Neutron Lifetime Constrains the Gravitational Constant

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Abstract

We improved the neutron lifetime formula and the gravitational constant formula to generate stronger constraints between them. And improved their calculation accuracy, so that they are in better agreement with the experimental data. By calculating the improved formulas, the neutron lifetime is $877.7495\ s$, and the gravitational constant is $6.6742986 \times 10^{-11}\ m^3/(kg \cdot s^2)$.

Introduction

First, let's look at the neutron lifetime formula [1], as follows:

$$t_n = \frac{\pi h}{\Delta mc^2}\frac{m_p}{m_e}\sqrt{\frac{ke^2}{Gm_pm_e}} \tag{1}$$

Where:

t_n is the neutron lifetime;	h is the Planck constant;
m_p is the mass of the proton;	m_e is the mass of the electron;
c is the speed of light in vacuum;	k is the Coulomb constant;
e is the elementary charge;	G is the gravitational constant;
π is the pi;	m_n is the mass of the neutron;

Δm is the mass difference between neutron and proton; $\Delta m = m_n - m_p$.

The neutron lifetime obtained by Equation (1) is $878.5753\ s$. The latest laboratory measurement [2] is $877.75\ s$. Their ratio is 1.0009403. We find that this ratio is very close to the value of $\frac{g_e^2}{4}\frac{m_p}{m_n}$, which is 1.0009409. And then, we improve Equation (1) as follows:

$$t_n = \frac{\pi h}{\Delta mc^2}\frac{m_p}{m_e}\frac{m_n}{m_p}\frac{4}{g_e^2}\sqrt{\frac{ke^2}{Gm_pm_e}} \tag{2}$$

Here g_e is the spin g-factor of the electron.

Simplifying Equation (2), we get a new neutron lifetime formula:

$$t_n = \frac{\pi h}{\Delta mc^2}\frac{m_n}{m_e}\frac{4}{g_e^2}\sqrt{\frac{ke^2}{Gm_pm_e}} \tag{3}$$

The neutron lifetime calculated by Equation (3) is $877.7494\,s$, which is in good agreement with the latest laboratory measurement [2].

According to Equation (3), we can get the value of the gravitational constant, which is $6.6742911 \times 10^{-11}\, m^3/(kg \cdot s^2)$. the gravitational constant recommended by the 2018 CODATA is $6.67430 \times 10^{-11}\, m^3/(kg \cdot s^2)$. Compared to the two, it is very close.

Now, let's look at a formula for calculating the gravitational constant as follows [3]:

$$G = \frac{ke^2}{g_n}\frac{\lambda_p^7}{a_0^7}\frac{m_n}{m_p^3}\frac{1}{9\alpha\pi^9} \tag{4}$$

Here, λ_p is the Compton wavelength of the proton; a_0 is the Bohr radius; g_n is the spin g-factor of the neutron; α is the fine-structure constant.

The gravitational constant calculated by Equation (4) is $6.67434039 \times 10^{-11} m^3/(kg \cdot s^2)$. Since its value is different from that of Equation (3), we improve Equation (4) as follows:

$$G = \frac{ke^2}{g_n}\frac{\lambda_p^7}{a_0^7}\frac{m_n}{m_p^3}\frac{g_e}{g_\mu}\frac{1}{9\alpha\pi^9} \tag{5}$$

Here g_μ is the spin g-factor of the muon.

The gravitational constant G calculated by Equation (5) is $6.6742986 \times 10^{-11} m^3/(kg \cdot s^2)$.

We substitute the result of Equation (5) into Equation (3), and the neutron lifetime is $877.7495\, s$. It agrees well with the result of Equation (3).

We can also substitute Equation (5) into Equation (3) to obtain a neutron lifetime formula without the gravitational constant G, as follows:

$$t_n = \frac{3\pi^2}{4\alpha^3}\frac{h}{\Delta mc^2}\frac{m_p^5}{m_e^5}\frac{1}{g_e^2}\sqrt{\frac{g_n g_\mu}{g_e}\frac{m_n}{m_p}} \tag{6}$$

The neutron lifetime calculated by Equation (6) is $877.7495\ s$. In Equation (6), g_μ/g_e It can be replaced, and it mainly depends on the measurement accuracy of neutron lifetime.

In Equation (2), we divide Equation (1) by $\frac{g_e^2}{4}\frac{m_p}{m_n}$, and now multiply them, then:

$$t_n = \frac{\pi h}{\Delta mc^2}\frac{m_p}{m_e}\frac{m_p}{m_n}\frac{g_e^2}{4}\sqrt{\frac{ke^2}{Gm_pm_e}} \tag{7}$$

The neutron lifetime calculated by Equation (7) is $879.402\ s$. It coincides perfectly with the mean lifetime of neutrons [4].

In fact, we can find that the neutron lifetime calculated by Equation (1) is the average of the results of Equations (3) and (7).

We improve Equation (1) because the value range of gravitational constant G in Equation (1) is a bit large, which affects the calculation accuracy of neutron lifetime. By improving Equation (1), the value range of the gravitational constant G can be better constrained, so that the calculation accuracy of the neutron lifetime is also improved, so that the experimental data is in good agreement with the theory.

References:

[1] viXra:2208.0070

[2] arXiv:2106.10375v2

[3] viXra:2208.0031

[4] Zyla, P. A. (2020) “n MEAN LIFE”. PDG Live: 2020 Review of Particle Physics. Particle Data Group. Archived from the original on 17 January 2021. Retrieved 25 February 2021.