Choice of Reference Frame to Determine Time Duration and Simultaneity in Special Relativity (v2, 2023-03-06)

Per Hokstad

phokstad@gmail.com

Abstract. Rather than adopting the *conventionality of simultaneity,* we discuss the possible identification of a 'correct' Reference Frame for analysing time duration and simultaneity of phenomena. Four standard examples are reviewed to illustrate the approach. The discussion includes use of the concepts, *time vector* and *event graph*[1].

Key words: Simultaneity, Symmetry, Time vector, Event graph, Twin paradox, μ-mesons.

1 Introduction

In the theory of special relativity moving reference frames provide different results both for the duration of a chain of events and for the simultaneity of events. Many authors discuss the *relativity of simultaneity*, *e.g.* Debs and Redhead (1996) who argue for a version of the *conventionality of simultaneity*: There will be a time *interval* of events, all being simultaneous to a (distant) event; *which* of these to choose is rather a matter of *convention*.

However, by properly specifying the phenomenon under investigation, one can identify *one* 'correct' perspective; *i.e.* a specific Reference Frame (RF) from which the events should be considered. This leads to *one* 'canonical' result for the duration and simultaneity of events.

Our discussion is based on well-known results of a few standard examples: the travelling twin (TT); two spacecrafts moving relative to each other in outer space, the case of the μ-mesons and the Michelson-Morley experiment.

2 Background

First some basic terminology, assumptions and tools are provided.

2.1 Notation

We start out from a RF denoted K, which for simplicity has just one space coordinate, x. The RF has synchronized, stationary clocks located at virtually any position. Further, there is an object moving at velocity, v relative to K along the x-axis. The object starts out from the origin of K when the clock at this position on K reads 0. Further, (we imagine that) this moving object brings with it a clock, and when the object passes the origin at time 0, this clock is synchronized with the clock on K at this position.

Three fundamental parameters are related to the movement of this object. First:

τ = Clock reading of the clock following the moving object. We would say that this is the 'internal time' of the object/event, but usually, this is referred to as the 'proper time'.

This proper time, τ is independent of which RF we choose as the basis for our observations. Further, we have two parameters (t, x), which specify events on the chosen RF, K:

x = Position of the moving object relative to K, (when the passing clock reads τ);

t = Clock reading of the clock permanently located at position x on K, when the moving clock reads τ; (this t is usually referred to as 'calendar time').

[1] A version of this paper appears in J Mod App Phy. Vol 6 No 1 March 2023. https://www.pulsus.com/journal-modern-applied-physics/inpress.html

Further, we have

$v = x/\,t$, *i.e.* the velocity of the object relative to the RF, *K*.

c = speed of light,

Then - according to the so-called *time dilation* in special relativity, we have (*e.g*. Mermin (2005)) the fundamental relation

$$\tau = \sqrt{t^2 - (x/c)^2} = t\sqrt{1 - (v/c)^2} \tag{1}$$

2.2 Time Vector

Now, we proceed to introduce the time vector of an event (*t, x*), *cf.* Hokstad (2018):

$$\vec{t} = \binom{\tau}{x/c} = \binom{v/c}{\sqrt{1-(w/c)^2}}\, t \tag{2}$$

Note that the absolute value of this vector equals

$$\left|\vec{t}\,\right| = t = \sqrt{\tau^2 + (x/c)^2} \tag{3}$$

Thus, the time vector comprises the information on all the fundamental parameters of an event. Note that the three parameters τ, t and x/c all represent time. In particular, x/c equals the time required for light to traverse the distance *x,* (*i.e.* from the origin to the location of the event). According to *eq*. (3) it could seem rather sensible to denote *t* for the 'total time'; as it is split into two components of time.

2.3 Event Graph

We will illustrate the discussion with an event graph. This will provide a graphical representation of the change in the time vector from a certain initiating event; *cf.* Hokstad (2023).

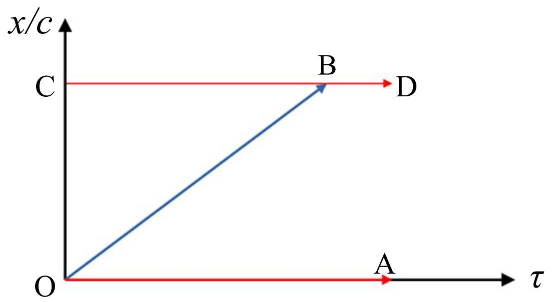

Figure 2.1. Following the three relevant clocks of the Twin paradox; (travel *to* the star). The perspective of the earth.

Thus, a graph illustrates a chain of events, say, a clock moving at a constant velocity relative to a specific RF, *K*. Fig. 2.1 illustrates the 'movements' in spacetime for three relevant clocks in the well-known Twin paradox. The graphs are given relative to the earth's RF, *K*; considering just the period from the departure from the earth until the TT's arrival at the star. In Fig. 2.1 the 'distance' to the star equals OC; the event vector OB represents the travel of the TT. The vector OA is the corresponding change in the time vector of the earthbound twin, and CD represents the (imagined) clock permanently located at the star. For the last two clocks, there are (of course) no change in *x/c*. Further, the three vectors OA, OB and CD have the same length; *i.e.* the time required for the TT to reach the star, (according to the RF, *K*).

3 Choosing *the* right RF of the event.

Simultaneity within a single inertial reference frame (RF), *K* is verified by synchronized clocks; *e.g.* see standard textbooks like Giulini (2005) and Mermin (2005). For events having the same clock reading, *t* on this RF we have simultaneity *in the perspective of K*.

Clocks on different RFs will not agree on simultaneity, and this has led to the concept of *relativity of simultaneity* (and *conventionality of simultaneity*). However, even if there is an infinite number of RFs to describe the chain of events, there is (typically) one that stands out as the most appropriate one. Our focus is that this RF gives the 'correct' result both for durations and simultaneity of events.

We now illustrate this statement on time duration and simultaneity in the theory of special relativity by looking at four standard examples.

3.1 Example 1: The twin paradox

The travelling twin (TT) example is discussed *e.g.* by Debs and Redhead (1996) and Schuler and Robert (2014). We apply a standard numerical example, (*e.g.* Mermin (2005)): The TT leaves the earth in a rocket of velocity, $v = 0.6c$ (with respect to the earth). This gives $\sqrt{1-(v/c)^2} = 0.8$. He travels to a 'star' at a distance of 3 light years from the earth. Thus, $x/c = 3$. Further, the earth's RF has a clock located at the star, and by the arrival of the TT, this clock will read

$$t = x/v = 3c/0.6c = 5 \text{ years.}$$

At this instant the TT's clock will – see *eq.* (1) - read

$$\tau = t\sqrt{1-(v/c)^2} = 5 \cdot 0.8 = 4 \text{ years.}$$

In summary, the arrival of the rocket to the star represents the following event (in the earth's RF):

$$\tau = 4 \text{ years}, \quad x/c = 3 \text{ years}, \ t = 5 \text{ years},$$

corresponding to the time vector

$$\vec{t} = \begin{pmatrix} \tau \\ x/c \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}, \text{ with } |\vec{t}| = t = 5$$

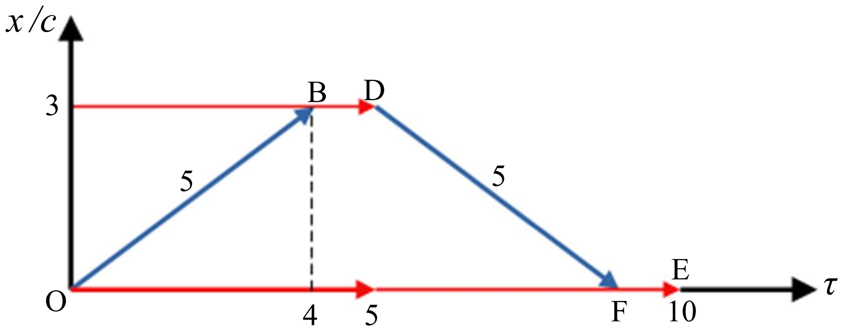

Figure 3.1. Event graphs of the twins in the perspective of the earth-based RF, *K*. The blue graphs describe the TT's journey. The red line, OE follows the earthbound twin, having x/c ≡ 0.

As discussed above, Fig. 2.1 illustrates the travel *to* the star. In Fig. 3.1 we now illustrate the total travel, using the above numerical values. This includes an abrupt change in the velocity of the TT; requiring a new initiating point. Actually, we should rather include a second space ship moving towards the earth with velocity -*v*.[2] The returning spaceship starts out from the initial position, D, (*i.e.,* at local time, $t = 5$ years). It will, however, calibrates its inner clock with that of the TT, *i.e.* $\tau = 4$ years. The returning TT (or at least the new calibrated clock) is illustrated by the blue line DF. Thus, the TT's travel is actually given as two distinct travels; OB + DF. The total length of these two travels equals the change in the clock rate of the earthbound twin, *i.e.* 10 years. Here the red line OE represents the total event graph for the earthbound twin.

[2] This presentation corresponds to the 'three brothers' approach', suggested by Lords Halsbury, being referred in Debs and Redhead (1996).

It remains to argue why this RF (of the earth) must be the correct one here. Actually, both phenomena (series of events for the two twins) take place relative to this RF. In particular, when the TT returns to the earth the clock of the TT *will* read 8 years and that of the earthbound twin *will* read 10 years, as predicted by the perspective given in Fig. 3.1. Observers moving relative to the phenomenon will obtain different results! It is essential that both the starting/ending point and turning point of the event chains are fixed relative to this RF. - Also see discussion in Section 3.2.

When the RF is specified, the correct results concerning time durations and simultaneity follow. In particular, we could ask which event on the earth is simultaneous with the arrival of the TT at the star. Now the answer is that this arrival is simultaneous with the earthbound twin's clock reading $t = 5$ years (identical to the clock reading at the star). The symmetry of the travel to and from the star demonstrated in Fig. 3.1 supports this.

3.2 Example 2: two spacecrafts

Now consider two spacecrafts in outer space. They start out simultaneously in opposite directions from the same position. Our intention is that the experiment shall be completely symmetric with respect to the two spacecrafts. So as our RF we now introduce K_S with its origin at their common starting point; and the spacecrafts move relative to this RF at velocities w and $-w$, respectively. Further, we choose $w = c/3$, which means – using the standard formula for adding velocities in special relativity – that the relative velocity between the two spacecrafts is given by

$$v/c = \frac{2\cdot(1/3)}{1+(1/3)^2} = 0.6.$$

This is in fact the same velocity as that of the TT relative to the earthbound twin, (see Section 3.1), giving an obvious link to that example. To further link these two cases, we let K_S move relative to K (of Section 3.1) at speed w. Thus, one spacecraft is stationary with respect to K, and the other has speed v relative to K, (in complete analogy with the two twins)!

When $\tau = 4$ years we have (since τ is independent of the RF), that the spacecraft moving to the right has travelled the distance to the star (relative to K). As its speed relative to K_S equals $w=c/3$, simple calculations give that this corresponds to the event $t_S = 3\sqrt{2}$, $x_S/c = \sqrt{2}$ in K_S.

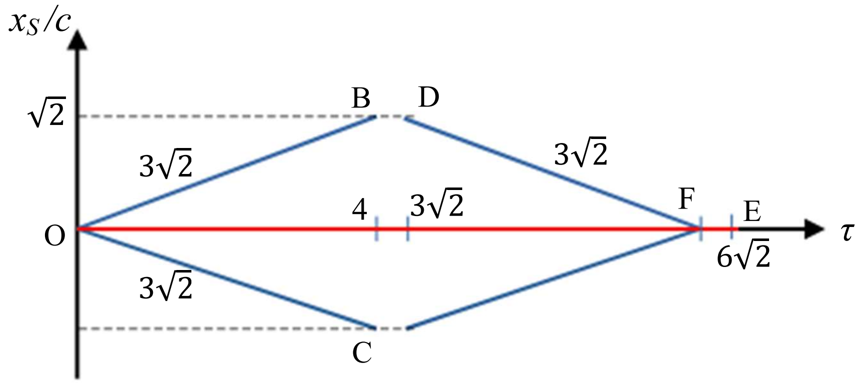

Figure 3.2 Event graphs for the two rockets; in the perspective of the symmetric RF, K_S.

Fig. 3.2 now illustrates the event graphs relative to K_S. The line OB represents the spacecraft moving the distance from the origin to the star (relative to K), and OC the corresponding travel of the other spacecraft. At this point the two spacecrafts will start on their travel to reunite. In order to maintain a complete symmetry, they both reverse their velocity, and doing so, we define new initiating events, corresponding to the local clock readings, $t_S = 3\sqrt{2}$. Thus, the line DF represents the travel of the first spacecraft back to the origin of K_S, and similarly for the other one (see the blue lines). The red line OE represents events describing the origin of K_S, (from departure of the two spacecrafts, to their return).

Trying now to ignore the various technicalities, Fig. 3.2 can in principle be seen as the event graph of two phenomena: of course the two spacecrafts, but also the TTs. But as these graphs obviously fail to give a proper illustration of the TT case, (definitely giving wrong results for the clock reading when the twins reunite), it very well describes the symmetry of the spacecraft example. In particular, it gives the same 'age' as they reunite. An essential difference in the two cases is the difference in the 'stopping rule' (*i.e.* the decision of when/how to change the object's movements, in order for them to get reunited).

This symmetry also fits the discussion of simultaneity at the turning of the spacecrafts. Here we have simultaneity when the clocks of K_S read $t_S = 3\sqrt{2}$, and the internal clocks of the spacecrafts read $\tau = 4$ years. In summary, by choosing the correct RF for the actual phenomenon under investigation, we identify one correct answer.

3.3 Example 3: The case of the μ-mesons

Next we discuss the example of the μ-mesons. As stated for instance in Mermin (2005): The μ-mesons are produced by cosmic rays in the upper atmosphere. When 'at rest' they have a lifetime of about 2 microseconds, so if their internal clocks ran at a rate independent of their speed, even if they traveled at the speed of light about half of them would be gone after they had traveled 2.000 feet. Yet about half of the μ-mesons produced in the upper atmosphere (about 100.000 feet up) manage to make it all the way down to the ground. This is because they travel at a speed so close to the speed of light that the slowing down factor equals 1/50, and they can survive for 50 times as long as they can when being stationary.

This phenomenon is easily described by the Lorentz transformation. We have two RFs; that of the earth, K, and that of the μ-mesons, named K_v (with parameters (t_v, x_v)). We will refer to the creation of the μ-mesons in the upper atmosphere as the initiating event A. At this event, $x = x_v = 0$ and $t = t_v = 0$. Further, we refer to the arrival at the ground as event B. At this event $x_v = 0$ and $t_v = 2\cdot 10^{-6}$ sec. Thus, in K_v we then have $\tau = t_v = 2\cdot 10^{-6}$ sec, and we get the time vector

$$\overrightarrow{t_v} = \binom{\tau}{x_v/c} = \binom{2\cdot 10^{-6}}{0}\ sec;\ (\text{with } \left|\overrightarrow{t_v}\right| = t_v = \tau = 2\ 10^{-6}\ \text{sec})$$

Further, this event B has the same τ- value in the RF K of the earth, and the μ-mesons have then gone the distance $x = 10^5$ feet in K, giving $x/c \approx 10^{-4}$ sec. Thus, the time vector of event B, as specified in K equals

$$\vec{t} = \binom{\tau}{x/c} = \binom{2\cdot 10^{-6}}{10^{-4}}\ sec;\ (\text{with } \left|\vec{t}\right| = t \approx 10^{-4}\ \text{sec})$$

So, these two time vectors verify the given observations, stating that $t \approx t_v \cdot 50 \approx 10^{-4}$ sec.

In summary, the given observations are in full agreement with the theory. It is an experimental fact that the (imagined) clock following the μ-mesons goes slower by a factor 50, *when it is compared with two clocks,* being *stationary relative to the earth*. But, how shall we interpret /refer to this? It is somewhat surprising that *e.g.* Mermin (2005) formulates the findings as: "The atomic particles can go much further because their internal clocks that govern when they decay are running much more slowly in the frame in which they rush along at speed close to *c*. This is a real effect, and it plays a crucial role in the operation of such particles accelerators".

However, rather than referring to this as a *real* effect, I suggest it should be referred to as an *observational* effect; caused by the observer (at K) moving relative to the occurring phenomenon taking place on K_v.

Similarly, regarding use of the concept *internal clocks* (of the atomic particles): Actually, if the observer (the observing clock) is at rest with respect to the phenomenon, then he will find that the average 'lifetime' of the μ-mesons equals 2 microseconds 'as usual'. So, this value truly expresses the 'internal clock' of the μ-mesons; while passing observers - at various speeds, v - will make 'all kind' of observations, (depending on v). And the 'inner clock' is not affected by passing observers. So, we see $\tau = 2 \cdot 10^{-6}\ sec$ to represents the 'true' lifetime of the μ-mesons also in the present observations. I note that *e.g.* Serret (2018) supports this view.

So, this case exemplifies the argument that the correct RF to describe the phenomenon is the one which 'follows' the event. It is the phenomenon itself – and not the observer – that should have priority.

3.4 Example 4: The Michelson - Morley experiment

Finally, we take look at some standard results, which we choose to relate to the well-known Michelson-Morley experiment, (Michelson and Morley, 1887). They designed an interferometer to compare the speed of light in the direction of the earth's motion and the speed of light at right angles to earth's motion. No difference in speed was found; (thereby, discrediting the so-called ether theory; and one could conclude that the speed of light is a universal constant).

In particular the beams of light in *both* directions (of an *x-y* plane) returned to the same location, at the same instant. Now making observations in an RF, K_v moving at speed v in the x- direction, relative to the RF of the experiment (the earth), we would observe the trajectories of the beams as indicated in Fig. 3.3; (here with a negative v). The red trajectory, OAC equals the beam along the x-axis, and the blue one, OBC that along the y-axis.

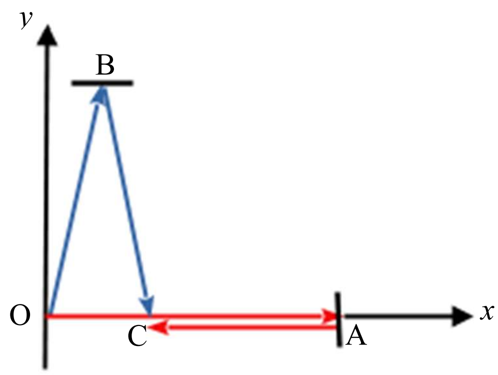

Figure 3.3 Two light rays in the x-y plane, measured in an RF moving along x-axis (*cf.* Michelson-Morley experiment)

Thus, both beams start in O, and they return simultaneously to the point C (on the moving RF, K_v). At this instant of return the clock on K_v reads $2t_v$. Thus, the distances OB and BC both equal ct_v. Further, the distance OC equals $2vt_v$. It directly follows that the time t^* when the 'red beam' arrives at A equals $t^* = (1+v/c)t_v$.

Of course, the 'blue beam' arrives at B at the time, t_v, and so the arrival to A and B are simultaneous if and only if v=0. But in case we actually observe that the two beams return simultaneously at the same point they left (*i.e.* C = O in Fig. 3.3), then we should obviously apply the RF that conforms with this observation, that is having $v = 0$, (giving that the arrivals of the beams to A and B are simultaneous).

4 Summary and Conclusions

In the above discussions (of standard results) we pinpoint that it *does* exist one RF, being best suited to describe a phenomenon; and thus, providing 'correct' time durations and of simultaneity of events 'at a distance'. We now outline the following rules for specifying the framework of the analysis.

1. We choose an RF that follows the phenomenon as closely as possible; in particular we apply the RF where the chain of events occurs; not that of an observer moving relative to the process.

2. The RF shall secure that any inherent (or chosen) symmetry of the situation is accounted for; (but not any 'unwanted' symmetry).
3. When an initiating event (or a set of events) is defined, the 'internal' clocks of these events are calibrated with the local clock on the RF; (focusing on time differences, only).
4. When there is a change in the velocity of one or more moving object/event involved, we specify a new initiating event, restarting from the current time, t of the 'local clock'. There is, however, no update on the internal clock (τ) of the chain of events.
5. We stick to the chosen RF throughout the experiment; *i.e.* we do not change RF during the course of events.

When we discuss simultaneity, the chains of events involved, typically start out from one common initiating event. The choice of RF will depend on process conditions, and by defining a rule for bringing the moving objects/clocks together again, we may identify the correct RF, (which rightfully predicts this event of reunion). Thereby we may provide a true simultaneity also for events 'at a distance'.

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