(z-1)\,ln(z+1)} * \frac{ln(z-1+r)\,ln(z+1+r)}{(z+1+r)\,ln(z-1+r) - (z-1+r)\,ln(z+1+r)}$$

Here, we treat z as a constant and r as the only variable. So the left side of the multiplication of the right side of the equation, we can treat it as a positive constant, given that z is a positive odd number larger than 1. Let us look at the right-most fraction above:

$$\frac{ln(z-1+r)\,ln(z+1+r)}{(z+1+r)\,ln(z-1+r) - (z-1+r)\,ln(z+1+r)}$$

The numerator of this fraction goes to infinity as r goes to infinity. Meanwhile, the denominator goes to zero as r goes to infinity. That means, as r goes to infinity, the fraction above goes to infinity, because an infinity divided by zero is infinity. And infinity times a positive constant is an infinity. And infinity is large than one. We just proved that POP function is a decreasing function. We'll name this proven fact as 'decreasing POP lemma',[29] as it will play a crucial role later in our proof of Goldbach's conjecture.

3. <u>Product Rule and Independence in Probability Theory</u>

Let's say we have a blue deck of cards on the left and red deck of card on the right, each of which an odd number is written on:

1 3 5 7 9 11   11 13 15 17 19 21
| | | | | | | | |

[29] See https://en.wikipedia.org/wiki/Lemma_(mathematics) .

A deck of cards contains consecutive odd numbers. The two decks of cards may be far apart, adjacent, or even overlap in the coverage of the consecutive odd numbers.

Now let us calculate the probability of a random blue card contains a prime number, that is, Probability of an Odd number being a Prime ("POP") in this window between 1 and 11 inclusive:

$$POP([1, 11]) = 5 / 6$$

Next, let us calculate the same POP for the red deck of cards:

$$POP([11, 21]) = 4 / 6$$

Next, let us calculate the probability where a random blue card is a prime, and also a random red card is a prime:

$$POP([1, 11] \; \& \; [11, 21]) = 5 / 6 * 4 / 6 = 5 / 9$$

Now, it is time to verify if the application of the product rule above is correct, as we did not use the Bayesian conditional probability, because we assumed independence of the two events.

The author recalls his middle school, in Seoul, South Korea, mathematics teacher back in 1992 advised him, and I'm paraphrasing, "Huhnkie, if a homework probability problem confuses you and if you have enough time to solve it, systematically enumerate all the cases till you see a repeating pattern." This author uses systematic enumeration method quite often in his independent research and study of number theories.

So, let us enumerate all the possibilities. First, let us see all the odd pairs from the blue decks and the red decks:

$\underline{1\ 11}$	$\underline{1\ 13}$	$1\ 15$	$\underline{1\ 17}$	$\underline{1\ 19}$	$1\ 21$
$\underline{3\ 11}$	$\underline{3\ 13}$	$3\ 15$	$\underline{3\ 17}$	$\underline{3\ 19}$	$3\ 21$
$\underline{5\ 11}$	$\underline{5\ 13}$	$5\ 15$	$\underline{5\ 17}$	$\underline{5\ 19}$	$5\ 21$

7 11	*7 13*	*7 15*	*7 17*	*7 19*	*7 21*
9 11	*9 13*	*9 15*	*9 17*	*9 19*	*1 21*
11 11	*11 13*	*11 15*	*11 17*	*11 19*	*11 21*

POP([1, 11] & [11, 21]) = 20 / 36 = 5 / 9

The reason why we can bypass the complexity of Bayesian conditional probability is that the two event are independent. We have two separate decks of cards (representing two ranges of consecutive odd numbers) and pulling a random card from one deck is an independent event from pulling a random card from the other deck. That is why we can safely multiply the two probabilities to find the joint probability.[30] We will name this proven fact as 'independent POP lemma', as it will play an instrument role later in our proof of Goldbach's conjecture.

4. Recursive Binary Fission Model

The methodology in this section is designed to help our distinguished readers understand our approach and concept in our proof of Goldbach's conjecture.

So far, we covered three main concepts. First, if a Goldbach pair exists, it is a member of a set of symmetric odd number pairs that adds up to N. Second, we can calculate the asymptotic probability of an odd number to be a prime, in any given window (i.e., a range of consecutive odd numbers). Third, we can safely use product rule to calculate the joint probability of the double occurrences of odd primes in two different windows of consecutive odd numbers, by simply multiplying the two probabilities, bypassing Bayesian complexity.

Now, let us combine the three aforementioned concepts. Consider the array of all the natural odd numbers from 1 to N-1 (recall N is even and we only consider odd numbers in this paper). The set of lower left numbers, we'll call it L, and the set of the other half larger right numbers, we'll call it R:

1 *N/2* *N*

L R

[30] See https://corporatefinanceinstitute.com/resources/data-science/joint-probability/ ; https://byjus.com/maths/joint-probability/ ; https://en.wikipedia.org/wiki/Conditional_probability ; https://en.wikipedia.org/wiki/Independence_(probability_theory) .

We can calculate the probability of a random odd number from the set L, another from R, and multiply the two probabilities. But the problem there is that both L and R are big sets and if we pick random numbers from both, most likely they will not add up to N, disqualifying them as Goldbach pair (i.e., two odd primes adding up to N). So, we divide each set into two again:

1 *N/4* *N/2* *3N/4* *N*

LL LR RL RR

Now we have four sets: left left, left right, right left, right right. Then we pick an odd number from LL, another from RR, and multiply their probabilities of being an odd prime. LL-RR paring would be the outer pair.

Next, we pick an odd number from LR, another from RL, and do the same. This would be the inner pair. As we can see, this second binary fission would give us a better chance than the first binary fission, because both the inner pair and the outer pair will have better chances to sum up to N. Let's do one more:

1 *N/8* *N/4* *3N/8* *N/2* *5N/8* *3N/4* *7N/8* *N*

LLL LLR LRL LRR RLL RLR RRL RRR

The total number of conducting the recursive binary fission would be:

$$\log_2 N$$

After that, each set (i.e. window or range) will contain a single odd number and each symmetric pair will add up to N, and if there is a Goldbach pair, it will be one of these symmetric odd number pairs. Basically we are encoding the "summing to N" requirement into this algorithmic procedural model of recursive binary fission methodology.

But, this model is for illustrative purpose only. When we actually calculate the probability of there being at least one Goldbach pair in the set of symmetric pairs, we will merely match odd numbers in symmetric fashion successively, like one by one, as we will see in the next section.

5. Consideration of Combined Probability

So, we have a set of symmetric odd number pairs. And how many symmetric pairs do we have?

N / 4

It is because, there are N natural numbers ranging from 1 to N and one half of them are odd numbers. So, with the "N / 2" odd numbers, we are making symmetric pairs and the pair contains two odd numbers, so there are "N / 4" odd pairs.

Now, if there is or are Goldbach pair(s) in this symmetric pair set, there can be one or two or more of them. For each odd pair, we can calculate the probability that both odd numbers in the pair are primes, i.e., the probability that the odd pair is a Goldbach pair.

Next, to find the probability that there is at least one Goldbach pair, we can't merely sum up the probabilities of the pairs being Goldbach pairs, due to inclusive-exclusive rule.[31] It is because two or more symmetric odd pairs can be Goldbach pairs simultaneously.

But, fortunately for us, it will later turn out that we can bypass the complexity of the inclusive-exclusive expansion of probability. So, for demonstration purpose only, we will use sigma summation, and then, we will look at only the k-th term in that sigma, and let the k go to infinity, to observe the probability's asymptotic behaviorism.

[31] See https://en.wikipedia.org/wiki/Inclusion%E2%80%93exclusion_principle .

6. Probability of an Odd Prime Pair Summing to N ("POPS(N)" as its acronym)

First, let us consider a symmetric odd pair summing to N. Next, we will later consider a odd prime pair summing to N. Remember, N is an even number. So the last odd number under N is N-1, and the first odd number under N is 1. And the half of the numbers between 1 and N are odd numbers, so there are N/2 odd numbers. Let us look at a symmetric pair of odd numbers summing to N. Since such pair contains two odd numbers, there are N/4 pairs.

1 3 5 7 9 …. N/2 … N-9 N-7 N-5 N-3 N-1

Let us label the pairs. There are N/r pairs. Let us say, we have this sigma variable 'k'. 'k' would range from 1 to N/4. Then, the k-th odd number would be '2k-1', which would be the smaller left number of a symmetric pair. And the larger right number of the pair would be:

$$N-(2k-1)$$
$$= N-2k+1$$

Next, let us calculate the probability of the k-th odd number (i.e., '2k-1') being a prime:

$$POP(z) = \frac{(z+1)\ln(z-1)-(z-1)\ln(z+1)}{\ln(z-1)\ln(z+1)}$$

$$POP(2k\text{-}1) = \frac{2k\ln(2k-2)-(2k-2)\ln 2k}{\ln(2k-2)\ln 2k}$$

And, the probability of the k-th odd number's twin sibling (i.e., 'N-(2k-1)') being a prime would be:

$$POP(N\text{-}2k\text{+}1) = \frac{(N-2k+2)\ln(N-2k)-(N-2k)\ln(N-2k+2)}{\ln(N-2k)\ln(N-2k+2)}$$

Next, the Probability of the Odd Prime pair Summing to N ("POPS(N)" for short) would be, by means of 'independent POP lemma',

*POPS(N) = POP(2k-1) * POP(N-2k+1)*

$$= \frac{2k\ln(2k-2)-(2k-2)\ln 2k}{\ln(2k-2)\ln 2k} * \frac{(N-2k+2)\,ln(N-2k)-(N-2k)\,ln(N-2k+2)}{ln(N-2k)\,ln(N-2k+2)}$$

Let us consider when 'k' is the maximum, that is, when 'k' is N/4. Then the right side of the above 'tank' equation would look like:

$$\frac{0.5N\ln(0.5N-2)-(0.5N-2)\ln\ \ .5N}{\ln(0.5N-2)\ln 0.5N} * \frac{(0.5N+2)\,ln\,0.5N-0.5N\,ln(0.5N+2)}{ln\ \ .5N\,ln(0.5N+2)}$$

As N goes to infinity, the product above becomes zero:

$$\frac{0}{\infty} * \frac{0}{\infty} = 0 * 0 = 0$$

Next, let us examine the right side of the 'tank' equation when k is in the middle of 1 and N/4, i.e., when k is N/8:

$$\frac{.25N\,\mathrm{l}\ \ (.25N-2)-(.25N-2)\ln.25N}{\ln(.25N-2)\ln.25N} * \frac{(.75N+2)\,ln\,.75N-.75N\,l\ \ (.75N+2)}{ln(.75\ \ \ )\,ln(.75N+2)}$$

As N goes to infinity, the product above becomes:

$$\frac{0}{\infty} * \frac{0}{\infty} = 0 * 0 = 0$$

Next next, let us examine the right side of the 'tank' equation when k is at its minimum, i.e., when k is one:

$$\frac{2\ln 0 - 0 * \ln 2}{\ln 0 * \ln} * \frac{N\,l \quad (N-2) - (N-2)\,ln\,N}{ln(N-2)\,ln\,N}$$

As N goes to infinity, the product above becomes:

$$\frac{-\infty}{-\infty} * \frac{0}{0} = \frac{\infty}{\infty} * \frac{0}{0}$$

So, when 'k' is at its maximum of N/4, the POPS(N) becomes zero as N goes to infinity.[32] And when 'k' is at its minimum of 1, the POPS(N) becomes indeterminate.[33]

Out of curiosity,[34] let us explore L'Hopital's rule (if we need later on), which can be used to simplify things using differentiation, when an indeterminate form is in the shape of infinity/infinity or zero/zero.[35] Here is the 'tank' product again:

$$\frac{2k\ln(2k-2) - (2k-2)\ln 2k}{\ln(2k-2)\ln 2k} * \frac{(N-2k+2)\,ln(N-2k) - (N-2k)\,ln(N-2k+2)}{ln(N-2k)\,ln(N-2k+2)}$$

Let us look at the numerator of the left term:

$$2k\ln(2k-2) - (2k-2)\ln 2k \;\; = \ln(2k-2)^{2k} - \ln 2k^{2k-2}$$

[32] No worries, folks, one k-th term going to zero is 'a' okay. What we want to prove is that at least one k-th symmetric odd pair has the probability of both of its members being prime, more than zero. If the such odd pair with probability of being primes more than zero, exists, that means there is at least one Goldbach's pair, which will constitute the proof of Goldbach's conjecture. We got plenty of other k-th terms that will turn out to be non-zeros, later on //:-)

[33] See https://en.wikipedia.org/wiki/Indeterminate_form .

[34] Curiosity may kill a cat but curiosity doesn't kill a mathematician //xD Many mathematicians are known for their longevity. See https://en.wikipedia.org/wiki/Paul_Erd%C5%91s ; https://en.wikipedia.org/wiki/Henri_Cartan ; https://en.wikipedia.org/wiki/Andr%C3%A9_Weil ; https://en.wikipedia.org/wiki/Charles_Jean_de_la_Vall%C3%A9e_Poussin ; https://en.wikipedia.org/wiki/Jacques_Hadamard . By the way, an eccentric traveling mathematician Prof. Erdos once said, and I'm paraphrasing, "probability theory may be a fruitful approach for number theory." We agree //:-)

[35] See https://www.mathsisfun.com/calculus/l-hopitals-rule.html ; https://en.wikipedia.org/wiki/L%27H%C3%B4pital%27s_rule .

$$= \ln \frac{(2k-2)^{2k}}{2k^{2k-2}} = \ln \frac{(2k-2)^{2k}}{2k^{2k}} + 2 \ln 2k = \ln \left(\frac{2k-2}{2k}\right)^{2k} +2 \ln 2k$$

$$= \ln \left(1 - \frac{1}{k}\right)^{2k} +2 \ln 2k = 2\ln \left(1 - \frac{1}{k}\right)^{k} +2 \ln 2k$$

As k goes to infinity, the expression above becomes:

$$\lim_{k\to\infty} (2ln \left(1 - \frac{1}{k}\right)^{k} + 2\, ln\, 2k) = \lim_{k\to\infty} (2ln(\frac{1}{e}) + 2\, ln\, 2k)$$

$$= \lim_{k\to\infty} (2\, ln\, 2k) - 2$$

If we take the limit of the denominator part too, the fraction goes to zero. We can expect the same zero convergence for the right side part of the product. This makes intuitive sense because as k goes to infinity, primes get rarer and rarer, so the probability of finding an odd prime pair goes to zero for big numbers.

Next, let us explore the limit of the 'tank' product as k goes to one.[36]

7. <u>POPS(N) as 'k' Goes to 1, and 'N' Goes to Infinity</u>

Let us investigate a special case of limit of POPS(N) as 'k' goes to 1 and 'N' goes to infinity. 'k' goes to 1 from the right, meaning, k is approaching 1 from numbers larger than 1, like a count down like 10 9 8 7 6 5 4 3 2 1. 'k' is getting smaller and closer to 1 forever and we call this "1 plus infinitesimality," where infinitesimality is not a zero as a constant, but actually is a variable that gets smaller and smaller infinitely. And we will let 'N' approach infinity from the

[36] Later on this paper, we will use online graphing calculator. "Proof by computer" concept used to be regarded as illegitimate, but these days it is an accepted method of mathematical proof and we will use such methodology later on in this paper. See https://en.wikipedia.org/wiki/Four_color_theorem ; https://en.wikipedia.org/wiki/Computer-assisted_proof . Online limit calculators are still in development stage as of now, but online graphing calculators are of reliable quality, as the history of graphing calculator is a rather long one, more than a century by now. See https://en.wikipedia.org/wiki/Graphing_calculator . And the functions that we will use in online graphing calculators later, are not very complex functions but rather elementary ones. So, it is a reasonable 'assumption' that the results from online calculators are reliable. We will call this new methodology, 'proof by online graphing calculator', which will be embraced as a subbranch of proof by computer someday, if not today (in fact, mathematicians have used graphing calculators for decades in their peer-reviewed journal papers, and copied/pasted the computer-generated graphs into their papers like we will do so later in this paper). Well, in humanology, we are ahead of time, always //:-)

left, meaning 'N' is increasing forever, which also means that infinity is not constant number but a variable that gets larger and larger infinitely.[37]

In both infinity and infinitesimality, what distinguishes the two infinities or infinitesimalities is the speed of approaching infinity or infinitesimality. For instance, as N goes to infinity, '$\ln N$' goes to infinity slower than 'N^2' goes to infinity, which in turn is slower than '3^N' going to infinity. Infinitesimality is the same way. So, when two infinities or two infinitesimalities go at the same speed, then we apply algebraic operation like subtraction or division in order to simplify an equation or a formula.

So, let's go ahead and grab our good ole ironside tank equation, shall we?

*POPS(N) = POP(2k-1) * POP(N-2k+1)*

$$= \frac{2k\ln(2k-2)-(2k-2)\ln 2k}{\ln(2k-2)\ln 2k} * \frac{(N-2k+2)\,ln(N-2k)-(N-2k)\,ln(N-2k+2)}{ln(N-2k)\,ln(N-2k+2)}$$

The left side of the product contains only 'k'. Let's send 'k' to 1 from the right side.

$$\lim_{k\to 1^+}\frac{2k\,ln(2k-2)-(2k-2)\,ln\,2k}{ln(2k-2)\,ln\,2k} = \frac{2^+\,ln(2^+-2)-(2^+-2)\,ln\,2^+}{ln(2^+-2)\,ln\,2^+}$$

$$= \frac{2^+\,ln(0^+)-(0^+)\,ln\,2^+}{ln(0^+)\,ln\,2^+} = \frac{2^+\,ln(0^+)-(0^+)\,ln\,2^+}{ln(0^+)\,ln\,2^+} = \frac{2^+}{ln\,2^+} \approx 2.89$$

Well, there must have been some error somewhere, because a probability should be between zero and 1 inclusive.[38] But, the development of science including meta-science like mathematics is all about trial and error and error correction, so it is actually a progress that we discover our errors.

Now, let's look at the right side of the product that involves both k and N. Here, we will let k to be truly 1, and send N to infinity, to observe the right-term's asymptotic behaviorism. This is a mathematical experiment, a brainstorming session, a brain exercise of sort.

[37] See this author's former papers about infinity at https://vixra.org/abs/2201.0135 .
[38] See https://en.wikipedia.org/wiki/Probability .

$$\lim_{N\to\infty} \frac{(N-2k+2)\,ln(N-2k) - (N-2k)\,ln(N-2k+2)}{ln(N-2k)\,ln(N-2k+2)}$$

$$= \lim_{N\to\infty} \frac{N\,ln(N-2) - (N-2)\,ln\,N}{ln(N-2)\,ln\,N}$$

We previously thought that the numerator part goes to zero, but it turns out that it does infinity and the whole fraction becomes an indeterminate term of infinity/infinity, which enables us to utilize L'Hopital's rule. Now let us examine why the numerator goes to infinity.

$$N\,ln(N-2) - (N-2)\,ln\,N = \ln \frac{(N-2)^N}{N^{N-2}}$$

Intuitively, we can tell the numerator of the fraction inside the logarithm above, 'N-2' to the Nth grows faster than N to the 'N-2'th, because the size of the exponent part than the base part. Empirically, if N is 4, then the numerator and the denominator becomes the same number:

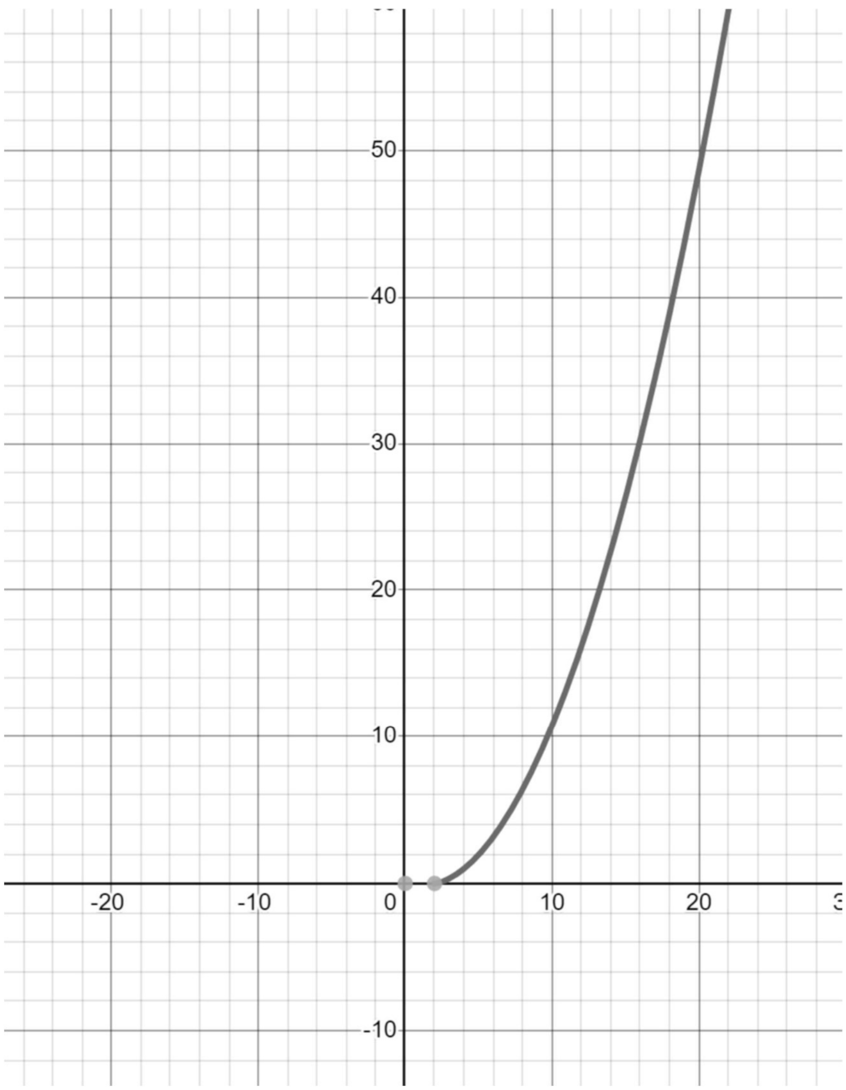
$(4\text{-}2)^4 = 4^{4-2} = 16$

But, when N is 5 or larger, numerator is bigger than denominator. This author tried to prove it by taking derivative of the fraction above and prove that derivative is positive for all N larger than 4. This author also tried to prove it by mathematical induction, but his math is too shallow and short at this point, and he was not able to prove it so far and we will leave the algebraic proof thereof to the future generations of both amateur and professional mathematicians. We can generalize this problem and let's name it as 'reciprocal base-exponent lemma':

$(N - r)^N > N^{N - r}$ *when* $N > q$

So this author resorted to online graphic calculators[39] and found out that, when r is 2, q is 4 and the graph looks like this:

[39] See https://www.desmos.com/calculator . Using an online graphic calculator may sound cutting the corner, but in this case, it is a legitimate cheating, because this author is not in school. He can time all his time solving this problem of Goldbach's conjecture proof and can use calculator because he is not taking a school exam with time limit or rules against calculators. Since this author does not belong to an academic institution, he even makes jokes in his academic papers and use unorthodox paper formats and structures, which is a freedom that he enjoys as an independent scholar //:-)



The reciprocal fraction function for the graph is:

$$f(x) = \frac{(x-2)^x}{x^{x-2}}$$

As we can see in the graph, f(2) is zero and f(4) is one. Let's algebraically play with the function a bit.

$$f(x) = \frac{(x-2)^x}{x^{x-2}} = \frac{x^2(x-2)^x}{x^x} = x^2 * \left(1-\frac{2}{x}\right)^x = x^2 * \left(\left(1-\frac{2}{x}\right)^{\frac{x}{2}}\right)^2$$

Then, let's take the limit of f(x) as x goes to infinity.[40]

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty}\left(x^2 * \left(\left(1-\frac{2}{x}\right)^{\frac{x}{2}}\right)^2\right) = \lim_{x\to\infty}\left(x^2 * \left(\left(1-\frac{1}{\frac{x}{2}}\right)^{\frac{x}{2}}\right)^2\right)$$

$$= \lim_{x\to\infty}\left(x^2 * \left(\frac{1}{e}\right)^2\right) = \lim_{x\to\infty}\left(\frac{x^2}{e^2}\right) = \infty$$

So, we did indeed prove the reciprocal base-exponent lemma for the case when r is 2, which is good enough for our purpose of Goldbach's conjecture proof.

Well, the original numerator has logarithm around f(x). So let us put logarithm around it and differentiate it once, as we'll use L'Hopital's rule later on:

$$\lim_{x\to\infty} \ln f(x) = \lim_{x\to\infty} \ln\left(\frac{x}{e}\right)^2 = \lim_{x\to\infty} 2(\ln x - 1)$$

$$\lim_{x\to\infty} (\ln f(x))' = \lim_{x\to\infty}\frac{2}{x} = 0$$

Next, we will examine the denominator part of the right side of the tank product:

$$g(x) = \ln(x-2)\ln x$$

[40] For the popular and powerful methods to calculate the variation of limits using Euler's number 'e', see https://www.quora.com/How-do-you-evaluate-the-limit-of-1-1-x-x-as-x-approaches-0 . Let's name the technique as "Euler's shoulder method" and we'll use it often in this paper //:-)

We plan to use L'Hopital's rule, so let us differentiate g(x):

$$\left(g(x)\right)' = \frac{\ln x}{x-2} + \frac{\ln(x-2)}{x} = \ln x^{\frac{1}{x-2}} + \ln(x-2)^{\frac{1}{x}}$$

Then,

$$\lim_{x\to\infty}\left(g(x)\right)' = \lim_{x\to\infty}(ln\, x^{\frac{1}{x-2}} + ln(x-2)^{\frac{1}{x}}) = \lim_{x\to\infty}\,(ln\, e^{\ln\,(x^{\frac{1}{x-2}})} + ln\, e^{\ln((x-2)^{\frac{1}{x}})})$$
$$= \ln 1 + \ln 1 = 0$$

So we have once again 0/0 situation, which allows us to conduct yet another round of L'Hopital's rule.

8. The Right Half of POPS(N) Product

Alright. Let's start this section by reminding us about the Probability of Odd Prime pair Summing to N, the POPS(N) function:

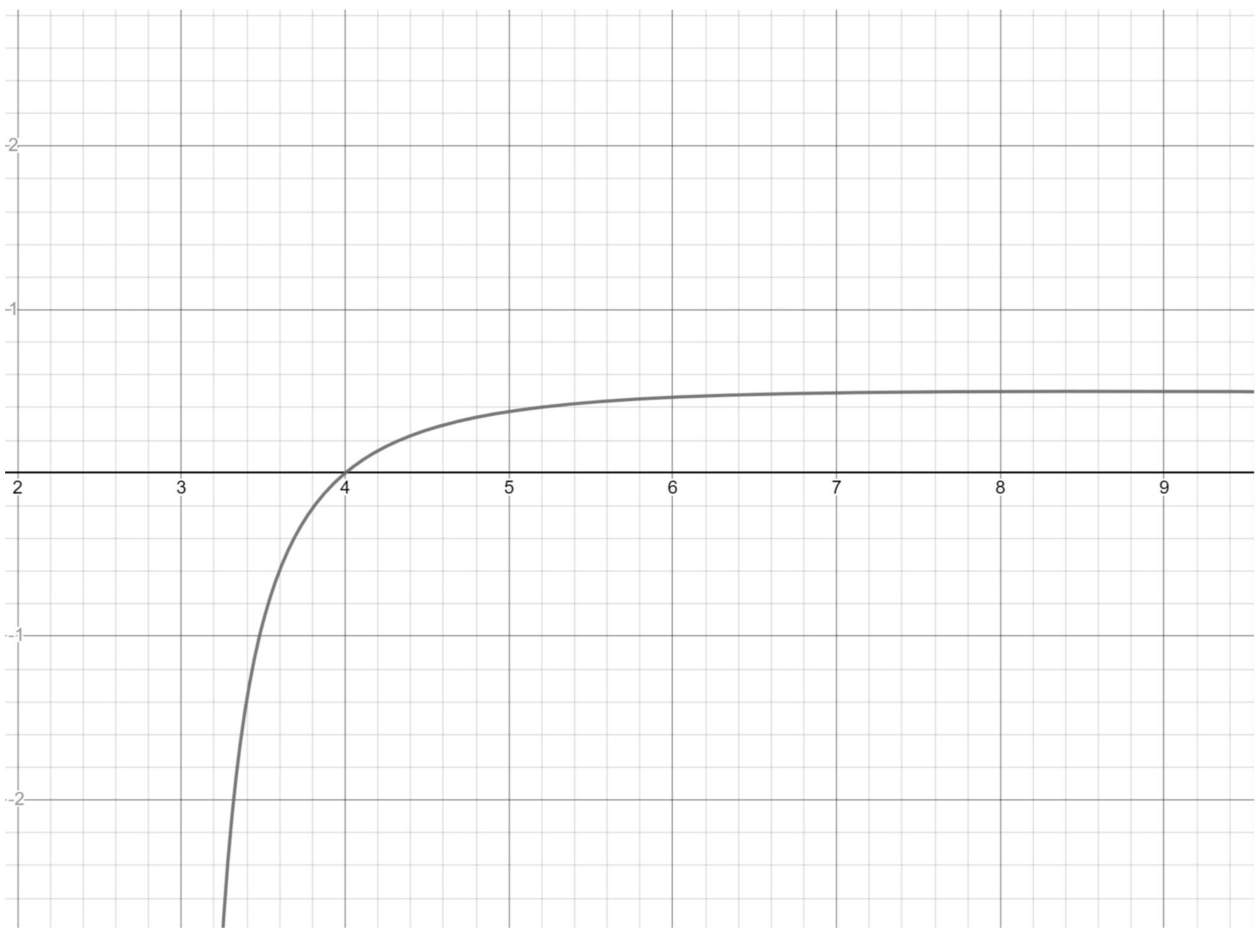
*POPS(N) = POP(2k-1) * POP(N-2k+1)*

$$= \frac{2k\ln(2k-2)-(2k-2)\ln 2k}{\ln(2k-2)\ln 2k} * \frac{(N-2k+2)\, ln(N-2k)-(N-2k)\, ln(N-2k+2)}{ln(N-2k)\, ln(N-2k+2)}$$

For now, we are trying to understand what the function above is about, how it behaves. So, for simplicity, let's look at the case when k is 1. Let's look at the right side of the product above when k is one. And we will plot that right half of the right side of the equation above in online graphic calculators. So let's substitute x for N when k is 1. We will name this as "tank right function", as it is the right half of the tank product:

$$f(x) = \frac{x\, ln(x-2) - (x-2)\, ln\, x}{ln(x-2)\, ln\, x} = \frac{\ln \frac{(x-2)^x}{x^{x-2}}}{\ln x \ln(x-2)}$$

If we plot the tank-right function in an online graphic calculator, we get the following picture:



$$\boxed{\frac{\left(\ln\left(\frac{(x-2)^x}{x^{(x-2)}}\right)\right)}{\ln(x-2)\ \ln x}}$$

When x is 4, f(x) is zero. As x goes to infinity, it seems that f(x) converges to 0.5, which is a good news for us, as a legitimate probability is supposed to be a fixed number between 0 and 1, inclusive.

Now, our next task ahead is to algebraically prove that the tank-right function converges to a number close to 0.5. Again, our goal is to prove that there is at least one Goldbach's pair for any even number N, which is equivalent of saying that the probability of finding a Goldbach's pair under N is more than zero.

Since tank-right function is infinity over infinity type as x goes to infinity, we can apply L'Hopital's rule, i.e., we can differentiate both numerator and denominator of the tank-right function, and the limit would be the same. So let us call the numerator part a(x), and the denominator part b(x). Then,

$$\mathrm{a(x)} = \ln\frac{(x-2)^x}{x^{x-2}}$$

As a reminder in calculus,

$$\left(\frac{\mathrm{m(x)}}{n(x)}\right)' = \frac{\mathrm{m'(x)}}{n(x)} - \frac{m(x)n'(x)}{\left(n(x)\right)^2} = \frac{n(x)m'(x) - m(x)n'(x)}{\left(n(x)\right)^2}$$

Then,

$$(\mathrm{a(x)})' = \frac{x^{x-2}}{(x-2)^x} * \left(\frac{(x-2)^x}{x^{x-2}}\right)'$$

Now, let us use "Euler's Shoulder Method" to differentiate the numerator part in the right half of the product above:

$$(x-2)^x = e^{\ln(x-2)^x} = e^{xln\,(x-2)}$$

So,

$$((x-2)^x)' = (e^{xln\,(x-2)})' = e^{x\ln(x-2)} * (\ln(x-2) + \frac{x}{x-2})$$

$$= (x-2)^x(ln(x-2) + \frac{2}{x-2} + 1)$$

Next, let us use "Euler's Shoulder Method" to differentiate the denominator part in the right half of the product above (we notice that the denominator and numerator is kinda symmetric in duodualistic way like in humanology):

$$x^{x-2} = e^{\ln x^{x-2}} = e^{(x-2)ln\ x}$$

So,

$$(x^{x-2})' = (e^{(x-2)ln\ x})' = e^{(x-2)\ln x} * (\ln x + \frac{x-2}{x})$$

$$= x^{x-2}(ln\,x - \frac{2}{x} + 1)$$

In sum,

$$m(x) = (x-2)^x$$

$$m'(x) = (x-2)^x(ln(x-2) + \frac{2}{x-2} + 1)$$

$$n(x) = x^{x-2}$$

$$n'(x) = x^{x-2}(ln\,x - \frac{2}{x} + 1)$$

Then,

$$\left(\frac{\mathrm{m(x)}}{n(x)}\right)' = \frac{\mathrm{m'(x)}}{n(x)} - \frac{m(x)n'(x)}{\left(n(x)\right)^2} = \frac{n(x)m'(x) - m(x)n'(x)}{\left(n(x)\right)^2}$$

$$= \frac{x^{x-2} * \ (x-2)^x \left(ln(x-2) + \frac{2}{x-2} + 1\right) - (x-2)^x * x^{x-2}(ln\, x - \frac{2}{x} + 1)}{(x^{x-2})^2}$$

$$= \frac{x^{x-2} * \ (x-2)^x \left(ln(x-2) + \frac{2}{x-2} - \ln x + \frac{2}{x}\right)}{(x^{x-2})^2}$$

$$= \frac{(x-2)^x \left(ln(x-2) + \frac{2}{x-2} - \ln x + \frac{2}{x}\right)}{x^{x-2}}$$

Next,

$$(\mathrm{a(x)})' = \frac{x^{x-2}}{(x-2)^x} * \left(\frac{(x-2)^x}{x^{x-2}}\right)'$$

$$= \frac{x^{x-2}}{(x-2)^x} * \frac{(x-2)^x \left(ln(x-2) + \frac{2}{x-2} - ln\, x + \frac{2}{x}\right)}{x^{x-2}}$$

$$= ln(x-2) + \frac{2}{x-2} - ln\, x + \frac{2}{x} = \ln\frac{x-2}{x} + 2\left(\frac{1}{x} + \frac{1}{x-2}\right)$$

$$= \ln\left(1 - \frac{2}{x}\right) + 2\left(\frac{1}{x} + \frac{1}{x-2}\right)$$

The above function goes to zero as x goes to infinity.

Alright. Let's now look at the denominator part:

b(x) = $\ln x \ln(x-2)$

b'(x) = $\frac{ln(x-2)}{x} + \frac{\ln}{x-2} = \ln(x-2)^{\frac{1}{x}} + \ln x^{\frac{1}{x-2}}$

Let's look at the left half of the equation above. Since ln(x) is slower than x, as x goes to infinity, b'(x) goes to zero.

Therefore, we can use L'Hopital's rule again, because both a'(x) and b'(x) goes to zero and we have zero/zero situation. Let's go.

$$a''(x) = (\ln(1-\frac{2}{x}) + 2(\frac{1}{x} + \frac{1}{x-2}))'$$

$$= \frac{x}{x-2} * \frac{2}{x^2} - 2\left(\frac{1}{(x-2)^2} + \frac{1}{x^2}\right) = \frac{2x(x-2) - 2(x^2 + (x-2)^2)}{x^2(x-2)^2}$$

$$= \frac{-2x^2 + 4x - 8}{x^2(x-2)^2} = \frac{-2(x^2 - 2x + 4)}{x^2(x-2)^2}$$

Next,

b''(x) = $\left(\frac{ln(x-2)}{x} + \frac{\ln x}{x-2}\right)'$

$$= \frac{2}{x(x-2)} - \frac{ln(x-2)}{x^2} - \frac{\ln x}{(x-2)^2}$$

$$= \frac{2x(x-2) - (x-2)^2 \ln(x-2) - x^2 \ln x}{x^2(x-2)^2}$$

Then,

$$\frac{a''(x)}{b''(x)} = \frac{-2(x^2 - 2x + 4)}{2x(x-2) - (x-2)^2\, ln(x-2) - x^2\, ln\, x}$$

$$= \frac{2(x^2 - 2x + 4)}{(x-2)^2\, ln(x-2) + x^2\, ln\, x - 2x(x-2)}$$

As x goes to infinity, the fraction above goes to zero. But, it never becomes zero. It only approaches zero. The fraction above is always more than zero.

The tank-left function works the same way. If we the the L'Hopital's rule by differentiating numerator and denominator of the tank-left function twice, we will get similar result.

So, POPS(N) function is the product of tank-left function and tank-right function. If we multiply two positive small numbers, the result is a small positive number, which is the POPS(N) function. This way, POPS(N) is always a small positive number, which means the probability of finding a pair of odd primes summing up to N is always more than zero, which means there is at least one such pair. This is the proof of Goldbach's conjecture and the rest is history.

Q.E.D.[41]

9. Looking at the Big Picture

Alright. What we proved in the previous section is that the right half of the tank product converges to zero as N goes to infinity when k is one. But, if apply the same methodology to the tank-left function of POPS(N) function, it will converge to zero too, when k is one. If you recall, k ranges from 1 to N/4. For any k in that range, the methodology of two applications of L'Hopital's rule of differentiating both numerator and denominator leads to the convergence to zero, in all cases. It looks like a bad result, but it is a good result, because POPS(N) going to zero as N goes to infinity means it is never zero, but it is a number slightly over zero, meaning the probability is always a positive number. Also, please note that the total probability is not the simple summation of POPS(N) for all k's, but the summation of probabilities using inclusion-exclusion principle.

Now, let us make some concrete example, where N is 100. There are 25 pairs of odd numbers that sum to 100:[42]

[41] See https://en.wikipedia.org/wiki/Q.E.D. .
[42] See https://www.amathematics.com/2021/07/first-100-prime-numbers-prime-numbers.html .

1 99, 3 97, 5 95, 7 93, 9 91, 11 89, 13 87, 15 85, 17 83, 19 81, 21 79, 23 77, 25 75,

1 2 3

27 73, 29 71, 31 69, 33 67, 35 65, 37 63, 39 61, 41 59, 43 57, 45 55, 47 53, 49 51

4 5 6

Then,

POPS(100) = 6 / 25 = 0.24

So, the probability of a pair of odd numbers summing to 100 to be a pair of odd primes is about 25%. And this result does nicely fit into our prediction, as our online graphic calculator predicts that both the tank-right function and the tank-left functions converge to about 0.5. Our algebraic prediction indicates that both the tank-right function and the tank-left functions converge to zero. Well, no matter what the case is, the k-th term of the total probability is always always above zero and that constitutes Goldbach's conjecture proof, because the probability is never zero, which means there is at lease one Goldbach pair, i.e., a pair of odd primes that sum to N.

Now, let us look at the tank function, our popsicle function, a.k.a., POPS(N):

$$POPS(N) = POP(2k-1) * POP(N-2k+1)$$

$$= \frac{2k \ln(2k-2) - (2k-2) \ln 2k}{\ln(2k-2) \ln 2k} * \frac{(N-2k+2)\, ln(N-2k) - (N-2k)\, ln(N-2k+2)}{ln(N-2k)\, ln(N-2k+2)}$$

$$= \textit{tank-left} \quad * \quad \textit{tank-right}$$

Our beloved online graphic calculators[43] look as follows when both N and k goes to infinity when k is an intermediate value of N/8.

[43] See https://www.desmos.com/calculator ; https://www.derivative-calculator.net/ .

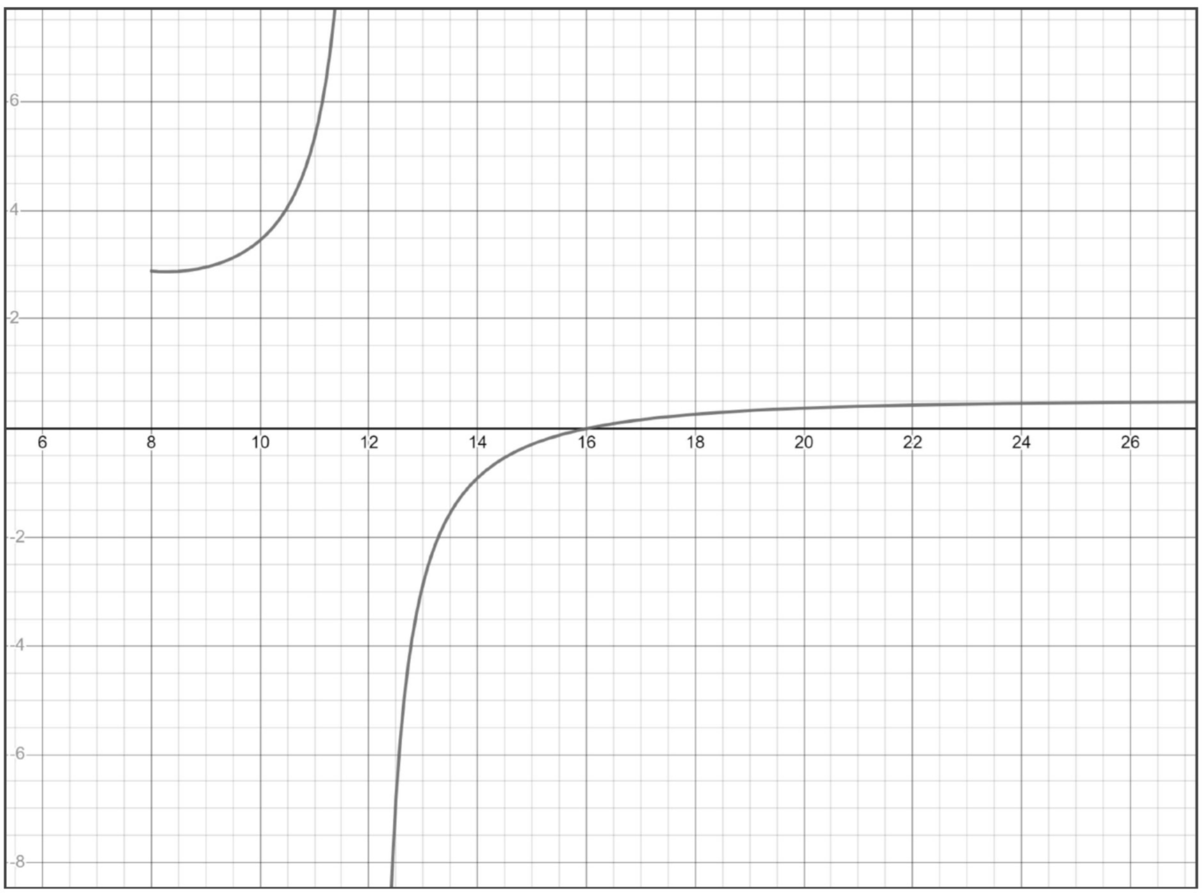
Tank-left:

1

$$\frac{\left(0.25x\ln(0.25x-2)-(0.25x-2)\ln\ (0.\right.}{\ln(0.25x-2)\ln(0.25x)}$$

2

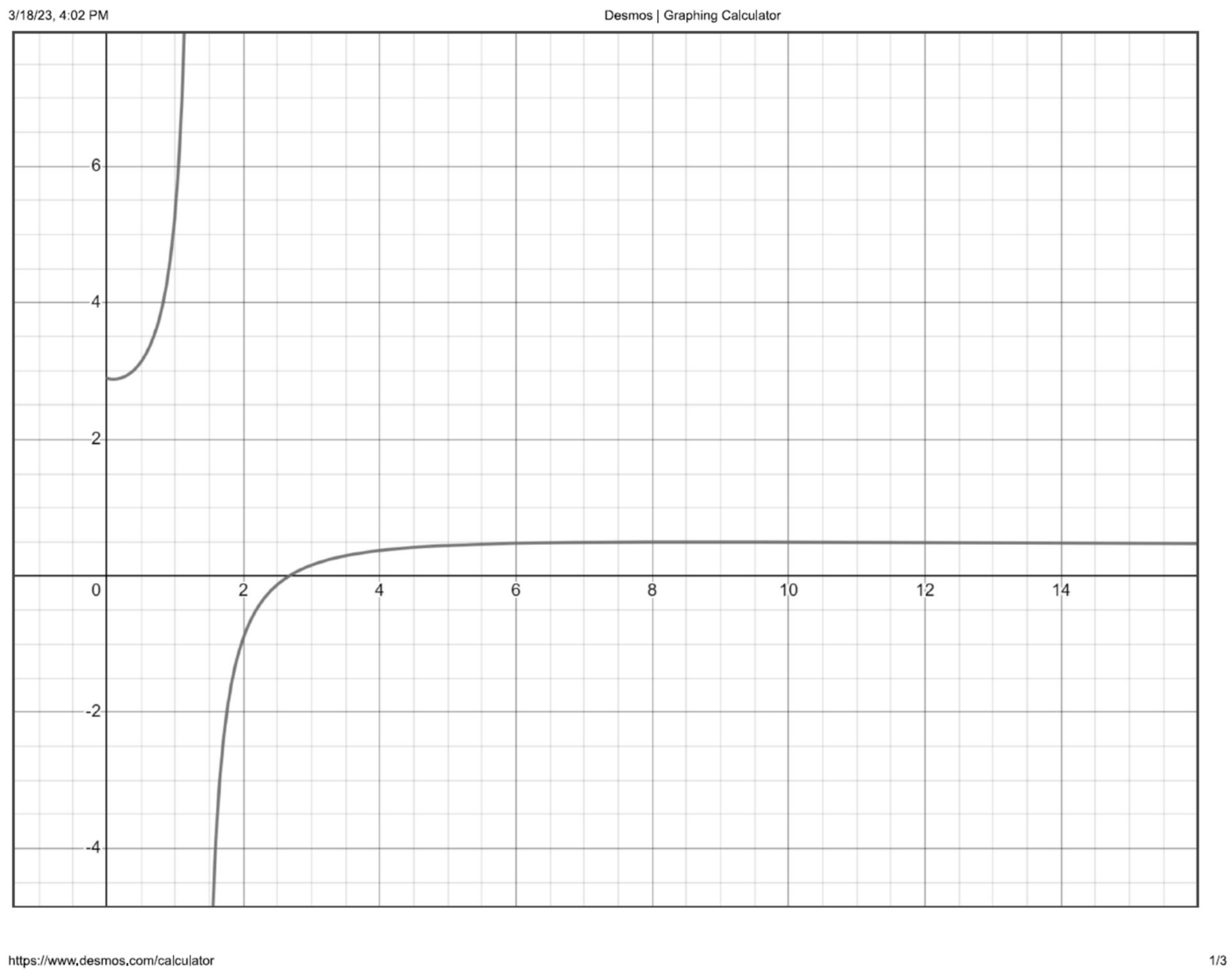
3/18/23, 4:07 PM Desmos | Graphing Calculator



https://www.desmos.com/calculator 1/3

Tank-right:

$$\frac{((0.75x+2)\ln(0.75x)-0.75x\ln\ (0.75x}{\ln(0.75x)\ln(0.75x+2)}$$

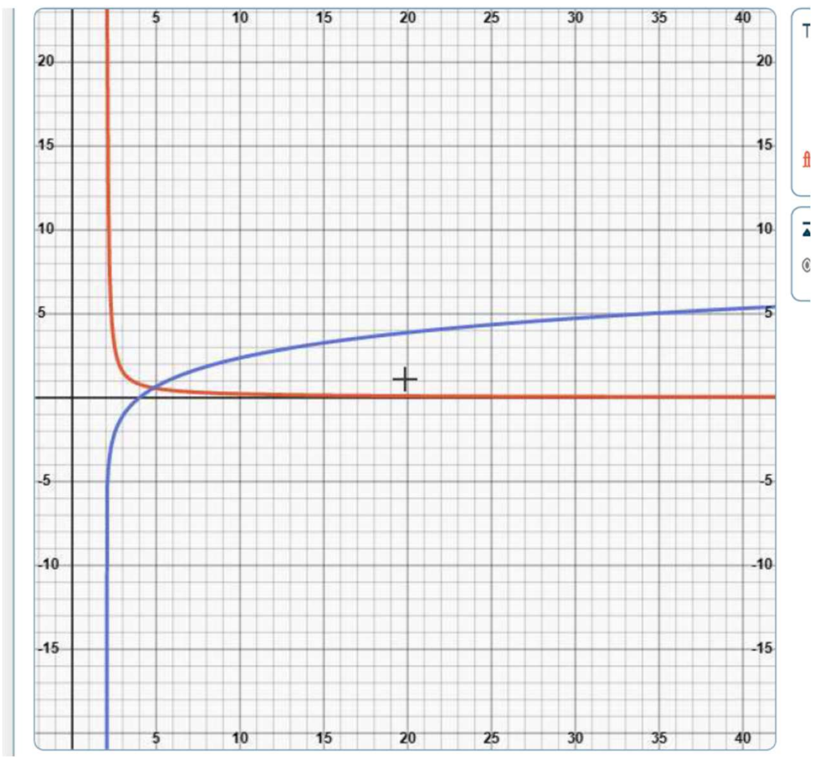


The prediction is that both tank-left and tank-right converges to 0.5, making the POPS(N) to be 0.5 times 0.5, equalizing 0.25, or 25%, when N is not too big. When is real big, our algebraic prediction is that POPS(N) is a small number slightly over 0.

When k is 1, the tank-right function (in blue) and its derivative (in red) look as follows.

Derivative of tank-right when k is 1:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[x\ln(x-2)-(x-2)\ln(x)\right]$$



Of course, the scaling above is a bit off, but we got the idea, where the slope of POPS(N) converges to zero as POPS(N) become slower and slower in its increase as N goes to infinity.

III. Mathematical Philosophy

1. <u>The Importance of Optimism</u>

Some mathematicians thought that Goldbach's conjecture might be impossible to prove.[44] Mathematical logicians like Kurt Goedel,[45] Alonzo Church,[46] and Alan Turing[47] came up with beautiful formal system that gives us some pessimistic predictions. Though this author appreciates the beauty and the intellectual entertainment perspective of their theoretical system, this author opines that their formal system is too limited to make realistic prediction in the real world. Their theories hinges on their limited unrealistic assumptions and axioms.[48] Unlike their dire predictions and assumptions, the real world of mathematics is not limited. Everyday, everywhere, some mathematicians around the world come up with brand-new mathematical theories, expanding the horizons in the metaphysical world of mathematics.

Like many amateur mathematicians in the world, this author did not major in mathematics and he does not belong to an academic institution either. This author is a PhD-dropout and does not even hold a doctoral degree in science.[49] But, this author, with kind and generous supports of friends like You, managed to solve Goldbach's conjecture that has been unproven for nearly three centuries. This carries a very important message for current and future generations: never give up and believe in God/Goddess and believe in Your Self, and dream often and dream big.

Many people from all around the world came to Alaska to find gold.[50] This author found Goldbach's conjecture in Alaska, of all places.

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[44] See also page 4 of http://www.ams.org/notices/200203/fea-knuth.pdf . See also https://en.wikipedia.org/wiki/Uncle_Petros_and_Goldbach%27s_Conjecture .
[45] See https://en.wikipedia.org/wiki/Kurt_G%C3%B6del .
[46] See https://en.wikipedia.org/wiki/Alonzo_Church .
[47] See https://en.wikipedia.org/wiki/Alan_Turing .
[48] See https://plato.stanford.edu/entries/goedel-incompleteness/ .
[49] Well, he does hold Juris Doctor degree, however //:-)
[50] See https://en.wikipedia.org/wiki/Alaska_gold_rush .

Epilogue[51]

Hello everyone, thank you for your kind and generous readership //:-D We hope you enjoyed the show. Our next article to write and publish will be titled, "Recent Development in Humanology". There, we'll introduce some interesting concepts in science and religion and anything in between.[52]

Thank you for your time and see you later, kind and generous ladies and gentlemen //:-)

[51] This paper was started being written on 3/14/2023. It was finished being written on 3/26/2023 //:-)
[52] See https://en.wikipedia.org/wiki/The_Road_Not_Taken .