# Collatz tree as subtree of a perfect binary tree

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**Abstract**

We transform a perfect binary tree with node 1 as its root node to have the Collatz tree as its subtree, and show that there is at least one node in a perfect binary tree that does not belong to the Collatz tree. Since all positive integer nodes are in the infinite perfect binary tree, then the Collatz conjecture is most likely not true.

## 1. Introduction

Define the iterating function introduced by R. Terras[1] :

$$a_{n+1} = (3^b a_n + b)/2 \qquad (1)$$

where $b = 1$ when $a_n$ is odd, and $b = 0$ when $a_n$ is even. The Collatz conjecture asserts that by starting with any positive integer $a_0$, there exists a natural number k such that $a_k = 1$.

## 2. The Collatz tree

The Collatz tree is defined as follows:

Start with number 1 as a root node , with each node as a parent node it can have one or two children nodes. If node $i \equiv 2 \bmod 3$, then its two children nodes are $2i$ and $(2i-1)/3$. If node $i \equiv 0 \bmod 3$ or $i \equiv 1 \bmod 3$ , then its children node is $2i$. A 5- levels Collatz tree is shown in Figure 1.

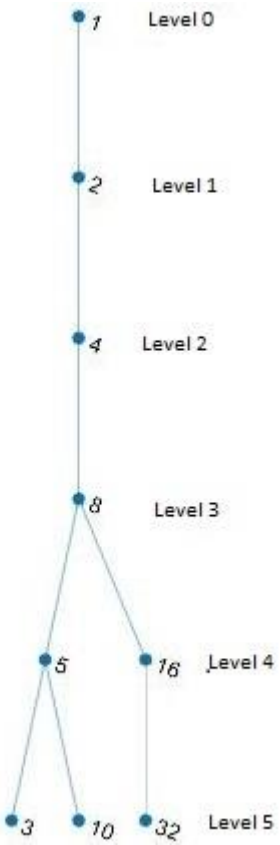


Figure 1. 5- levels Collatz tree

## 3. **A Perfect binary tree**

A perfect binary tree is a binary tree in which all interior nodes have two children and all leaves have the same depth. Let number 1 as a root node , two children nodes of a node i are 2i and 2i+1. An example of a 5- levels perfect binary tree is shown in Figure 2.

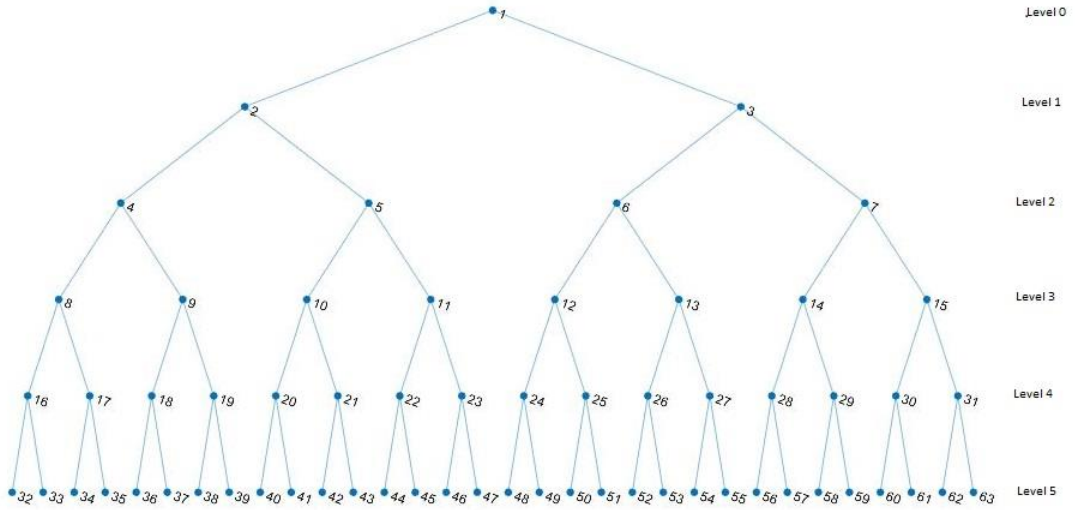


Figure 2  5- levels perfect binary tree.

By interchanged a pair of nodes in a perfect binary tree, the Collatz tree can be obtained as a subtree of a perfect binary tree. Since each node in the Collatz tree can have one or two children nodes then there are nodes in a perfect binary tree that are not in the Collatz tree. As an example, the transformed 5- levels perfect binary tree that has the Collatz tree as its subtree is shown in Figure 3.

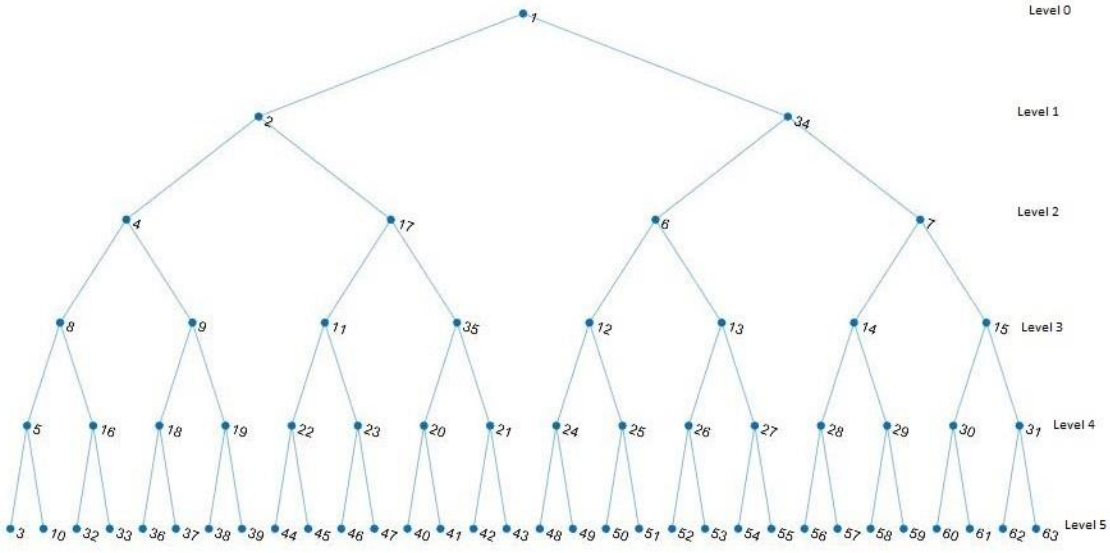


Figure 3. A transformed 5- levels perfect binary tree.

The important property of a binary tree is that there is a unique path from each node to the root node. Since at least one node is not in the Collatz tree, then the Collatz conjecture is most likely not true.

**Reference**

[1] R . Terras, (1976). " A stopping time problem on the positive integers". Acta Arithmetica, 30(3), 241-2