New Research on Electromagnetic Induction

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[Abstract] The electromotive force is divided into motional electromotive force and induced electromotive force. Through theoretical analysis and calculation, this paper reveals that the electrostatic field loop theorem can be extended to the general electric field loop theorem: In any electric field, the line integral of the electric field intensity along any closed path is equal to zero. Maxwell's mathematical expression of Faraday's law of electromagnetic induction is incorrect. Faraday's law of electromagnetic induction should be modified to Faraday's law of open circuit electromagnetic induction. Based on Helmholtz coil, a uniform alternating sinusoidal magnetic field is formed in the central area of the coil. Two different shapes of induction coils, round and square, are used. Their sectional area and the number of coils are equal. The two induction coils with different shapes are respectively placed in the center of Helmholtz coil, and their open circuit induced electromotive force is measured. The experimental results show that the induced electromotive force is not only related to the change rate of magnetic fluxes passing through the loop, but also to the shape of the loop, so Faraday's law of open circuit electromagnetic induction not correct. This paper further reveals the metal wire law of induced electromotive force: The induced electromotive force of any metal wire is proportional to the magnetic induction intensity, angular frequency of the magnetic field wave and the projection length of the wire in the normal direction of the magnetic field.

[Key words] motional electromotive force, induced electromotive force, Maxwell’s equations, Faraday's law of open circuit electromagnetic induction, electrostatic field loop theorem, general electric field loop theorem, Helmholtz coil, metal wire law of induced electromotive force, metal wire law of motional electromotive force, Lorentz magnetic field force.

1. introduction

In 1820, the Danish physicist Oster found that electric current can produce a magnetic field. Many scientists began to explore and study whether the magnetic field can also produce electric current. In 1831, Faraday revealed for the first time through experiments that the change of magnetic fluxes in the metal circuit can produce induced current and voltage in the metal circuit.

As shown in Figure 1-1, a n-turn metal coil C is connected in series with a galvanometer A and a load resistance R_L to form a closed loop, and the load resistance R_L is connected with a voltmeter V in parallel. When the magnetic rod B is inserted or pulled out of the metal coil C, the coil C generates current and the pointer of the galvanometer A deflects; The coil C also generates voltage， and the pointer of voltmeter V deflects. The faster the insertion or withdrawal speed, the greater the current and voltage generated by coil C; When the magnetic rod stops in the coil, there is no current in the circuit and no voltage on the load resistance R_L.

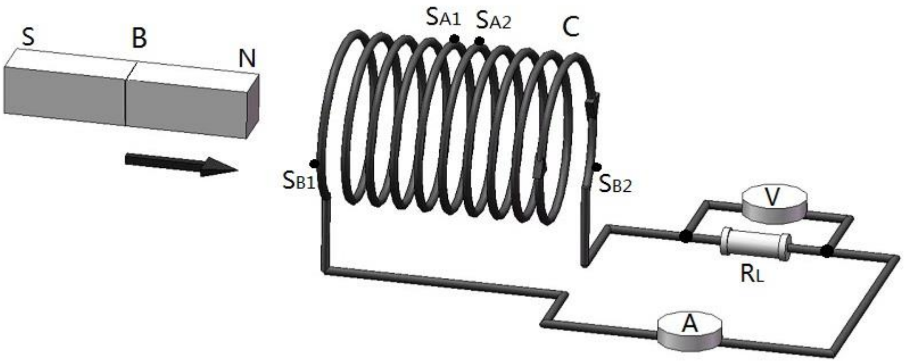

Figure 1-1

With a large number of experiments，Faraday revealed: the magnitude of the induced current changes with the resistance of the circuit, but the magnitude of the induced electromotive force does not change with the resistance and structure of the circuit. It is concluded that the induced electromotive force generated in a closed metal circuit is directly proportional to the change rate of the magnetic fluxes passing through the circuit. This conclusion is called Faraday's law of electromagnetic induction, which is expressed as

$$\varepsilon_1 = -\, d\Phi/dt \tag{1-1}$$

Where "-" indicates the direction of the induced electromotive force, which is determined by Lenz's law.

As shown in Figure 1.1, formula (1-1) is a single turn coil and it is the induced electromotive force between S_{A1} and S_{A2}. For the n-turn metal coil C, the induced electromotive force between the two ends S_{B1} and S_{B2} of the coil：

$$\varepsilon_n = -\, n\, d\Phi/dt \tag{1-2}$$

According to the definitions of electric potential and electric field, Maxwell expressed the above induced electromotive force as the line integral of the electric field intensity at each point of the circuit to the closed metal circuit. Faraday's law of electromagnetic induction in Maxwell integral form is

$$\oint_L E \cdot d\ell = -\frac{\mathrm{d}\Phi_B}{dt} \tag{1-3}$$

Faraday's law of electromagnetic induction points out that no matter what the reason, as long as the magnetic fluxes passing through the enclosed area of the circuit change, the induced electromotive force will be generated in the circuit. There are two ways in which the magnetic fluxes change: The first is that the magnetic field is unchanged, while the whole or part of the closed circuit moves in the magnetic field, resulting in the change of the magnetic fluxes in the circuit. The electromotive force generated in this way is called motional electromotive force; The second is that the closed circuit does not move, and the space magnetic field changes, resulting in the change of magnetic fluxex in the circuit. The electromotive force generated in this way is called induced electromotive force.

The following is a more detailed analysis of electromagnetic induction through a example, which belongs to motional electromotive force.

As shown in Figure 1-2, it is common in physics textbooks[2][3]. In the uniform magnetic field B, place a metal wireframe ABCD. The wireframe has two parts. The fixed U-shaped part of the wireframe is composed of metal wires AD, AB and BC. The metal wire CD can slide left and right, and its length is L_{CD}.

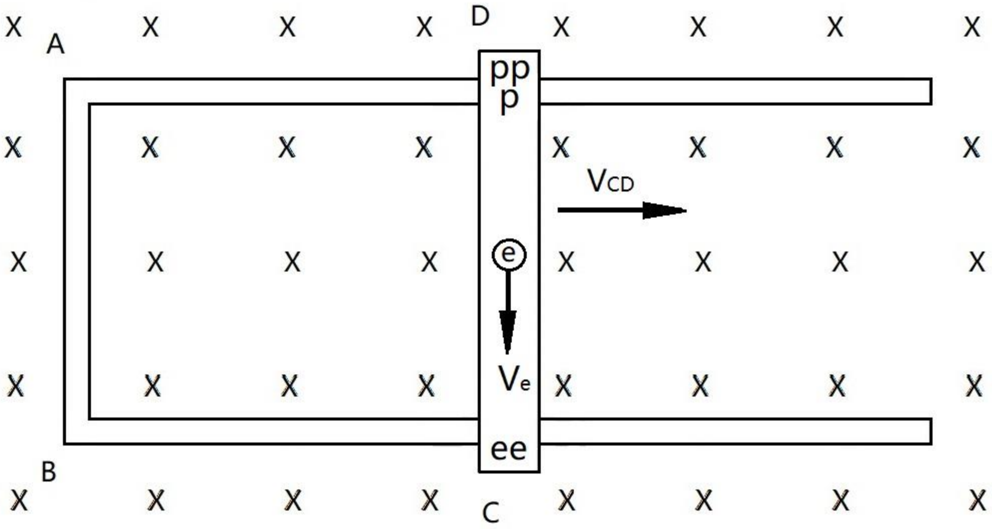

Figure 1-2 Motional electromotive force

When the metal wire CD is not in contact with the fixed U-shaped part of the wireframe, that is, the circuit is disconnected, the metal wire CD moves to the right at the speed of V_{CD}, and the free electron e in the metal wireframe CD moves along the wire to the C end at the speed of V_e under the action of Lorentz magnetic field force, so that the negative electron e accumulates at the C end, and the positive charge p accumulates at the D end, thus generating an electric field from the D end to the C end. The the free electron e on the metal wire CD is affected by both the Lorentz magnetic field force and the electric field force. When the Lorentz magnetic field force and the electric field force balance, the accumulation of the electron e and the positive charge p stops.

The metal wire CD moves to the right at the speed of V_{CD}, and the magnetic fluxes increased by closing the wireframe ABCD in dt time

$$d\Phi = \mathbf{V}_{CD} L_{CD} B\, dt \qquad （1\text{-}4）$$

The change rate of magnetic fluxes

$$d\Phi/dt = \mathbf{V}_{CD} B L_{CD} \qquad （1\text{-}5）$$

According to equations (1-1) and (1-5), the electromotive force of metal conductor CD

$$\varepsilon_{DC} = \mathbf{V}_{CD} B L_{CD} \qquad （1\text{-}6）$$

The metal wire CD is equivalent to a battery. The D end is positive and the C end is negative. The induced electromotive force ε_{DC} is directly proportional to the velocity V_{CD} of the metal wire CD, that is, the change rate of the magnetic fluxes.

2. Faraday's law of open circuit electromagnetic induction

Faraday's law of electromagnetic induction reveals that the induced electromotive force in a closed metal circuit is directly proportional to the change rate of the magnetic fluxes passing through the circuit, and has nothing to do with the shape and structure of the closed metal circuit. Due to the experimental conditions at that time and the lack of systematic understanding of the electric field and magnetic field, Faraday, as a great experimental physicist, lacked mathematical knowledge, and his three volumes of experimental Monographs on electromagnetism did not have a mathematical formula [4]. Faraday himself lacked a profound understanding and clear and complete expression of his experimental results, while Maxwell, as a mathematical genius, also lacked a systematic understanding of electromagnetism, Maxwell did not fully understand Faraday electromagnetism experiment in the mathematical expression and expansion of Faraday electromagnetic induction law.

According to the electrostatic field loop theorem, the line integral of electric field intensity along any closed path in electrostatic field is equal to zero. That is, the left side of equation (1-3) in the electrostatic field must be equal to zero. In fact, the left side of equation (1-3) represents the induced voltage of the metal coil at a certain time, and the time interval at a certain time is zero. Even in an alternating electric field, at a certain time when the time interval is zero, the electric field intensity at any point in the closed loop is a fixed value, and the electrostatic field loop theorem is also true in an alternating electric field. The electrostatic field loop theorem can be extended to the general electric field loop theorem: Nn any electric field, the line integral of the electric field intensity along any closed path must be equal to zero.

In fact, for Figure 1-1, in the closed loop formed by metal coil C, galvanometer A and load resistance R_L, the induced electromotive force is the voltage value of load resistance R_L.

Because the internal resistance of coil C is far less than the load resistance R_L, the induced electromotive force of metal coil C in Faraday's law of electromagnetic induction is actually the open circuit induced voltage of metal coil C. For the induced electromotive force of single turn coil according to the formula (1-1) ε_1 is the induced voltage when S_{A1} and S_{A2} are open circuit. And formula (1-2) induced electromotive force ε_n of n-turn metal coil C. Is the induced voltage at the open circuit of the two ends S_{B1} and S_{B2} of the coil.

Therefore, Faraday's law of electromagnetic induction actually expresses the induced electromotive force of the open circuit metal coil. Faraday's law of electromagnetic induction should be revised to Faraday's law of open circuit electromagnetic induction: the magnitude of the induced electromotive force generated in the open circuit metal coil is directly proportional to the change rate of the magnetic fluxes passing through the coil.

3. Metal wire electromagnetic induction law

The modified Faraday's law of open circuit electromagnetic induction reveals that the induced electromotive force generated in the open circuit metal coil is directly proportional to the change rate of the magnetic fluxes passing through the coil. Does the shape of the metal coil affect the induced electromotive force of the coil?

As shown in Figure 3.1， the verification experiment of metal coil induced electromotive force uses Helmholtz coil, which can form a uniform magnetic field in its central area. The Helmholtz coil in the experiment has two coils, with an inner diameter of 180mm and an outer diameter of 286mm. A uniform magnetic field area of 50mmx50mmx50mm in the center of the Helmholtz coil can be formed, with a uniformity of 99%. Moreover, the magnetic induction intensity in the center area has a linear relationship with the input current of the coil. With calibration of the Helmholtz coil, the relationship between magnetic induction intensity B and current I is as follows:

$$B = 0.00114\ I \qquad (3\text{-}1)$$

The unit of magnetic induction intensity B is Tesla(T), and the unit of current is Ampere(A). When the Helmholtz coil is input with an angular frequency of ω Sinusoidal alternating current， a uniform sinusoidal alternating magnetic field will be formed in the central area of the Helmholtz coil， and its sinusoidal alternating magnetic field intensity：

$$B(t) = B_{max} \sin\omega t$$

Where B_{max} is the maximum magnetic induction intensity.

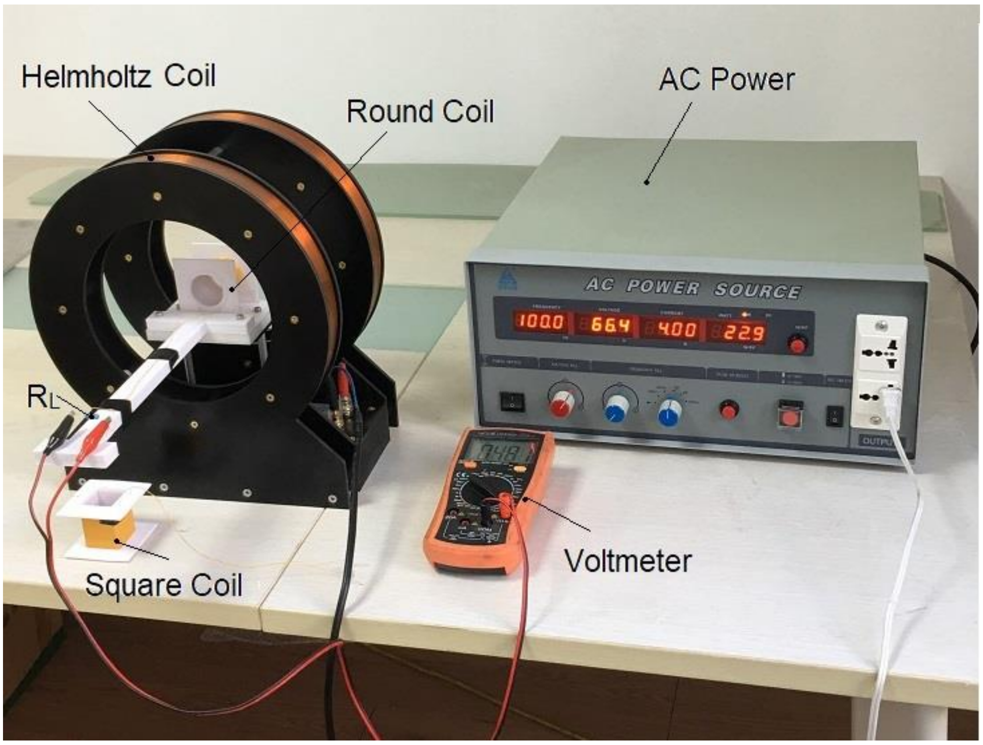

Figure 3.1 Verification experiment of
the metal coil induced electromotive force with Helmholtz coil

In the experiment in Figure 3.1, two different shapes of magnetic induction coils, round and square, are used for comparative measurement. The diameter of the round coil is 35.0mm, and its sectional area is 962.1mm^2. The side length of the square coil is b=31.0mm, and its sectional area is 961.0mm^2. The sectional areas of the two magnetic induction coils are almost equal, taking both of them S_1=960.0mm^2 within the allowable range of error. The height of the two magnetic induction coils is the same, both of them are 40mm. Both coils are wound n=172 turns with 0.5mm magnetic wires in double layers. The load resistance R_L connected at two ends of the coil is 220 kΩ, which is much greater than the internal resistance of the coil and the coil is equivalent to an open circuit. The two different shapes of magnetic induction coils are respectively placed in the central area of the Helmholtz coil, and the central axis of the induction coil coincides with the central axis of the Helmholtz coil. As shown in Figure 3.1. Then the magnetic fluxes of the induction coils:

$$\Phi(t) = n\, S_1\, B(t)$$

$$= n\, S_1\, B_{max} \sin\omega t$$

According to Faraday's law of open circuit magnetic induction, the induced electromotive force of the coil:

$$\varepsilon = -\, d\Phi\,(t)/dt$$

$$\varepsilon = -n\,S_1\,\omega B_{max}\cos\omega t$$

The induced electromotive force of the coil is also a sine wave, and its effective value

$$\varepsilon_{rms} = \quad n\,S_1\,\omega B_{rms}$$

Here n=172, S_1= 960mm^2 = 9.6x10^{-4}m^2， then the effective value of its induced electromotive force:

$$\varepsilon_{rms} = 0.1651\,\omega B_{rms} \qquad (3\text{-}2)$$

In the experiment, the AC power supply is used to supply power to the Helmholtz coil. The frequency is set to 50Hz and 100Hz, and the effective value of the input current is set to 2.00A and 4.00A. The round and square induction coils are respectively placed in the central area of the Helmholtz coil, as shown in Figure 3.1. Compare the value of induced voltage measured on the load resistance R_L. Table 3.1 shows the effective value of the induced electromotive force calculated from the formula (3-2) and that of actual experimental measurement results.

Table 3.1 Induced electromotive force (rms) of the round and square induction coils

Frequency /Current (rms)	Calculation according to formula (3-1) Magnetic induction intensity (rms)	Calculation according to formula (3-2) Induced electromotive force (rms)	Measurement of round coil Induced electromotive force (rms)	Measurement of square coil Induced electromotive force (rms)
50Hz/2.00A	0.00228 T	0.118V	0.120V	0.130V
50Hz/4.00A	0.00456 T	0.236V	0.240V	0.259V
100Hz/2.00A	0.00228 T	0.0376V	0.240V	0.259V
100Hz/4.00A	0.00456 T	0.4721V	0.481V	0.519V

It can be directly concluded from Table 3.1 that, within the allowable range of calculation and experiment errors, the theoretical value of the induced electromotive force calculated by the modified Faraday law of open circuit electromagnetic induction and the measured value of the induced electromotive force obtained by the experiment of the round induction coil can be considered equal. However, the round and square induction coils have the same sectional area and the same change rate of the magnetic fluxes passing through them, but the experimental measurement values of the induced electromotive force are obviously different. Therefore, the induced electromotive force generated in the metal coil is related to the shape of the metal coil, and Faraday law of open circuit electromagnetic induction is incorrect.

Table 3.1 reveals that induced electromotive force ε_{rms} of the coil is proportional to the magnetic feild intensity B_{rms} and the angular frequency ω. In addition, the induced electromotive force of a coil is the line integral of the electric field intensity at each point of the coil. Therefore, the induced electromotive force of the round induction coil can be expressed as:

$$\varepsilon_{rms}=k\int_C \omega\, B_{rms}\, d\ell \qquad (3\text{-}6)$$

Where k is the proportional coefficient of induced electromotive force. According to Table 3.1, at 100Hz/4.00A, the magnetic field intensity $B_{rms}=0.00456T$, the radius of round coil r=17.5mm, the number of coils n=172, and the induced electromotive force $\varepsilon_{rms}=0.480V$. Substitute these data into (3-6) and obtain the proportional coefficient of induced electromotive force $k=0.00886\ VST^{-1}m^{-1}$.

In Experiment 3.1, the central axis of the magnetic induction coil coincides with the central axis of the Helmholtz coil, if there is an angle φ between the two central axes, the induced electromotive force of the round induction coil can be expressed as:

$$\varepsilon_{rms}=k\int_C \omega\, B_{rms}\sin\varphi\, d\ell \qquad (3\text{-}7)$$

Where φ is also the angle between B_{rms} and $d\ell$. In Experiment 3.1, φ is 90°.

According to formula (3-7), the metal wire law of induced electromotive force can be obtained: The induced electromotive force of any metal wire is proportional to the magnetic induction intensity, angular frequency of the magnetic field wave and the projection length of the wire in the normal direction of the magnetic field.

Equations (3-6) and (3-7) are valid for round induction coils, but they are not directly applicable to square induction coils. The induced electromotive force of square induction coil needs further study.

There are two ways to calculate the motional electromotive force. The first is to calculate the change rate of magnetic fluxes passing through the coil with Faraday's law of open circuit electromagnetic induction. The second way is to calculate the motional electromotive force of any metal wire with Lorentz magnetic force:

$$\varepsilon=\int_L (\mathbf{V}\times\mathbf{B})\cdot d\ell \qquad (3\text{-}8)$$

Comparing equations (3-7) and (3-8), the two equations are similar, both of them are proportional to the magnetic induction intensity and the length of the metal wire. The angular frequency in equation (3-7) represents the number of sine waves passing through the wire in unit time, while the velocity V in equation (3-8) represents the number of magnetic wires passing through the wire in unit time.

According to the above analysis, the induction electromotive force is divided into motional electromotive force and induced electromotive force. The motional electromotive force is excited by the metal wire moving relative to the magnetic field and passing through the magnetic line of force. The induced electromotive force is excited by the magnetic field wave moving relative to the metal wire and passing through the wire. Either the motional electromotive force or the induced electromotive force is, in essence, the electric potential formed by the uneven distribution of charges accumulated on the wire under the action of Lorentz magnetic force. Equations (3-7) and (3-8) are the physical essence of induced electromotive force and motional electromotive force. Faraday's law of open circuit electromagnetic induction is an empirical formula. The experiment in Figure 3.1 reveals that Faraday's law of open circuit electromagnetic induction is not correct.

5. Conclusion

The electromotive force is divided into motional electromotive force and induced electromotive force. Through theoretical analysis and calculation, this paper reveals that the electrostatic field loop theorem can be extended to the general electric field loop theorem: In any electric field, the line integral of the electric field intensity along any closed path is equal to zero. Maxwell's mathematical expression of Faraday's law of electromagnetic induction is incorrect. Faraday's law of electromagnetic induction should be modified to Faraday's law of open circuit electromagnetic induction.

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