Axioms for a Differential Geometric approach to the von Neumann's Theory of Quantum Measurement

By

Dr. Moninder Singh Modgil

Spiritual Applications Research Center (SpARC)
Gyansarovar, Mount Abu, Rajasthan, Pin 307501
India

Email: msmodgil@gmail.com
Date: 25th December 2022
ORCID: 0000-0002-1890-2841

Abstract

This papers introduces axioms for von Neumann's theory of quantum measurement in a differential geometric framework. Our visual perception places us (observers) at the origin of a 3-dimensional cartesian coordinate system. So the **first axiom** is that Observers can be regarded as point objects/particles. Our **second axiom** is that perceptual experiences can be regarded as occurring on a tangent space at the point where the Observer is located. Due to our psychological experience of time asymmetry, we have the **third axiom** that, the observer is traversing irreversibly along the future time-like direction, on the (3+1) dimensional space-time manifold. Interaction between Observers is discussed at classical and quantum mechanical levels. Quantum mechanical experiments such as those of Schrodinger's Cat, and the Quantum Zeno Effect, are examined; keeping in view, the information processing and flicker rate, with in the brain. It is anticipated that as the spatial and temporal resolution, of real time brain investigation techniques, such as the functional Magnetic Resonance Imaging (fMRI), Superconducting Quantum Interference Device (SQUID), Magneto-Encephalo-Gram (MEG), Electro-Encephalo-Gram (EEG), reach a critical level – the Quantum Measurement Processes within the brain, would be open to study experimentally.

Keywords: Differential Geometry, Quantum Measurement, von Neumann's Interpretation, Many Worlds Interpretation, Quantum Zeno Effect, Neuroscience, Axiom of Choice, Flicker rate, fMRI, SQUID, MEG, EEG

1. Introduction.

Here in, we introduce axioms for a differential geometric frame work for von Neumann's Theory of Quantum Measurement [1]. In his seminal work, von Neumann emphasized that collapse of the wave function occurs in the brain, at the level of Observer. This point of view has also been advocated by Wigner [2]. In this paper, we present the axioms, which localize the Observer, on the space-time manifold - and introduce the Observer's Perceptual Tangent Spaces. We investigate how the information from the object of measurement propagates through the environment; onto the human senses and therein to the brain. We consider quantum wave functions for this classical description and examine von Neumann's concept of wave function collapsing at the level of the observer, within the brain.

Differential Geometry [3] has been used successfully in modeling objects such as Black Holes [4], and Universes with various metrics and matter distributions [5,6]. In recent years, Differential Geometry has also made inroads in Quantum Mechanics. Ciaglia, Ibort and Marmo [7] have developed Differential Geometry of Quantum States, Observables and Evolution. They suggest, that this approach could lead to a unification of General Relativity and Quantum Mechanics, i.e., a new theory of Quantum Gravity. Kycia [8] has developed Cartan Connection for Schrodinger equation. Martini and Santamato [9] have investigated Nonlocality, No-Signalling and Bell's Theorem in Weyl's Conformal Geometry [10]. Interestingly, differential geometric methods have also been developed for visual and audio, processeing - both with in the brain, as well as in computers. Role of neuroscience [11] in this framework comes to fore. Hoffman [12] showed that Visual Cortex (brain area of visual processing), is a contact bundle. Subsequently, a number of applications of differential geometry to neuro-vision were made – see for instance Petitot [13], Dmitri, Alekseevsky and Andrea [14]. Montobbio, Sarti and Citti [15], developed a metric model for visual cortex. Boscain et. al. [16] adapted Hoffman's model to computer Audio processing. Zucker [17] has used differential geometry in vision processing. Another example of Neurogeometrically inspired computer vision is that given by Baspinar [18].

. It is anticipated that as the spatial and temporal resolution, of real time brain imaging techniques, such as the functional Magnetic Resonance Imaging (fMRI) [19], Superconducting Quantum Interference Device (SQUID) [21, 22], Magneto-Encephalo-Gram (MEG), Electro-Encephalo-Gram (EEG), reach a critical point – the Quantum Measurement Processes within the brain, would be open to study experimentally.

2. The Axioms.

We introduce the axioms below -

- Our visual perception places us at the origin of a 3-dimensional cartesian coordinate system. On a cosmological scale, we can indeed be considered as points. So, Our **first axiom** is that the Observer, denoted as O, is considered to be localized on to a point p.
- Our **second axiom** is that the space of the perception of an Observer, can be regarded as existing on a tangent space whose origin is located at the point p. We will denote this Tangent space as T_p^O.
- Due to our psychological experience of time asymmetry [22], we have the **third axiom** that, O is traversing irreversibly along the future time-like direction – on the (3+1) dimensional space-time manifold $M_{3,1}$ with the metric $g_{\mu\nu}$. For irreversible future evolution of O we can specify its "temporal displacement rate", as N Planck Time units per second. 'N' can vary due to special relativistic or gravitational time dilation. The space-time manifold, can have a non-trivial topology. We note that a number of physical phenomenon, including collapse of wave function in quantum mechanics - also show time asymmetry [23,24,25].

In general, multiple Observers –

$$O_1, O_2, \ldots, O_k \tag{1}$$

can be regarded as located on a series of points –

$$p_1, p_2, \ldots, p_k \tag{2}$$

on $M_{3,1}$.

3. The Perceptual Tangent Space.

Our approach of perceptual tangent space of an Observer, has some similarity with dualistic view of Sir Ecclles [26] – the dual entities being –

(1) Objects made up of the Energy-matter existing on $M_{3,1}$, and
(2) The $T_{p_i}^{C_i}$, - perceptual tangent spaces of the Observers.

The point p_i is the common point of tangency between $M_{3,1}$ and $T_{p_i}^{O_i}$. We define a map f_i which takes points on $M_{3,1}$ to points on $T_{p_i}^{O_i}$. Under this mapping, a neighborhood $N_{p_i}^{M_{3,1}}$ of p_i , on $M_{3,1}$ can be mapped on to a neighborhood of p_i on $T_{p_i}^{O_i}$ –

$$f_i \colon N_{p_i}^{M_{3,1}} \to N_{p_i}^{T^{O_i}} \tag{3}$$

The physical body associated with O_i on $M_{3,1}$ will be referred to as $\mathfrak{I}_3^{O_i}$ – where the subscript 3 indicates the dimensions. The geometrical (spatial) extent of $\mathfrak{I}_3^{O_i}$ would be referred to as $G_3^{O_i}$, whose 2 dimensional surface is $\partial G_3^{O_i}$. Their mapping onto the Tangent space is $f_i(G_3^{O_i})$ and $f_i(\partial G_3^{O_i})$. Due to - (1) Our motor abilities, and (2) Irreversible passage of time; these objects are time dependent and may be written as $\mathfrak{J}_3^{O_i}(t), G_3^{O_i}(t), \partial G_3^{O_i}(t), f_i(G_3^{O_i}(t))\ f_i(\partial G_3^{O_i}(t))$; where t is the time. As the Observer O_i moves around in the space-time continuum $M_{3,1}$, $G_3^{O_i}$, $T_{p_i}^{O_i}(t)$ also move accordingly. We can associate a world line l^{O_i}, parameterized by t , for temporal evolution of Observer O_i. Thus, we have the fiber bundle [27] –

$$\mathcal{F}_i = T_{p_i}^{O_i} \times l^{O_i} \tag{4}$$

In the above equation, the subscript *i* identifies the observer, and the point p_i is co-moving with the observer O_i.

On T_p^O we define a special class of cartesian coordinate systems, whose origin is the point of tangency, i.e. – the origin is touching p .

In general, we have five senses, namely – sight, sound, taste, smell and touch [28]. Each of them differs in –

(1) The physics of the interface between external world and the body, and
(2) Processing of the input by the brain, and its extension – the nervous system.

For the sensory aspects of the Observer, we can define a sensory tangent space for each of the sense, namely –

$$T_{p_i}^{Vision}, T_{p_i}^{Audio}, T_{p_i}^{Touch}, T_{p_i}^{Taste}, T_{p_i}^{Smell} \tag{5}$$

Then, there exists the reverse interaction – actions by a person through the motor system – modifying the external world. Accordingly, we define a tangent space for motor actions and propioception –

$$T_{p_i}^{Motor} \tag{6}$$

Various structures need to be defined on these spaces – according to the corresponding functions of brain, nerves and sensors. The total Perceptual Tangent Space (PTS), can be written as their tensor product, i.e., -

$$T_{p_i}^{O_i}(t) = T_{p_i}^{Vision}(t) \otimes T_{p_i}^{Audio}(t) \otimes T_{p_i}^{Touch}(t) \otimes T_{p_i}^{Taste}(t) \otimes T_{p_i}^{Smell}(t) \otimes T_{p_i}^{Motor}(t) \tag{8}$$

In the rest of this paper, we will use the following abbreviations for the superscript identifying the Tangent Space -

$Vision \rightarrow V$

Audio → *A*
Touch → *To*
Taste → *Ta*
Smell → *S*
Motor and Propioception → *M* (9)

Accordingly, the perceptual tangent space, can be re-written as –

$$T_{p_i}^{O_i}(t) = T_{p_i}^{V} \otimes T_{p_i}^{A}(t) \otimes T_{p_i}^{To}(t) \otimes T_{p_i}^{Ta}(t) \otimes T_{p_i}^{S}(t) \otimes T_{p_i}^{M}(t) \tag{10}$$

4. Properties of the Perceptual Tangent Spaces.

Here, we discuss the properties of the Visual and Audio Perceptual Tangent Spaces.

4.1 The Visual Tangent Space $T_{p_i}^{V}$

Let $V_{p_i}^{+}$ and $V_{p_i}^{-}$ be the future and the past light cones at p_i respectively. Our visual, perceptual experience is limited to absorption of light emitted in the past – and subjected to the processing by the eyes and the brain. Further, it is a subset of the solid angle of $4\,\pi$ Sr. What is coming into eyes is a stream of photons. Thus, if $\mathcal{A}^V$ is our visual experience, of an object O_i located at p_i on $M_{3,1}$, we have –

$$\mathcal{A}^V \subset T_{p_i}^{V} \subset f_i^{V}(V_{p_i}^{-}) \tag{11}$$

i.e., Visual perception $\mathcal{A}^V$ of an object, is a subset of the Visual Perceptual Tangent Space $T_{p_i}^{V}$, which is a subset of the mapping of the past light cone $V_{p_i}^{-}$.

Let,

$$Q_1^V, Q_2^V, \dots, Q_{N_V}^V \tag{12}$$

be objects lying in $V_{p_i}^{-}$. Here, N_V stands for number of visual object in the field of vision of the Observer O_i. Let, $\mathcal{P}_l^i(Q_l^V)$ be the set of photons streaming from Q_l^V into the eyes of the Observer O_i. Let $\mathcal{B}_i^V$, be the processing by eyes and brain – leading to a sensed object in the Visual Tangent Space. Alternatively, $\mathcal{B}_i^V$ stands for processing which starts in retina; then information going via optic nerves to Optic Chiasm; splitting into left half and right half of the visual field; then optic radiation going into occipital cortex via lateral geniculate nuclei; on to visual Brodmann areas; then coming forward and finally coming into the conscious awareness. Then for each Q_l^V we have –

$$\mathcal{B}_i^V\left(\mathcal{P}_l^i(Q_l^V)\right) \subset T_{p_i}^{V} \tag{13}$$

And reciprocally the Observer's Perceptual Tangent Space is the union of all such images of the objects –

$$T_{p_i}^{V} = \bigcup_{l=1}^{N_V} \mathcal{B}_i^{V}\left({}^{V}\mathcal{P}_l^{i}(Q_l^{V}) \right) \tag{14}$$

i.e., the Perceptual Visual Tangent Space of observer O_i, is union of perception of all the Visual Objects Q_l^V, which is obtained through the brain processing $\mathcal{B}_i^V$, whose information is carried by the light waves or alternatively, the set of photons. ${}^{V}\mathcal{P}_l^{i}$, from point of origin p_l, to the point of location of observer O_i. $\mathcal{B}_i^V$ is akin to, visual differential geometric processing described, by Hoffman [12].

Note has to be made of the visual perception of the written words – which is processed in association areas of visual and lingual areas.

4.2 The Audio Tangent Space $T_{p_i}^{A}$

Audio and Visual perception have a certain similarity, in the sense that both are based upon incoming physical waves – electromagnetic waves in case of vision, and air pressure waves in case of Audio. Just as vision has association areas for reading and writing (motor areas) similarly, audio has association areas for listening and speaking (motor areas again).

Consider a rock band playing. The summed air pressure waves are impinging onto the ear of the listener. Then the processing starts at ear drums, then Cochlea, Auditory Nerve, and finally audio cortex, and its association areas. This processing will be represented as $\mathcal{B}_i^A$. The hairs in Cochlea perform a Fourier Transform of the incoming air pressure waves. An inverse Fourier Transform is done with in the brain, and an audio-scape of the Rock band is re-created

Analogous to light cones $V_{p_i}^{\pm}$, we define past and future "Audio Cones" –

$${}^{A}V_{p_i}^{-}, \text{ and } {}^{A}V_{p_i}^{+} \tag{15}$$

We have –

$$\mathcal{A}^A \subset T_{p_i}^A \subset f_i^{\,A}(\,{}^{A}V_{p_i}^{-}) \tag{16}$$

i.e., Audio perception $\mathcal{A}^A$of an audio object, is a subset of the Audio Perceptual Tangent Space $T_{p_i}^A$, which is a subset of past Audio-cone ${}^{A}V_{p_i}^{-}$, located at p_i.

Let the audio source objects be –

$$Q_1^A, Q_2^A, \dots, Q_{N_A}^A \tag{17}$$

Analogous to Visual Tangent Space, we have –

$${}^A\mathcal{B}_l^i\left({}^A\mathcal{P}_l^i\left(Q_{p_l}^A\right)\right) \subset T_{p_i}^A \tag{18}$$

where, ${}^A\mathcal{P}_l^i$ stands for air pressure Waves, originating at point p_l and reaching the ears of the Observer located at p_i. ${}^A\mathcal{B}_l^i$ stands for Audio processing within the brain. And then,

$$T_{p_i}^A = \bigcup_{l=1}^{N_A} {}^A\mathcal{B}_i^i\left(\mathcal{P}_l^i(Q_l^A)\right) \tag{19}$$

i.e., the Perceptual Audio Tangent Space of observer O_i, is union of perception of all the Audio Objects Q_l^A, which is obtained through the brain processing ${}^A\mathcal{B}_i^i$, whose information is carried by air pressure waves $\mathcal{P}_l^i$, from point of origin p_l to point p_i, where the observer is located. ${}^A\mathcal{B}_i^i$ is akin to, the Audio differential geometric processing described, by Boscain [16].

As in case of vision, in Audio also, we have to note the speaking, reading and writing.

4.4. An Example of the Interplay between the Tangent Perceptual Spaces.

Consider an Observer, watching a dance, listening to the music, and keeping the beat with his/her feet. This is an interplay between $T_{p_i}^V$, $T_{p_i}^A$ and $T_{p_i}^M$. Futher, if this Observer is eating - the Tangent spaces corresponding to taste $T_{p_i}^{Ta}$, and aroma $T_{p_i}^S$ come into play. Holding the food in hand allows $T_{p_i}^T$ to be part of the perceptual process. There exist mappings of subset of each pair of the Tangent Spaces to provide the rich, complex experience to the observer.

4.3 Interpreting Vectors and Tensors on the Perceptual Tangent Spaces.

Our complete conscious experience can be encoded as vectors and tensors defined within the Tangent Spaces. This is a result of processing within the brain. These vectors and tensors are time dependent.

5. Interaction Between Observers.

Here, we examine interaction between two Observers O_i and O_j – both at classical level and Quantum Mechanical level.

5.1.The Classical Interaction:

Let $V_{p_i}^-$ and $V_{p_j}^-$ be the past light cones of O_i and O_j respectively. Let both Observers lie on the same space-like surface $\mathcal{F}$ at time instant t_o. We require –

$$V_{p_i}^- \cap V_{p_j}^- \neq \emptyset \tag{20}$$

i.e., the two Observers can be causally connected by light signals – because their past light cones intersect.

Consider the mappings, from the visual object Q_l to observer O_i's perceptual visual tangent space–

$$\tau_i: Q_l \to {}^{V}T_{p_i}^{O_i} \tag{21}$$

and similarly for O_j,

$$\tau_j: Q_l \to {}^{V}T_{p_i}^{O_i} \tag{22}$$

The mapping -

$$\tau_{ij}: \tau_i^{-1}\left({}^{V}T_{p_i}^{O_i}\right) \to \tau_j \left({}^{V}T_{p_j}^{O_j}\right) \tag{23}$$

is between the perceptual tangent spaces of the two observers. It is required that on both sides of the above equation, the perceived objects are compatible – though there could be differences of orientation, brightness etc.. Its required that p_l lie on the intersection of the past light cones of O_i and O_j,

$$p_l = V_{p_i}^- \cap V_{p_j}^- \tag{24}$$

5.1.1. Physical Touch.

The condition is –

$$\partial G_i \cap \partial G_j \neq \emptyset \tag{25}$$

i.e., body surfaces ∂G_i and ∂G_j, of O_i and O_j respectively, touch, and therefore, have a non-null intersection.

5.1.2. Eye Contact.

Clearly, it is not possible for O_i and O_j to exchange light signals at same time instant. For the interaction to happen, one Observer has to lie on past light cone of the other. So what is perceived in the eye contact is an image from the past.

5.1.3. Speech and Hearing.

As in case of eye contact, the speech heard by O_i, and uttered by O_j, comes from the past Audio-cone of the later.

6. Incorporating Hilbert Space Along Side the Space-Time Formalism.

The particles/objects propagate on $M_{3,1}$ and their quantum state is described as rays within a Hilbert Space. Therefore, for their representation, we need a larger manifold. This is -

$$M_{3,1} \otimes \mathcal{H}^{\infty} \tag{26}$$

where, $\mathcal{H}^{\infty}$ is an infinite dimensional Hilbert space, for incorporating wave function of particles propagating of the particles on $M_{3,1}$. A quantum mechanical description of the quantum measurement chain, perceptual process is as follows -

$$\Psi(T_{p_i}^{V}) = \Psi\left[\bigcup_{l=1}^{N_V} \mathcal{B}_i^V\left({}^{V}\mathcal{P}_l^i(Q_l^V)\right)\right] \tag{27}$$

$$= \bigcup_{l=1}^{N_V} \chi^{\mathcal{B}}(\mathcal{B}_i^V) \otimes \left(\chi^{\mathcal{P}}\left({}^{V}\mathcal{P}_l^i\right) \otimes \chi^{Q}(Q_l^V)\right) \tag{28}$$

These equations are interpreted as follows –

(i) $\Psi(T_{p_i}^{V})$ is the Quantum Mechanical Hilbert Space description of the Visual Perceptual Tangent Space of the observer located at p_i.

(ii) Wave function of the visual objects is $\chi^{Q}(Q_l^V)$,

(iii) Wave function of the photons propagating through space-time and conveying information of the object to the eyes, is $\chi^{\mathcal{P}}\left({}^{V}\mathcal{P}_l^i\right)$,

(iv) Finally, wave function describing the processing in the brain, of the visual information coming through the eyes is $\chi^{\mathcal{B}}(\mathcal{B}_i^V)$, leading to perception.

In terms of eigenvalues a_l^Q and eigenvectors ϕ_i^Q the wave function $\chi^{Q}(Q_l^V)$ of the visual object Q_l^V is -

$$\chi^{Q}(Q_l^V) = \sum_{l=1}^{N_Q} a_l^Q \phi_l^Q \tag{28}$$

Similarly, we have for the photons propagating from the object to the observer's eyes –

$$\chi^{\mathcal{P}}(\mathcal{P}_l^V) = \sum_{l=1}^{N_{\mathcal{P}}} a_l^{\mathcal{P}} \phi_l^{\mathcal{P}} \tag{29}$$

And for processing within the brain –

$$\chi^{\mathcal{B}}(\mathcal{B}_i^V) = \sum_{l=1}^{N_{\mathcal{B}}} a_l^{\mathcal{B}} \phi_i^{\mathcal{B}} \tag{30}$$

Corresponding to the collapse of the wave function, we have a Quantum Measurement Chain (QMC) of collapse of the above three wave functions, i.e., -

$$\chi^{Q}(Q_l^V) \rightarrow \phi_C^Q$$

$$\chi^{\mathcal{P}}(\mathcal{P}_l^V) \rightarrow \phi_C^{\mathcal{P}}$$

$$\chi^{\mathcal{B}}(\mathcal{B}_l^V) \rightarrow \phi_C^{\mathcal{B}}$$

where, the subscript 'C', (on the right hand side) of the above three expressions identifies the state to which the corresponding superposed wave function (on the left hand side), collapses to.

7. Information processing in the brain of the Observers, the Schrodinger's Cat and the Quantum Zeno Effect (QZE).

While our perceptual experience shows continuity of vision – however, because of the flicker rate, the monitoring is not continuous. There is a delay of about a few hundred milliseconds between the photons entering the eye, subsequent visual processing within the brain – and emergence of awareness in the Observer, of the state of the system [30]. QZE [31, 32] comes into play when an Observer is monitoring a state – such as that of the Schrodinger's cat. One can use a high speed camera, and the observer can, latter on watch the recording; played at a much slower pace. Accordingly, QZE state transition parameters would depend upon the shutter speed at the time of photo-shooting the cat. Here, again if the Observer detects a QZE, then we have Quantum Measurements effecting events in the past, i.e., - the transition rate of the quantum state of the cat, at the time, when the film is shot.

Let τ be the half life of the radioactive material in the Schrodinger's cat experiment. Let T_{Camera} be the shutter speed (reciprocal of the frames per second) of the camera which is filming the cat. Let T_{Brain} be the postulated time scale, for quantum processes within the brain of the observer. The time scale of firing of the neurons T_{Neuron} is a few millisecond.

8. Observers as Dirac δ Functions.

In a certain sense, an Observer can be regarded as a Dirac δ function, due to their properties of localization onto a point and singular value at that point. Let $\mathcal{R}$ be a space-like region in $M_{3,1}$ parameterized by $x \equiv (x_1, x_2, x_3)$, and containing N_O number of Observers – which are modelled as Dirac δ functions located at a set of points $\xi = \{p_1, p_2, \ldots, p_{N_O}\}$; all lying within $\mathcal{R}$.

$$O_i \rightarrow \delta(p_i) \tag{31}$$

We define a function g on $\mathcal{R}$, which is infinite (i.e., a local Dirac δ) on the elements of ξ and zero elsewhere. We have –

$$g(x) = \sum_{i=1}^{N_O} \delta(p_i) \tag{32}$$

and,

$$\int_{\mathcal{R}} g \, dV = N_O \tag{33}$$

where, dV is the volume element -

$$dV = dx_1 dx_2 dx_3 \tag{34}$$

The integral in eq. (33) gives the number of Observers with in $\mathcal{R}$. Introducing time into this picture, we have –

$$O_i \to \delta(t_i, p_i) \tag{35}$$

Let $Q_{p_j}^V$ be a visual object located at p_j in $M_{3,1}$. Let its superposed quantum state be described by a wave packet –

$$\Psi_{Q_{p_j}^V}(\mathrm{x}, \mathrm{t}) = \sum_{i=1}^{n} a_i \psi_i \tag{36}$$

which is lying in an n dimensional Hilbert space $\mathcal{H}^n$. We will consider interaction of this wave packet with the set of observers lying in $\mathcal{R}$ at positions given by ξ. Collapse of the wave function is given by the Projection operators –

$$P_k: \mathcal{H}^n \ni \Psi(\mathrm{x}, \mathrm{t}) \to \psi_k \in \mathcal{H}^1 \tag{37}$$

What is the physics behind the geometric concept of Projection Operators in Hilbert Space? Here, we give an answer in terms of properties of the Dirac δ function. As Ψ propagates in space-time, it need not approach all Observers simultaneously – even though they are lying on the same space-like hypersurface. However, for simplicity we will assume that this is indeed the case. Let the Fourier components incident onto the observer be υ_α, $\alpha = 1, \dots n$. Quantum Measurement Process may be described as follows. Choose, those Fourier components of the Dirac δ Observer – which which are identical to the incoming components. Then choose a phase difference of π between these incoming components and the corresponding components of the Observer's Dirac δ - except the component which is the result of the measurement, i.e., υ_i. Thus, we get the required Quantum Measurement. The Dirac δ, as shown below, is sufficiently robust to carry on doing these measurements indefinitely.

$$\delta_N = \frac{1}{\pi}\left(\sum_{n=1}^{N} \cos nx + \frac{1}{2}\right) \tag{38}$$

$$\delta = \lim_{N \to \infty} \delta_N \tag{39}$$

Now –

$$\delta(x) \times \delta(x) = \delta(x) \tag{40}$$

i.e., Dirac δ can self multiply but still remain unchanged. And -

$$\delta(x) + \delta(x) = \delta(x) \tag{41}$$

i.e., Dirac δ can be added to itself, but still remain unchanged. These properties are similar to those of infinite cardinals [33]. Its clear that we can have as many copies of the Dirac δ, which are non-zero, on a single point x, of the space-time, as required. So, if certain components of Dirac δ are used to annihilate the incoming frequencies, it does not effect this capability of the Dirac δ, to continue to do so, further on.

9. Electroganetic and Gravitational Properties of the Observer.

9.1. Electromagnetic Properties of the Observer.

Electro-Encephalo-Gram (EEG) and Magnet-Encephalo-Gram (MEG) studies show how state of awareness of an observer, depends upon the electromagnetic activity within the brain. To interact with the brain states, the point-like observer must have some electromagnetic properties, while at the same time be electromagnetically neutral – for stability. An electromagnetic multipole meets this criteria.

9.3. Gravitational Aspects - Singularities and Black Holes Associated with the Observers.

Our next question is - does the point like observer have mass? General relativistic considerations indicate that we have a curvature singularity at that point, if the Observer has mass. Further, mass would allow gravitational force to act on it and make its position unstable. Unless, the observer is a naked singularity – it will have an event horizon because of which –

(1) It takes an infinite amount of time, for an object outside, to cross the event horizon, as seen by a stationary observer, outside the horizon. Proper time for an object initially at infinity - to cross the horizon, and reach the curvature singularity, would be finite,

(2) No signal carrying energy would come out of the event horizon. Tachyonic signals could penetrate the horizon and come out of the Black Hole associated with Observer, and influence brain function and other physical processes.

A naked singularity would avoid the two problems, mentioned above.

10. Discussion and unresolved questions.

10.1. Unconscious steps to conscious perceptions.

We do not have the perception of a superposed state. Nor do we have a firsthand information or conscious experience, of how the collapse of the wavefunction occurs, within the brain. If there is a physical mechanism causing collapse with in the brain, as described in the previous section, then the interaction between the observer and the superposed state must also be unconscious. However, in brain there are important processes in which computation precedes conscious perception. For example, a few seconds before we apparently and consciously, make a choice, the brain has already computed what choice we will be making [34, 35].

We can pose the following questions –

- How many neurons in a quantum neural network, are needed, to define quantum state of an external object?
- Is the local neural network of the quantum state, physically isolated (temporarily) from other brain areas?
- How many maximum (and minimum) superposed states can be supported within the brain?
- How long (time wise) can a superposed quantum brain state exist before collapse?

10.2. The Quantum Mechanical Interaction.

von Neumann [1], pointed out that, while trying to measure a superposed microscopic state, the classical measuring device would also go into a superposed state. The observing human brain would also go into a superposed state. And collapse of the wave function would occur at the level of the Observer. So, we have, a **Quantum Measurement Chain (QMC)** with the radioactive material in Schrodinger's cat experiment, at start and Observers at its end. This implies that, for each Observer, we have to postulate a set of Projection operators which lead to the collapse of wave function at each measurement. Now what would happen, if two Observers O_i and O_j are observing the same state, e.g., - that of the Schrodinger's cat? Clearly O_i and O_j, both should see either the cat is dead or alive. We cannot have the situation, where for instance, O_i sees that the cat is alive and O_j sees that the cat is dead. But if both the Observers are seeing an identical state, and also causing the collapse of the wave function – then the corresponding projection operators of O_i and O_j should be compatible. Now, where from is this correspondence coming?

The possibilities for resolving this paradox are–

(1) One could say that there is only one Observer – present in all living beings, and it causes the universal collapse of wave function every where.

(2) There is of course the Everett's Many World Interpretation (MWI) [29] – in which O_i sees the cat alive in Universe 1 (say), and O_j see the cat dead in Universe 2 – with the two Observers inhabiting different universes. Let's say that at time $t = t_0$ the Universe U_0 splits into two branches U_1 and U_2 with the Schrodinger's cat being alive in U_1 and dead in U_2 . Note that this gives a non-Hausdroff topology to the Multiverse. However, in U_1 the body $\mathfrak{I}_3^{O_j}$ corresponding to O_j is not associated with any Observer, and similarly in U_2, the body $\mathfrak{I}_3^{O_i}$ corresponding to O_i is not associated with any Observer. Thus we run into a paradox.

(3) The first Observer who causes the collapse of the wave function, sends a signal (possibly Tachyonic), to all other Observers – and this leads to a universal collapse of the wave function.

(4) **Entangled Observers.** This possibility suggests that there is specific epoch of space-time, when all the Observers are together and consistent Projection operators are recorded within the observers. The Projection operators come into play when the corresponding quantum measurement arises.

10.3. The Neuro-Biological Quantum Zeno Effect.

In recent years, devices such as fMRI [19] and SQUID [21,22], Magneto-Encephalo-Gram (MEG), Electro-Encephalo-Gram (EEG), have come to fore. We anticipate that, as these instruments, reach a specific spatial-temporal resolution, light would be shed experimentally, on collapse of wavefunction within the brain. Monitoring at these high resolutions, could lead to a freezing of quantum mechanical brain state – a **Neuro-Biological Quantum Zeno Effect** (NBQZE) [36].

11 Conclusions:

We have introduced the ansatz that an Observer O can be considered as a point-like object propagating time irreversibly on a space-time manifold. Its Perceptual Spaces are the Tangent Spaces at the point of its location on the manifold. Properties of these Perceptual Tangent Spaces are discussed. Classical and Quantum Mechanical interaction between two Observers, is examined. The Schrodinger's cat experiment and the Quantum Zeno Effect for interaction between the Observer and microscopic quantum system is discussed – keeping in view, the time scales of the brain processes.

References

1. von Neumann, J. : *Mathematical Theory of Quantum Mechanics*, Princeton University Press, (1955).

2. Wigner, E. P.: *The Place of Consciousness in Modern Physics*, The Collected Works of Eugene Paul Wigner, Book Series, Volume B/6.

3. Kobayashi, S. and Nomizu, K. : *Foundations of Differential Geometry*, Wiley, (1963, Vol.1) and (1969, Vol. 2).

4. Chandrasekhar, S. : *The Mathematical Theory of Black Holes*, Oxford University Press, (1983).

5. Hawking, S. and Ellis, G. : *The Large Scale Structure of Space-time*, Cambridge University Press, (1973).

6. Stephani, H., Kramer, D., MacCallum, M., Hoenselaers, C. : *Exact Solutions of Einstein's Field Equations,* Cambridge University Press (2003).

7. Ciaglia, F. M., Ibort, A. and Marmo, G. : *Differential Geometry of Quantum States, Observables and Evolution*, arXiv: 1903.10465v1.

8. Kycia, R. A. : *Cartan Connection for Schrodinger Equation. The nature of Vacuum,* arXiv: 2004.04622v2.

9. Martini, F. D. and Santamato, E. : *Nonlocality, No-Signaling and Bell's Theorem investigated by Weyl's Conformal Differential Geometry*.

10. Weyl, H. : *Space*, *Time*, *Matter*, Dover Publications Inc., New York, (1952).

11. Kandel E. R., Koester J. D., Mack S. H., Siegelbaum S. A.: *Principles of Neural Science*, 6th ed. McGraw-Hill, New York, (2021).

12. Hoffman, W. C. : *The Visual Cortex is a Contact Bundle,* Applied Mathematics and Computation, Vol. 32, pp137-167 (1989).

13. Petitot: *Landmarks for Neurogeometry,* Neuromathematics of Vision, ed. Citti, G. and Sarti, A., Springer, (2014).

14. Dmitri V. Alekseevsky & Andrea F. Spiro : *Conformal models for hypercolumns and the visual V1 cortex,* arXiv:2202.10157v1.

15. Montobbio, N., Sarti, A. and Citti, G. : *A Metric Model for Functional Architecture of Visual Cortex,* arXiv:1807.02479v2.

16. Boscain, U. et. al. : *A bio-inspired geometric model for sound reconstruction,* Journal of Mathematical Neuroscience, (2021), 11:2.

17. Zucker, S. : *Differential Geometry from the Frenet point of view: Boundary detection, stereo texture and color*, Handbook of Mathematical Models in Computer Vision, pages 357-373, Springer (2006).

18. Baspinar, E.: *Multi-frequency image completion via a biologically-inspired sub-Riemannian model with frequency and phase,* arXiv:2110.14330v1.

19. Logothetis, N. K.; Pauls, Jon; Augath, M.; Trinath, T.; Oeltermann, A. : *A neurophysiological investigation of the basis of the BOLD signal in fMRI*. Nature. **412** (6843): 150–157, (July 2001).

20. D. Drung; C. Assmann; J. Beyer; A. Kirste; M. Peters; F. Ruede & Th. Schurig *Highly sensitive and easy-to-use SQUID sensors,* IEEE Transactions on Applied Superconductivity. **17** (2): 699–704, (2007).

21. Romani, G. L.; Williamson, S. J.; Kaufman, L. *: Biomagnetic instrumentation*. Review of Scientific Instruments. **53** (12): 1815–1845, (1982).

22. Zeh, H. : *The Physical Basis of the Direction of Time,* Springer, (1992).

23. Davies, P. : *The Physics of Time Asymmetry*, University of California Press (1977).

24. Sachs, R.G.: *The Physics of Time Reversal*, University of Chicago Press (1987).

25. Mackey, M. C. : *Time's Arrow: The Origins of Thermodynamic Behavior*, Spriger-Verlag (1992).

26. Sir Ecclles, J. C. and Popper, K. : *Self and Its Brain,* Routledge, (1984).

27. Steenrod, N. : *The Topology of Fiber Bundles,* Princeton University Press, (1950).

28. Hall, J. E. : *Guyton and Hall Textbook of Medical Physiology,* Elseiver (2016).

29. Hugh Everett: *Theory of the Universal Wavefunction*, Thesis, Princeton University, (1956).

30. E F Wells 1, G M Bernstein, B W Scott, P J Bennett, J R Mendelson : *Critical flicker frequency responses in visual cortex,* Exp. Brain. Res., 139(1):106-10, (2001).

31. Mishra, B. and Sudarshan, E. C. G. : J. Math. Phys., 756 (1977).

32. Grunbaum, A.: *Modern Science and Zeno's Paradoxes*, George Allen and Unwin Ltd, London (1967).

33. Sheppard, Barnaby, The Logic of Infinity, Cambridge: Cambridge University Press, (2014).

34. Libet, Benjamin; Wright Jr., Elwood W.; Feinstein, Bertram; Pearl, Dennis K. *"Subjective Referral of the Timing for a Conscious Sensory Experience – A Functional Role for the Somatosensory Specific Projection System in Man,* Brain. **102** (1): 193–224, (1979).

35. Chun Siong Soon, Marcel Brass, Hans-Jochen Heinze, John-Dylan Haynes: *Unconscious determinants of free decisions in the human brain,* Nature Neuroscience. **11** (5): pp 543–5, (2008).

36. Modgil, M. S. : *Geometry of Time, Axiom of Choice and Neuro-Biological Quantum Zeno Effect,* arXiv:0704.1054.