Flat Rotation Curves of Spiral Galaxies

Stefan Bernhard Rüster[1]*

Abstract

In this article, a simple but quite accurate model conforming to observations and solving the modified Poisson equation is utilized in order to compute the rotation curves as well as the radial acceleration relations of spiral galaxies being composed of several components: the central region of the bulge, the bulge, the disk, and dark matter. With this model, flat rotation curves of spiral galaxies are obtained, because the modified Poisson equation approximately meets the requirement of conservation of total energy in Newton's theory of gravity, wherefore the model contains the exact circular speeds of test particles which are caused by the dark matter component of the respective considered spiral galaxies. The computed rotation curves are fitted to the observed speeds in spiral galaxies taken from SPARC. It turns out, that the simple model in this article is able to explain the observed flat rotation curves as well as the increasing rotation curves at large distances from the center of spiral galaxies with a very good accuracy.

Keywords

Modified Poisson equation, Newton's theory of gravity, dark matter, spiral galaxy, rotation curve, observational data taken from SPARC, least square method, radial acceleration relation, MOND theory.

[1] *Am Wiebelsberg 12, 63579 Freigericht, Germany.*

***Corresponding author**: dr.ruester@t-online.de

Contents

1. Introduction

It has been proven in Ref. [1], that in the limit of weak gravitational fields, the modified Poisson equation

$$\Delta\Phi(\mathbf{r}) = 4\pi G\rho(\mathbf{r}) - \Lambda c^2 \tag{1}$$

approximately meets the requirement of conservation of total energy in Newton's theory of gravity. Therein, ρ denotes the baryonic mass distribution, whereas the cosmological constant

$$\Lambda = \frac{8\pi G\rho_\Lambda}{c^2} \tag{2}$$

is proportional to the density of dark matter, $\rho_{\mathrm{dm}} = \rho_\Lambda < 0$, or to the density of dark energy, $\rho_{\mathrm{de}} = \rho_\Lambda > 0$, respectively.

It has also been shown, that the cosmological constant Λ is no universal constant but a constant of integration and therefore a parameter, which is proportional to the total energy density with respect to the metric of the considered gravitational system [1, 2]. With this finding, dark matter is nothing else than a negative scalar curvature of space-time ($\Lambda < 0$), while dark energy is nothing else than a positive scalar curvature of space-time ($\Lambda > 0$). Initially, the cosmological constant is unknown and has to be determined by observations.

In Sec. 2, a simple but quite accurate model is shown in order to compute the rotation curves as well as the radial acceleration relations (RARs) of spiral galaxies. In Sec. 3, the results are discussed, which are obtained by fitting the computed rotation curves by using the model in Sec. 2 to the observed speeds taken from SPARC [3]. In Sec. 4, the conclusions are drawn, and an outlook to a lot of future work is given. Due to space restrictions, the figures and tables are demonstrated in the appendix.

2. Model of spiral galaxies

In this section, a simple but quite accurate model conforming to observations and solving the modified Poisson equation (1) is shown in order to compute the rotation curves as well as the RARs of spiral galaxies being composed of several components [4]:

- the central region of the bulge (bh),
- the bulge (b),
- the disk (d),
- and dark matter (dm).

2.1 Baryonic matter components

The baryonic mass M_{bh} in the central region of the bulge is considered to be point-like and probably containing a supermassive black hole (bh).

The baryonic matter of the bulge is considered to be exponentially and isotropically distributed,

$$\rho_{\mathrm{b}}(r) = \rho_0 \exp\left(-\frac{r}{R_{\mathrm{b}}}\right),$$

where ρ_0 is the central mass density of the bulge, and R_{b} is the bulge scale length. The mass of the exponential bulge inside the radius r is given by

$$\begin{aligned} M_\rho(r) &= 4\pi \int_0^r dr'\, r'^2 \rho_{\mathrm{b}}(r') \\ &= M_{\mathrm{b}}\left[1-\exp\left(-\frac{r}{R_{\mathrm{b}}}\right)\left(1+\frac{r}{R_{\mathrm{b}}}+\frac{r^2}{2R_{\mathrm{b}}^2}\right)\right], \end{aligned}$$

where

$$M_{\mathrm{b}} = 8\pi\rho_0 R_{\mathrm{b}}^3$$

is its total baryonic mass.

The disk of a spiral galaxy can considered to be infinitesimally thin with an exponentially mass distribution of baryonic matter [5, 6]. Hence, its surface mass distribution is given by

$$\Sigma(r) = \Sigma_0 \exp\left(-\frac{r}{R_{\mathrm{d}}}\right),$$

where Σ_0 is the central surface mass density of the disk, and R_{d} is the disk scale length. The mass of the disk inside the radius r amounts to [6]

$$M_\Sigma(r) = 2\pi \int_0^r dr'\, r'\Sigma(r') = M_{\mathrm{d}}\left[1-\exp\left(-\frac{r}{R_{\mathrm{d}}}\right)\left(1+\frac{r}{R_{\mathrm{d}}}\right)\right],$$

where

$$M_{\mathrm{d}} = 2\pi\Sigma_0 R_{\mathrm{d}}^2 \tag{3}$$

is the total baryonic mass of the disk.

The gravitational potentials of the respective baryonic matter components of a spiral galaxy are given by [4, 5, 6]

$$\Phi_{\mathrm{bh}}(r) = -\frac{GM_{\mathrm{bh}}}{r}, \tag{4a}$$

$$\begin{aligned} \Phi_{\mathrm{b}}(r) &= -\frac{GM_\rho}{r} - 4\pi G\int_r^\infty dr'\, r'\rho_{\mathrm{b}}(r') \\ &= -\frac{GM_\rho}{r} - 4\pi G\rho_{\mathrm{b}}R_{\mathrm{b}}^2\left(1+\frac{r}{R_{\mathrm{b}}}\right), \end{aligned} \tag{4b}$$

$$\Phi_{\mathrm{d}}(r) = -\pi G\Sigma_0 r[I_0(y)K_1(y) - I_1(y)K_0(y)], \tag{4c}$$

where

$$y = \frac{r}{2R_{\mathrm{d}}}.$$

Thereby, the squared circular speeds of a test particle caused by the respective components of a spiral galaxy can be determined by using the relation

$$v_i^2(r) = r\frac{\partial\Phi_i}{\partial r}$$

and hence read [4, 5, 6]

$$v_{\mathrm{bh}}^2(r) = \frac{GM_{\mathrm{bh}}}{r}, \qquad v_{\mathrm{b}}^2(r) = \frac{GM_\rho}{r},$$
$$v_{\mathrm{d}}^2(r) = 4\pi G\Sigma_0 R_{\mathrm{d}} y^2[I_0(y)K_0(y) - I_1(y)K_1(y)].$$

It is important to know, that Eq. (4c) is only valid in and not outside the disk.

2.2 Dark matter component

From the contribution of dark matter in the modified Poisson equation (1),

$$\Delta\Phi_{\mathrm{dm}} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi_{\mathrm{dm}}}{\partial r}\right) = -\Lambda c^2 = -8\pi G\rho_{\mathrm{dm}},$$

where because of Eq. (2)

$$\rho_{\mathrm{dm}} = \frac{\Lambda c^2}{8\pi G}$$

is the homogeneous mass density of dark matter, the squared circular speed of a test particle is obtained,

$$v_{\mathrm{dm}}^2(r) = r\frac{\partial\Phi_{\mathrm{dm}}}{\partial r} = -\frac{\Lambda c^2 r^2}{3} = -\frac{8\pi G r^2\rho_{\mathrm{dm}}}{3} = -\frac{2GM_{\mathrm{dm}}}{r},$$

where

$$M_{\rm dm}(r) = \frac{\Lambda c^2 r^3}{6G} = \frac{4\pi r^3 \rho_{\rm dm}}{3}$$

is the amount of dark matter inside the radius r. The centripetal acceleration on a test particle caused by dark matter is given by

$$g_{\rm dm}(r) = \frac{v_{\rm dm}^2}{r} = -\frac{\Lambda c^2 r}{3} = -\frac{8\pi G r \rho_{\rm dm}}{3} = -\frac{2GM_{\rm dm}}{r^2}. \quad (5)$$

The gravitational potential generated by dark matter reads

$$\Phi_{\rm dm}(r) = -\frac{\Lambda c^2 r^2}{6} = -\frac{4\pi G r^2 \rho_{\rm dm}}{3} = -\frac{GM_{\rm dm}}{r}.$$

2.3 Rotation curve and radial acceleration relation

The total baryonic mass of the spiral galaxy amounts to

$$M_{\rm bar} = M_{\rm bh} + M_{\rm b} + M_{\rm d}.$$

The gravitational potential of the spiral galaxy in the disk generated by baryonic matter reads

$$\Phi_{\rm bar}(r) = \Phi_{\rm bh}(r) + \Phi_{\rm b}(r) + \Phi_{\rm d}(r),$$

whereby the squared circular speed of a test particle in the disk caused by baryonic matter is given by

$$v_{\rm bar}^2(r) = r\frac{\partial \Phi_{\rm bar}}{\partial r} = v_{\rm bh}^2(r) + v_{\rm b}^2(r) + v_{\rm d}^2(r).$$

The centripetal acceleration on a test particle in the disk caused by baryonic matter is obtained by

$$g_{\rm bar}(r) = \frac{v_{\rm bar}^2}{r} = \frac{\partial \Phi_{\rm bar}}{\partial r}.$$

The gravitational potential of the spiral galaxy in the disk is determined by

$$\Phi(r) = \Phi_{\rm bar}(r) + \Phi_{\rm dm}(r).$$

The squared circular speed of a test particle in the disk caused by the spiral galaxy is obtained by

$$v^2(r) = r\frac{\partial \Phi}{\partial r} = v_{\rm bar}^2(r) + v_{\rm dm}^2(r),$$

the square root of which is the computed rotation curve, that can be fitted to the observed speeds $v_{\rm obs}$ by using the method of least squares [7],

$$\begin{gathered}\chi^2 = \sum_{i=1}^{n} [v_{\rm obs}(r_i) - v(r_i)]^2, \\ \frac{\partial \chi^2}{\partial M_{\rm bh}} = \frac{\partial \chi^2}{\partial M_{\rm b}} = \frac{\partial \chi^2}{\partial R_{\rm b}} = \frac{\partial \chi^2}{\partial M_{\rm d}} = \frac{\partial \chi^2}{\partial R_{\rm d}} = \frac{\partial \chi^2}{\partial \Lambda} = 0,\end{gathered} \quad (6)$$

where n is the number of observed speeds in the respective spiral galaxies. The standard deviation is determined by

$$\sigma_v = \sqrt{\frac{\chi^2}{n-6}}.$$

The centripetal acceleration on a test particle in the disk is given by

$$g(r) = g_{\rm bar}(r) + g_{\rm dm}(r) = \frac{v^2}{r} = \frac{\partial \Phi}{\partial r}, \quad (7)$$

from which the RAR

$$g = g(g_{\rm bar}) \quad (8)$$

is obtained. In order to plot the RAR it is appropriate to take the decadic logarithm of $g_{\rm bar}$ and g to avoid small numbers with decimal powers and to get a better resolution of the graph.

2.3.1 Asymptotic behavior of the rotation curve

Far away from the center of the exponential bulge the gravitational potential approximately is determined by

$$\tilde{\Phi}_{\rm b}(r) = -\frac{GM_{\rm b}}{r},$$

whereby the approximated squared circular speed of a test particle amounts to

$$\tilde{v}_{\rm b}^2(r) = r\frac{\partial \tilde{\Phi}_{\rm b}}{\partial r} = \frac{GM_{\rm b}}{r}.$$

At large distances from the center the gravitational potential generated by the disk approximately amounts to

$$\tilde{\Phi}_{\rm d_1}(r) = -\frac{GM_{\rm d}}{r} - \frac{3GM_{\rm d}R_{\rm d}^2}{2r^3}. \quad (9)$$

In order to obtain this result, formulas 9.7.1 and 9.7.2 of Ref. [8] and Eq. (3) have been utilized in Eq. (4c). Thereby, the approximated squared circular speed of a test particle in the disk far away from its center is determined by

$$\tilde{v}_{\rm d_1}^2(r) = r\frac{\partial \tilde{\Phi}_{\rm d_1}}{\partial r} = \frac{GM_{\rm d}}{r} + \frac{9GM_{\rm d}R_{\rm d}^2}{2r^3}, \quad (10)$$

so that the approximated squared circular speed of a test particle in the disk caused by the whole spiral galaxy reads

$$\tilde{v}_1^2(r) = v_{\rm bh}^2(r) + \tilde{v}_{\rm b}^2(r) + \tilde{v}_{\rm d_1}^2(r) + v_{\rm dm}^2(r).$$

At huge distances from the center only the monopole contributions in Eqs. (9) and (10) play a significant role,

$$\tilde{\Phi}_{\rm d_2}(r) = -\frac{GM_{\rm d}}{r}, \qquad \tilde{v}_{\rm d_2}^2(r) = \frac{GM_{\rm d}}{r},$$

so that the approximated squared circular speed of a test particle caused by the whole spiral galaxy becomes

$$\tilde{v}_2^2(r) = v_{\rm bh}^2(r) + \tilde{v}_{\rm b}^2(r) + \tilde{v}_{\rm d_2}^2(r) + v_{\rm dm}^2(r).$$

2.3.2 Asymptotic behavior of the radial acceleration relation

As a crude method, the total baryonic mass of the spiral galaxy can considered to be point-like far away from its center, so that the centripetal acceleration caused by the baryonic matter of the spiral galaxy on a test particle approximately reads

$$\tilde{g}_{\rm bar}(r) = \frac{GM_{\rm bar}}{r^2} \,. \qquad (11)$$

In order to obtain the RAR, one needs to know the centripetal acceleration as a function of the centripetal acceleration caused by the baryonic matter, see Eq. (8). The centripetal acceleration caused by dark matter can be expressed by the approximated centripetal acceleration caused by baryonic matter, which is achieved by solving Eq. (11) for the distance,

$$r = \sqrt{\frac{GM_{\rm bar}}{\tilde{g}_{\rm bar}}} \,,$$

by inserting this result in Eq. (5), and finally by renaming $g_{\rm dm}$ to $\tilde{g}_{\rm dm}$ in order to show that it is being about an approximation,

$$\tilde{g}_{\rm dm} = \frac{K}{\sqrt{\tilde{g}_{\rm bar}}} \,, \qquad K = -\frac{\sqrt{GM_{\rm bar}}\Lambda c^2}{3} \,.$$

The positive parameter K is initially unknown and depends on the point-like total baryonic mass as well as on the initially unknown negative scalar curvature of the spiral galaxy's metric in matter-free space-time, $R = 4\Lambda$. By using Eq. (7) the approximated RAR (aRAR) is obtained by

$$\tilde{g} = \tilde{g}_{\rm bar} + \frac{K}{\sqrt{\tilde{g}_{\rm bar}}} \,.$$

The expression for the aRAR can be divided by $\tilde{g}_{\rm bar}$ and be rearranged in order to get the aRAR in its dimensionless form, the adRAR,

$$\frac{\tilde{g}}{\tilde{g}_{\rm bar}} - 1 = K\tilde{g}_{\rm bar}^{-\frac{3}{2}} \,, \qquad (12)$$

whereby the parameter K can be determined by performing a linear regression through the origin with the method of least squares. In order to check the accuracy of the aRAR or the adRAR, respectively, the dimensionless RAR (dRAR) is introduced, where $g_{\rm bar}^{-3/2}$ is plotted on the abscissa and the dimensionless quantity $g/g_{\rm bar} - 1$ is plotted on the ordinate.

3. Results and discussion

By utilizing the model shown in Sec. 2 the computed rotation curves of different spiral galaxies are fitted to observational data taken from SPARC [3]. Instead of using the mass models of Ref. [3] the author utilizes the model given in Sec. 2 because in the latter the exact formulas of the dark matter component are demonstrated in contrast to the assumptions made in the mass models of Ref. [3].

For the computations of the model in Sec. 2 the author neither has needed special software nor any computer source codes, but he just has made use of the spreadsheet program `LibreOffice Calc` and the tool `Solver` therein, the latter of which successfully has fitted the rotation curves to the observed speeds by using Eqs. (6) and by specifying reasonable search intervals for the six model parameters $M_{\rm bh}$, $M_{\rm b}$, $R_{\rm b}$, $M_{\rm d}$, $R_{\rm d}$, and Λ.

The fitted flat rotation curves of several spiral galaxies are shown in Figs. 1–6. They are drawn as very thick gray solid lines and mostly lie in between the error bars of the observed speeds. Even increasing observed speeds at large distances from the galactic center can wonderfully be explained with the simple model in Sec. 2 as can be seen for example in case of the fitted flat rotation curve of UGC 3205. The very thick gray densely dotted lines show the rotation curves in case there were no dark matter components. The horizontal yellow thin solid lines demonstrate the circular speeds obtained by Milgrom's MOND theory [9]

$$v_{\rm M}^4 = GM_{\rm bar}a_0 \,, \qquad a_0 = 1.2 \cdot 10^{-10}\,{\rm m/s}^2 \,,$$

which are often in good agreement with the fitted flat rotations curves obtained by the model in Sec. 2. The deviations are explained by the fact, that in contrast to MOND, the model in Sec. 2 comprises the exact formulas for the dark matter component with Λ as a necessarily existing parameter, which is missing in MOND. Therefore, not all circular speeds obtained by MOND are in agreement with the model in Sec. 2.

The thick magenta densely dashed as well as the thick cyan densely dash-dotted lines represent the respective approximated circular speeds, which are not always reliable in comparison to the fitted flat rotation curves. The remaining lines show the circular speeds of the respective components of the spiral galaxies.

It is interesting to recognize, that the circular speed of a test particle caused by baryonic matter in NGC 3198 is almost completely described by that one of its disk component. For some spiral galaxies the contribution of the circular speed of the center of the bulge is negligibly small.

The values of the fitted parameters of the spiral galaxies under consideration, their total baryonic mass, and the standard deviation of the respective fits are shown in Tab. 1.

The RARs of the spiral galaxies under consideration are plotted in Fig. 7. At small values of $g_{\rm bar}$, which means at large distances from the galactic center, they come close to the fit formula from McGaugh [10]

$$g = \frac{g_{\rm bar}}{1 - \exp\left(-\sqrt{\frac{g_{\rm bar}}{a_0}}\right)} \,,$$

the latter of which is demonstrated by the very thick gray solid curve. The deviations are explained by the fact, that in contrast to McGaugh's fit formula, the model in Sec. 2 comprises the exact formulas for the dark matter component with Λ as a necessarily existing parameter, which is missing in McGaugh's fit formula.

The dRARs of the spiral galaxies under consideration are plotted in Fig. 8. At first sight, they seem to be straight lines through the origin, see upper plot in Fig. 8. However, by creating a double-logarithmic plot of the dRARs in order to get a better resolution – see lower plot in Fig. 8 – one clearly recognizes, that the dRARs in fact are no straight lines, whereof one immediately concludes, that the formulas of the aRAR and the adRAR are far from being precise. This is why linear regressions through the origin are not reliable in order to obtain the parameters K, cf. Eq. (12). Even constructing tangents at large values of $g_{\mathrm{bar}}^{-3/2}$ in the double-logarithmic plot of the dRARs in order to determine the parameters K is not recommended because the dRARs need not necessarily become linear in this region.

4. Conclusions and outlook

The flat rotation curves of spiral galaxies can be explained by using the model in Sec. (2) solving the modified Poisson equation (1), which approximately meets the requirement of conservation of total energy in Newton's theory of gravity. All the *approximated* formulas of the model in Sec. 2, such as the approximated circular speeds $\tilde{v}_1$ and $\tilde{v}_2$, the aRAR and the adRAR, are not reliable. This is why one needs to compute the rotation curves and the RARs with the *exact* formulas of the respective components of the spiral galaxies, which are represented in Sec. 2.

Not all circular speeds obtained by MOND are in agreement with the model in Sec. 2. The deviations are explained by the fact, that in contrast to MOND, the model in Sec. 2 comprises the exact formulas for the dark matter component with Λ as a necessarily existing parameter, which is missing in MOND. The same argument applies regarding the deviations of McGaugh's fit formula from the RARs, the latter of which are obtained by using the model in Sec. 2.

Each spiral galaxy has different parameters. Consequently, there is a lot of work to do in future, because there are many spiral galaxies the computed rotation curves have to be fitted to. The model in Sec. 2 is just a simple, but nevertheless an almost exact one, because it corresponds to the observed circular speeds. It is of great interest to find out the precise distribution of baryonic as well as of dark matter in spiral galaxies. The exact formulas of the latter are given in Sec. 2. Computer simulations of spiral galaxies can be performed by using the equations of motion shown in Sec. 2. One can utilize more precise models improving the fits and the computer simulations.

Acknowledgments

The author thanks Pavel Kroupa for bringing the author's attention to the RAR.

References

[1] S. B. Rüster, Parana J. Sci. Educ., v.**8**, n.6, (13-22), August 8, 2022, `http://tiny.cc/PJSE24476153v8i6p013-022`.

[2] S. B. Rüster, V. B. Morozov, Parana J. Sci. Educ., v.**7**, n.10, (181-184), December 1, 2021, `http://tiny.cc/PJSE24476153v7i10p181-184`.

[3] F. Lelli, S. S. McGaugh, J. M. Schombert, AJ, v.**152**, n.6, 157, 2016, `https://doi.org/10.3847/0004-6256/152/6/157`,
SPARC website: `http://astroweb.cwru.edu/SPARC`.

[4] Y. Sofue, Publ. Astron. Soc. Japan, Volume **69**, Issue 1, February 2017, R1, `https://doi.org/10.1093/pasj/psw103`.

[5] K. C. Freeman, ApJ, v.**160**, (811-830), June 1970, `https://doi.org/10.1086/150474`.

[6] J. Binney, S. Tremaine, *Galactic Dynamics*, Princeton University Press, 1987, Third printing with corrections, 1994.

[7] P. R. Bevington, D. K. Robinson, *Data Reduction and Error Analysis for the Physical Sciences*, McGraw-Hill, third edition, 2003.

[8] M. Abramowitz, I. A. Stegun, *Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables*, National Bureau of Standards, Applied Mathematics Series · 55, Issued June 1964, Tenth Printing, December 1972, with corrections, U.S. Government Printing Office Washington.

[9] M. Milgrom, ApJ, v.**270**, (371-383), July 15, 1983, `https://doi.org/10.1086/161131`.

[10] F. Lelli, S. S. McGaugh, J. M. Schombert, M. S. Pawlowski, ApJ, v.**836**, n.2, 152, 2017, `https://doi.org/10.3847/1538-4357/836/2/152`.

Appendix

Figures and tables

Due to space restrictions, the figures and the table are shown on the subsequent pages.

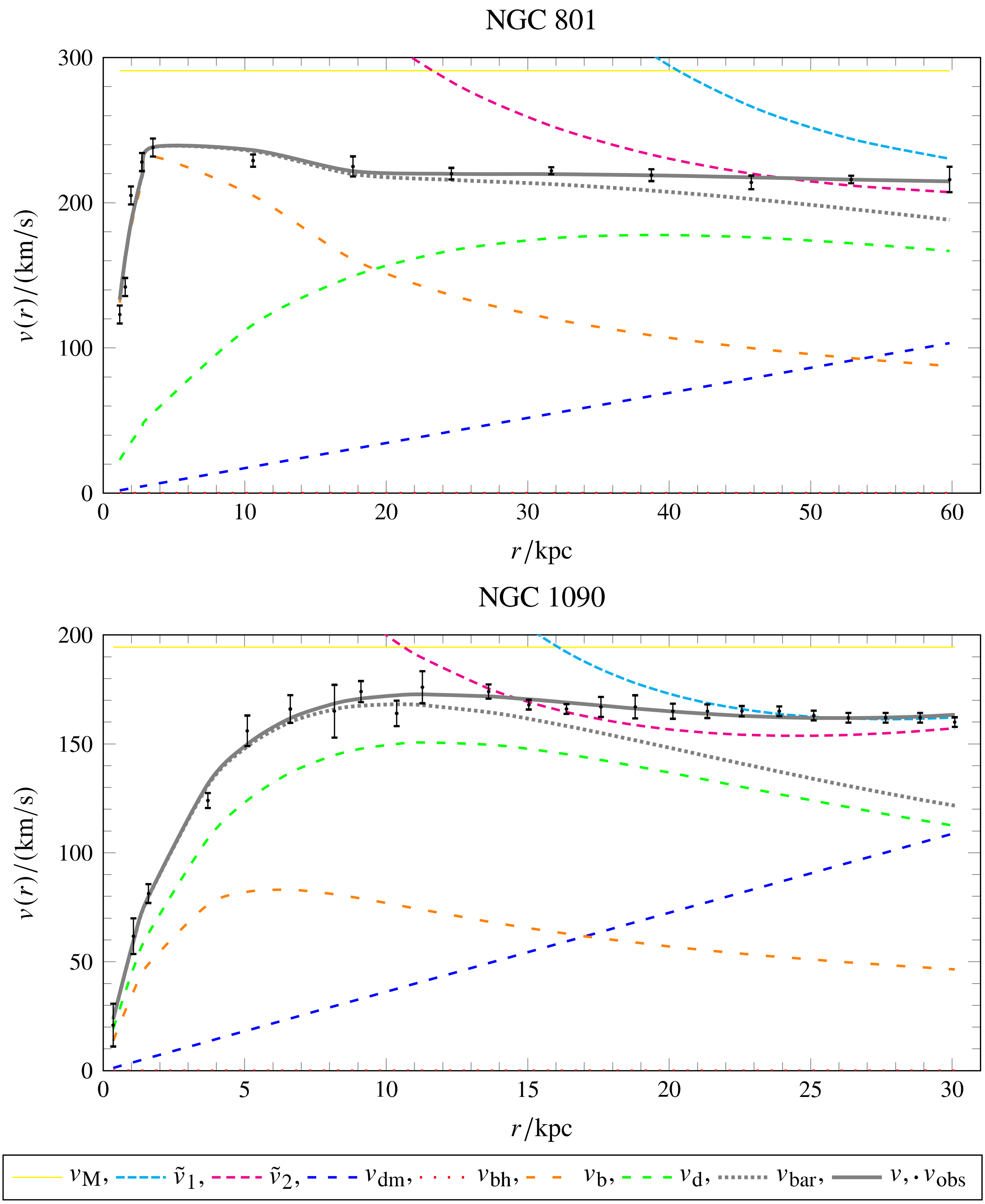

Figure 1. Rotation curves of NGC 801 and NGC 1090.

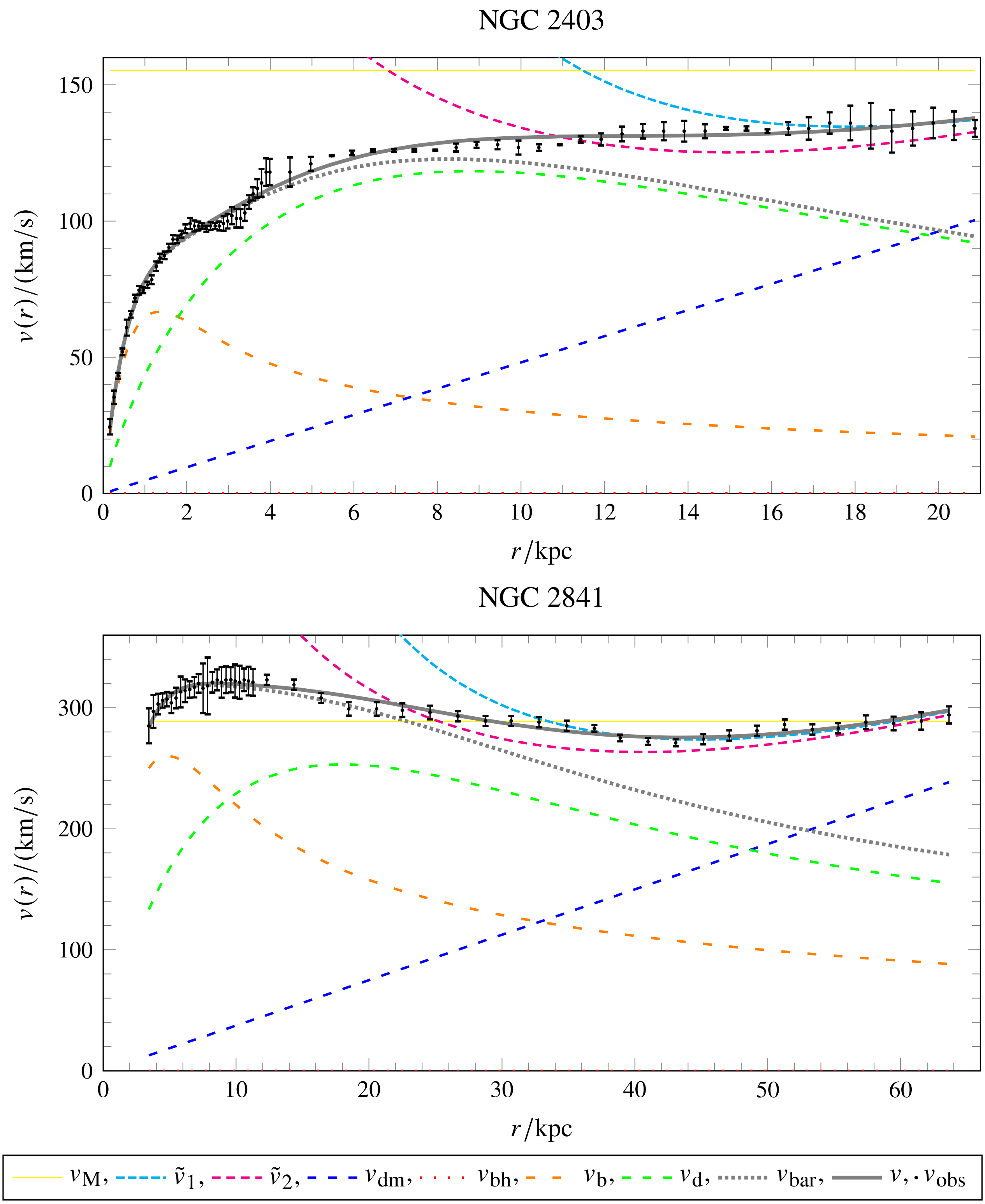

Figure 2. Rotation curves of NGC 2403 and NGC 2841.

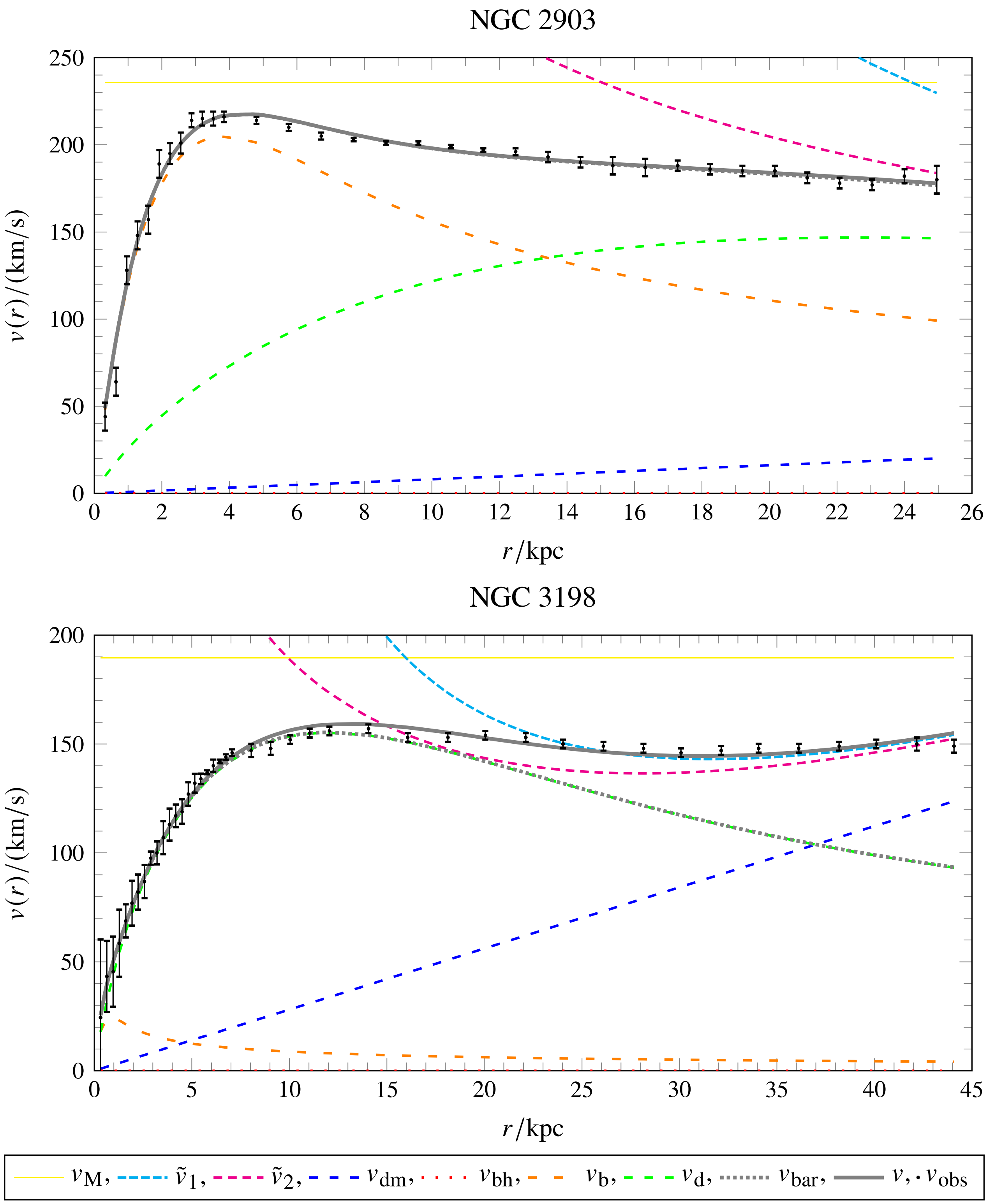

Figure 3. Rotation curves of NGC 2903 and NGC 3198.

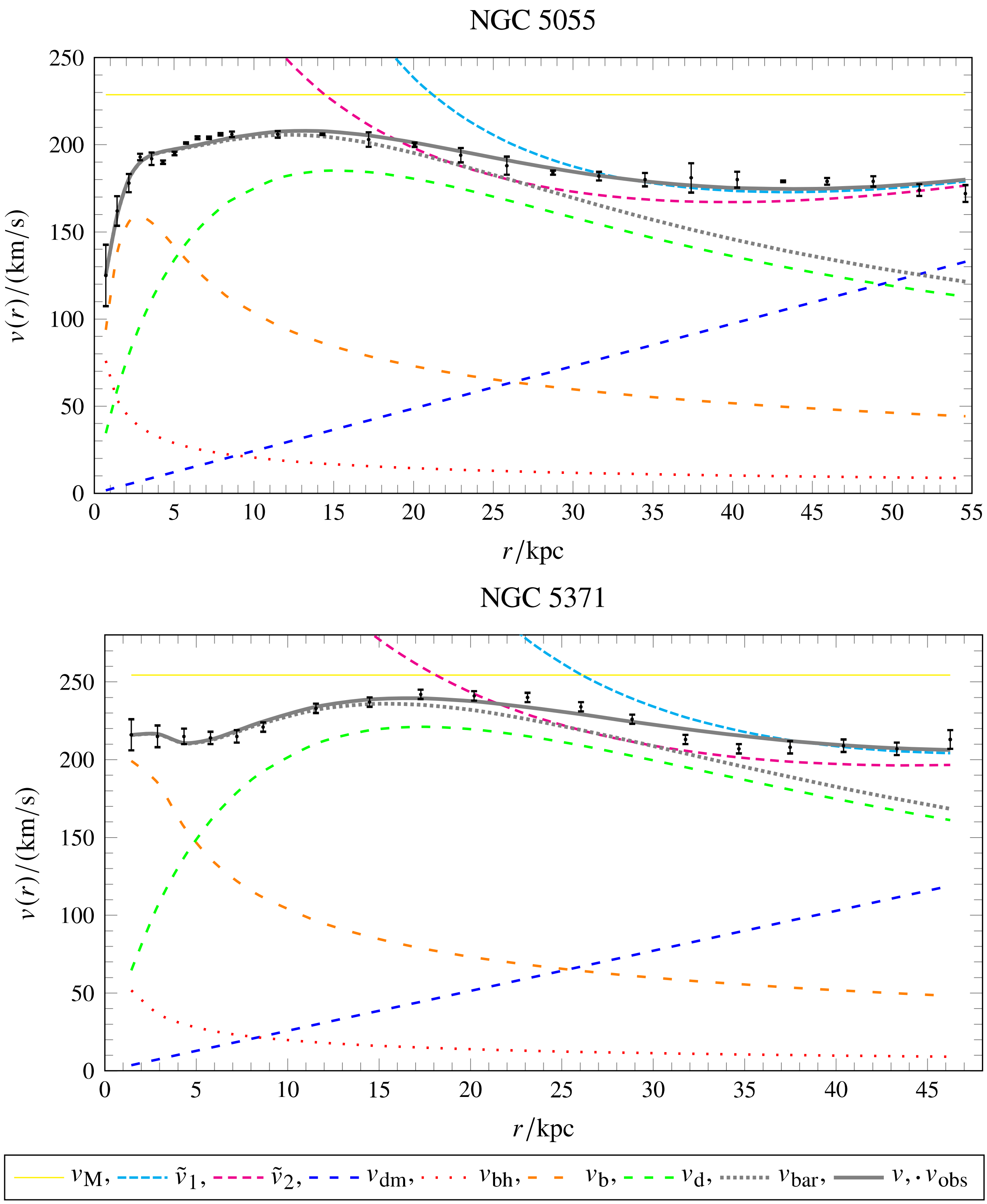

Figure 4. Rotation curves of NGC 5055 and NGC 5371.

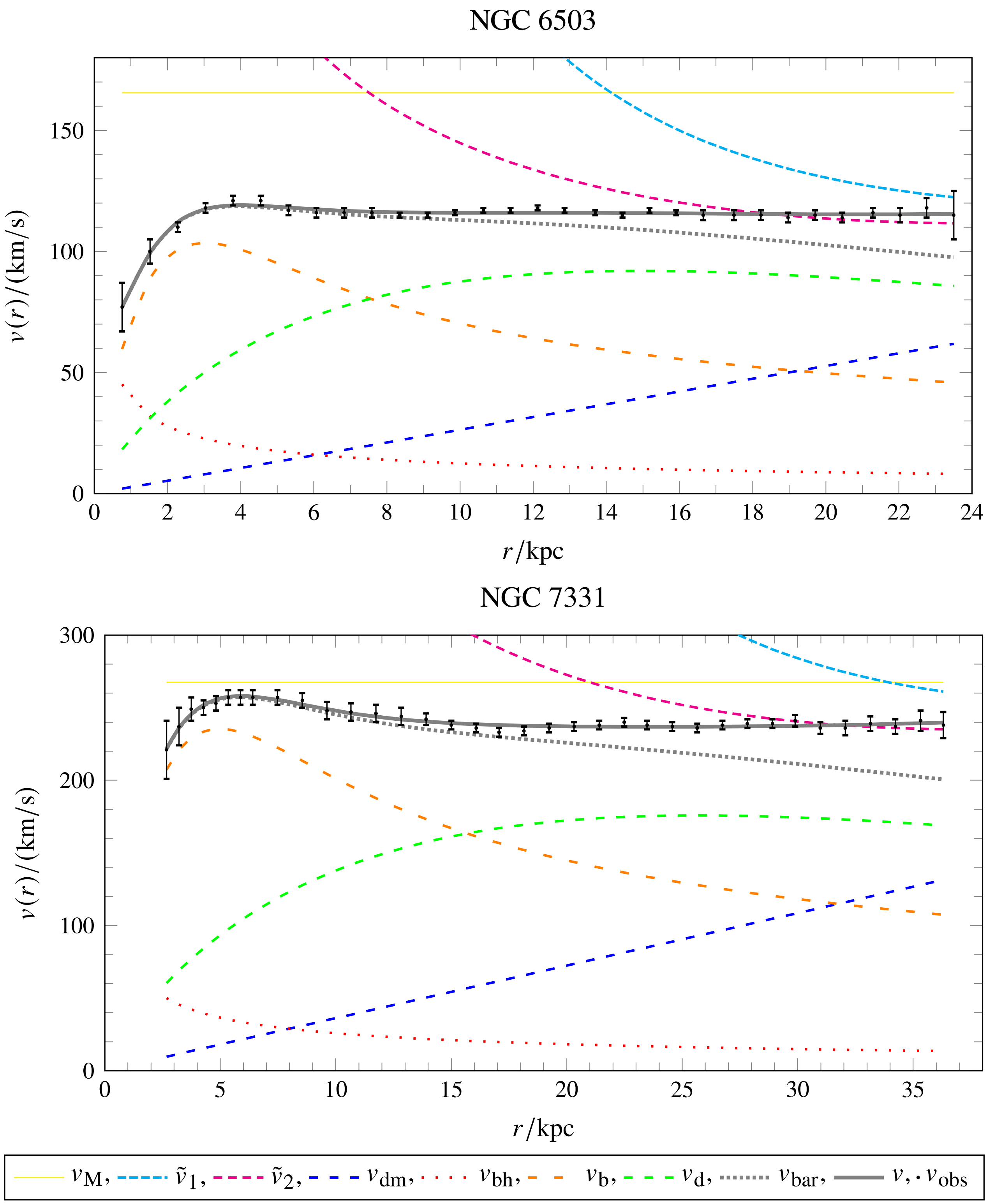

Figure 5. Rotation curves of NGC 6503 and NGC 7331.

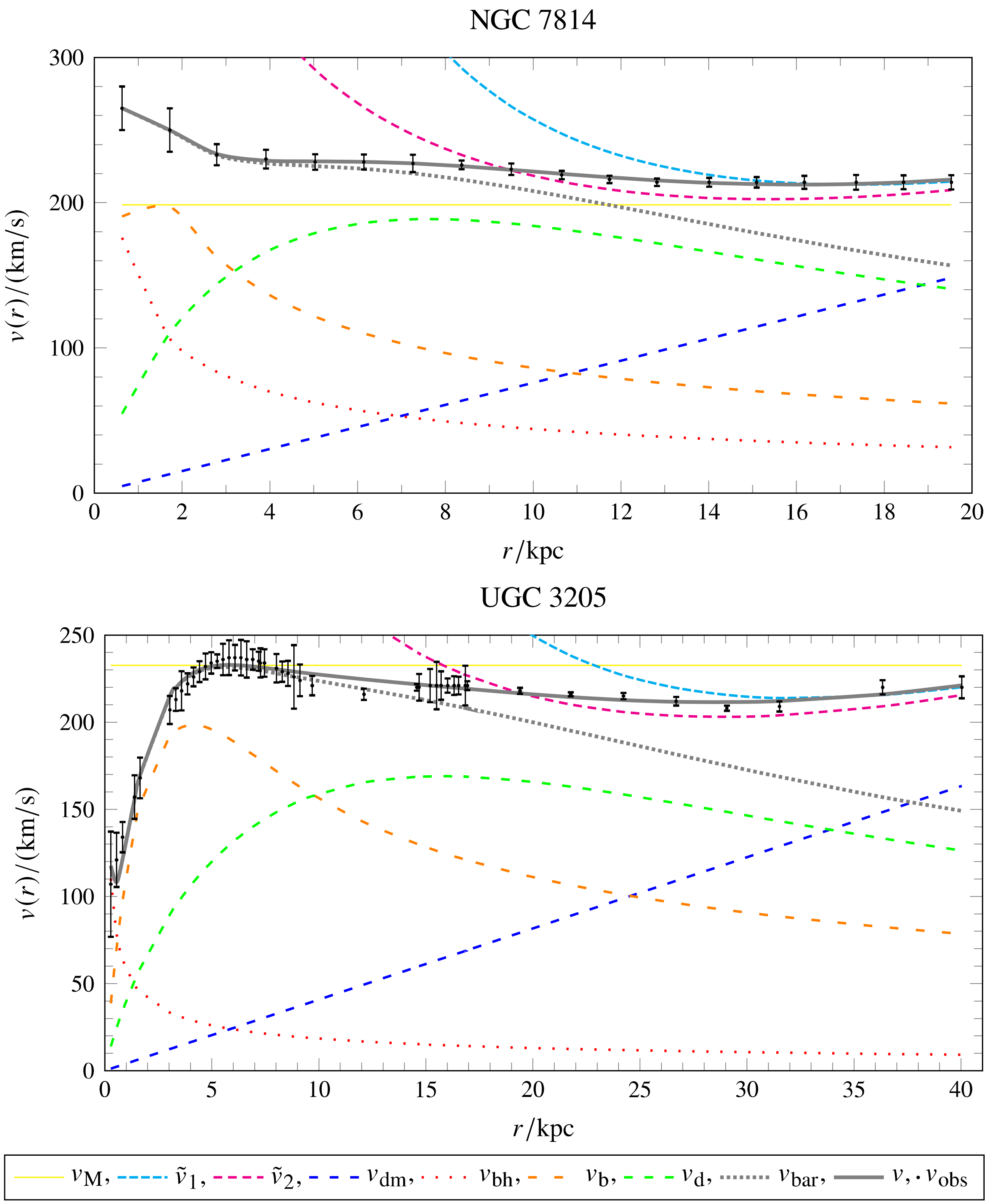

Figure 6. Rotation curves of NGC 7814 and UGC 3205.

spiral galaxy	M_{bh} $[10^9\,\mathrm{M}_\odot]$	M_{b} $[10^9\,\mathrm{M}_\odot]$	R_{b} [kpc]	M_{d} $[10^9\,\mathrm{M}_\odot]$	R_{d} [kpc]	Λ $[\mathrm{m}^{-2}]$	M_{bar} $[10^9\,\mathrm{M}_\odot]$	σ_v [km/s]
NGC 801	0.0000	106.4329	1.5084	342.5265	18.0344	$-1.0455\cdot10^{-49}$	448.9594	12.23
NGC 1090	0.0000	15.1426	1.8292	74.5877	5.4653	$-4.6000\cdot10^{-49}$	89.7303	3.92
NGC 2403	0.0000	2.1175	0.3980	34.4186	4.0927	$-8.1101\cdot10^{-49}$	36.5361	2.67
NGC 2841	0.0000	115.5831	1.4276	321.0818	8.3421	$-4.9228\cdot10^{-49}$	436.6649	4.00
NGC 2903	0.0000	57.0214	1.1385	136.6491	10.5603	$-2.2674\cdot10^{-50}$	193.6705	6.09
NGC 3198	0.0000	0.1797	0.2536	80.9231	5.6024	$-2.7621\cdot10^{-49}$	81.1028	2.85
NGC 5055	0.9688	24.7984	0.8200	145.8105	7.0786	$-2.0755\cdot10^{-49}$	171.5777	3.77
NGC 5371	0.9055	25.0186	0.5145	237.1728	8.0744	$-2.3223\cdot10^{-49}$	263.0969	4.93
NGC 6503	0.3600	11.4847	0.8955	35.3070	6.9474	$-2.4306\cdot10^{-49}$	47.1517	1.30
NGC 7331	1.5500	97.4700	1.4705	222.1078	11.9685	$-4.5920\cdot10^{-49}$	321.1277	2.22
NGC 7814	4.5317	17.2839	0.3246	75.7871	3.5439	$-2.0236\cdot10^{-48}$	97.6027	1.17
UGC 3205	0.7889	57.4323	1.2195	125.3369	7.3088	$-5.8438\cdot10^{-49}$	183.5580	4.75

Table 1. Fitted parameters of the spiral galaxies under consideration, their total baryonic mass, and the standard deviation of the respective fits.

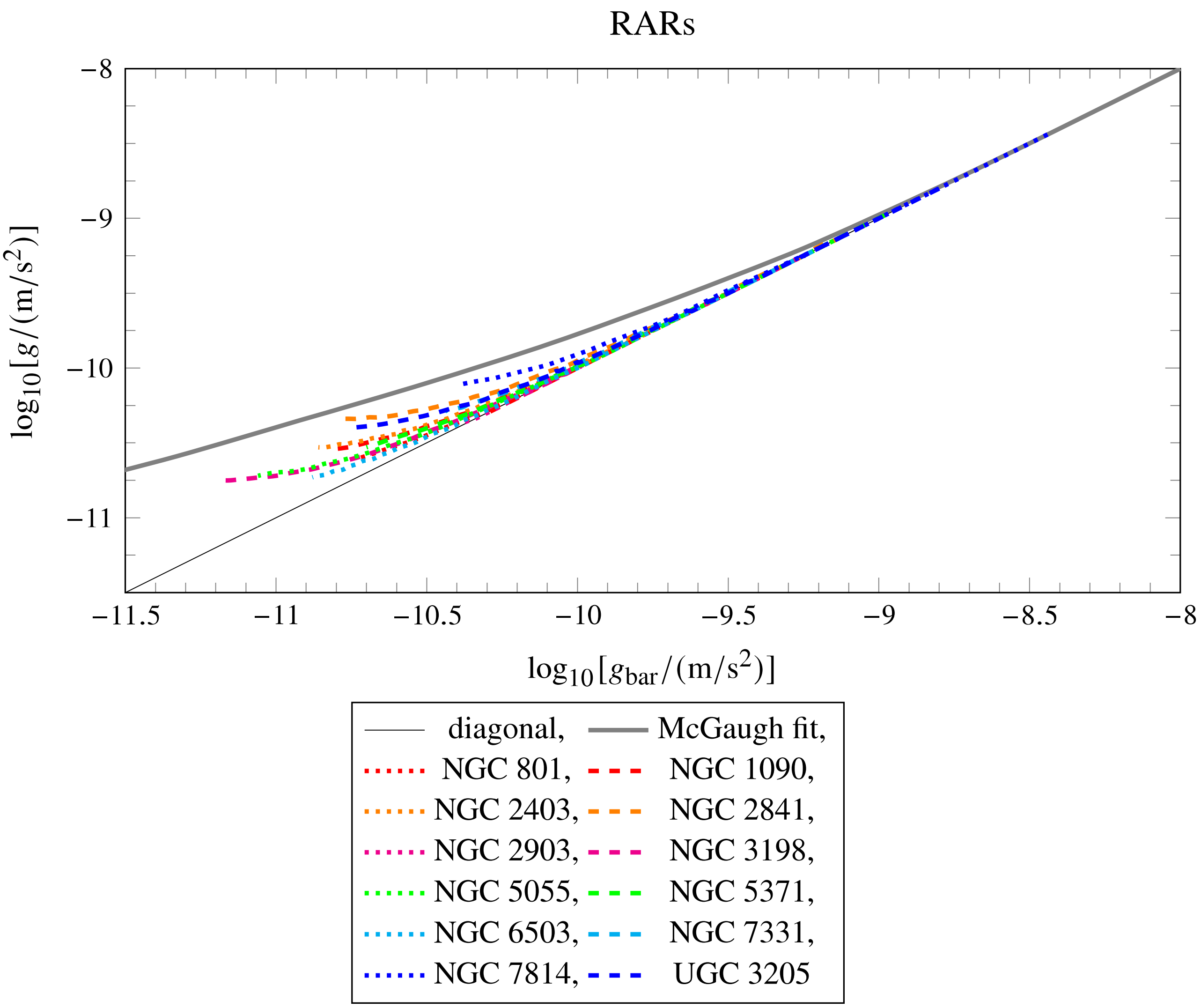

Figure 7. Radial acceleration relations (RARs) of the spiral galaxies under consideration.

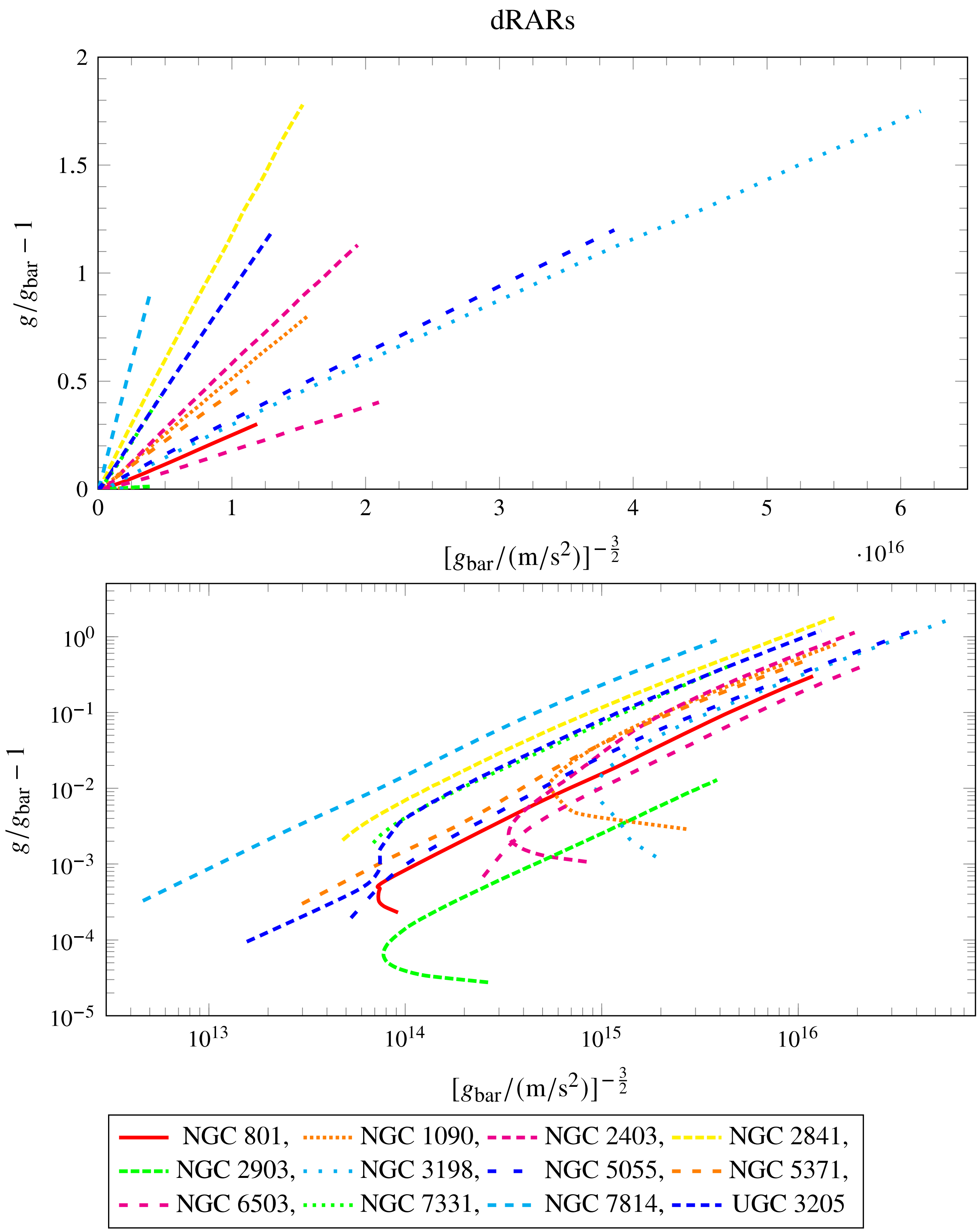

Figure 8. Dimensionless radial acceleration relations (dRARs) of the spiral galaxies under consideration.