Link between ΛCDM model and expanding De Sitter flat universe.

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Abstract

We establish a simple relation between the Hawking temperature and the mass of the Hubble sphere via the Hubble time which constitutes with a beginning of proof for the experimental validity of Stephen Hawking's theory of black hole evaporation. Adding the notion of Planck mass flow, we find that the mass of the Hubble sphere and thus its energy increases in accordance with the ΛCDM model. Then we show that the temperature arising from the Unruh effect at the Hubble radius in an expanding flat De Sitter space-time is identical to the Gibbons-Hawking temperature in this same space. Finally we show that the expanding De Sitter spacetime is related to the Hawking temperature of the Hubble sphere of the ΛCDM model.

Introduction

There are few physical correspondences between the ΛCDM model and an expanding flat De Sitter universe, We show that we can establish a definite relationship between these two models via different temperature approaches. We show that the thermal agitation of the ΛCDM model is twice as small as that of the expanding flat De Sitter universe for the Hubble sphere. This ratio can be attributed to only one difference: the presence of matter in the ΛCDM model.

1, The Hawking temperature and Unruh temperature in the ΛCDM model of the Hubble sphere.

The Hawking temperature T_{Haw} of a black hole of mass M_{Haw} ahas been defined by Stephen Hawking as follows:

$$T_{Haw} = \frac{1}{8\pi M_{Haw}} \cdot \frac{\hbar c^3}{G k_B} \tag{1}$$

Moreover, a Schwarzschild black hole of mass M_S has an associated Schwarzschild radius R_S associé tel que :such that :

$$R_S = \frac{2GM_S}{c^2} \tag{2}$$

Now the Hubble sphere of mass M_H and radius $R_H = c/H$, where *H* is the Hubble constant, can be equated to a Schwarzschild black hole because $R_H = R_S$. It is understood that the Hubble mass is defined as the mass of dark energy added to the mass of dark and baryonic matter, although it is more usual to speak in terms of energy rather than mass in this context.

For M_{Haw}= M_S = M_H and with $R_H = R_S$, we find the Hubble time$t_H = \frac{1}{H}$ as follows:

$$t_H = \frac{2GM_{Haw}}{c^3} \tag{3}$$

It becomes simple to demonstrate with the definitions of Planck time, $t_P = \sqrt{\frac{\hbar G}{c^5}}$, and Planck's temperature, $T_P = \sqrt{\frac{\hbar c^5}{Gk_B^2}}$, that :

$$\frac{T_P}{T_{Haw}} = \frac{4\pi t_H}{t_P} \tag{4}$$

This writing is not new: it simply translates (Eq.1) expressed in Planck units the well-known expression which, for $M_{Haw} = M_H$, gives :

$$\frac{T_P}{T_{Haw}} = \frac{8\pi M_H}{M_P} \tag{5}$$

where M_P is the Planck mass, replacing the notion of mass of (Eq.5) by the notion of time (Eq.4). The only "novelty" of this article is to consider the mass of the universe of my Hubble sphere as a black hole for the Hawking temperature and to be able to verify it numerically with the equations Eq.4 and Eq.5. In other words, to consider the universe at the Hubble radius as a relevant laboratory to validate the theory of Stephen Hawking.

Let us now consider :

$$\frac{Eq.4}{Eq.5} \tag{6}$$

Eq,6 is equivalent to :

$$M_H = t_H \frac{1}{2} \frac{M_P}{t_P} \tag{7}$$

In other words, the mass of the Hubble sphere increases by half the Planck mass flow, $\frac{M_p}{t_p}=\frac{c^3}{G}$,

and proportionally the Hubble time flow t_H. This would be the origin of dark energy and matter in the ΛCDM model .

The Unruh acceleration of the cosmological constant in the ΛCDMmodel at Hubble distance R_H is:

$$a_{Unruh\ \Lambda}=Hc(\Omega_\Lambda-\frac{1}{2}\Omega_m)R_H \tag{8}$$

The Unruh temperature is in this case :

$$T_{Unruh\Lambda}=\frac{\hbar a_{Unruh\ \Lambda}}{2\pi ck_B} \tag{9}$$

where $\hbar$ is the reduced Planck constant,c the speed of light in vacuum and k_B the Boltzmann constant.

As
$$R_H=c/H \tag{10}$$

Eq ,8 becomes :

$$a_{Unruh\ \Lambda}=H^2\frac{c}{H}(\Omega_\Lambda-\frac{1}{2}\Omega_m) \tag{11}$$

Eq,9 with Eq,11 then becomes :

$$T_{Unruh\Lambda}=\frac{\hbar Hc(\Omega_\Lambda-\frac{1}{2}\Omega_m)}{2\pi ck_B} \tag{12}$$

that is, simplifying by c :

$$T_{Unruh\Lambda}=\frac{\hbar H(\Omega_\Lambda-\frac{1}{2}\Omega_m)}{2\pi k_B} \tag{13}$$

2, Gibbons-Hawking temperature and Unruh temperature in a flat De Sitter space of the Hubble sphere.

Ulf Leonhardt used in an expanding flat Sitter space the Gibbons - Hawking temperature [1]:

$$T_{Gibb-Haw} = \frac{\hbar H}{2\pi k_B} \quad (14)$$

In his paper, he reaffirms that cosmological horizons radiate with the Gibbons-Hawking temperature (14) in a spatially flat expanding universe with H(t) > 0. He shows that this statement is an exact result for a Friedmann-Lemaître-Robertson-Walker universe with zero spatial curvature.

We can see that Eq.13 and Eq,.4 differ by a factor

$$\Omega_\Lambda - \frac{1}{2}\Omega_m = 1 \quad (15)$$

Now, in a flat De Sitter space-time there is neither matter nor radiation pressure. It contains only vacuum modeled by a cosmological constant Λ . $\Omega_m = 0$ Eq.15 is simplified to :

$$\Omega_\Lambda = 1 \quad (16)$$

We have for Eq.13 with Eq.16 :

$$T_{Unruh\Lambda} = \frac{\hbar H}{2\pi k_B} \quad (17)$$

i.e. in an expanding flat Sitter space, the temperature Gibbons - Hawking, Eq.14, is strictly equal to the Unruh temperature at the Hubble radius Eq.17.

3. Link with the Hawking temperature at the Hubble radius in the ΛCDM model .

M_{Haw} of Eq,1 is also in this case the mass,M_H, at the Hubble radius $R_H = c/H$

We define in the ΛCDM model , M_H, the mass of my Hubble sphere for a flat universe with ρ_c, the critical density and V_H, the volume of the Hubble sphere :

$$M_H = \rho_c V_H \quad \text{(Equation 18)}$$

with :

$$\rho_c = \frac{3H^2}{8\pi G} \quad \text{(Equation 19)}$$

$$V_H = \frac{4\pi}{3}\frac{c^3}{H^3} \quad \text{(Equation 20)}$$

thus

$$M_H = \frac{c^3}{2GH} \qquad \text{(Equation 21)}$$

with M_H=M_{Haw} , we have for Eq,1 :

$$T_{Haw} = \frac{\hbar c^3}{8\pi \frac{c^3}{2GH} G k_B} \qquad \text{(Equation 22)}$$

or after simplification :

$$T_{Haw} = \frac{\hbar H}{4\pi k_B} \qquad \text{(Equation 23)}$$

With Eq,14, Eq,17 and Eq,23 we have with the Hubble parameter H:

$$\frac{T_{Gibb-Haw}}{2} = \frac{T_{Unruh\Lambda}}{2} = T_{Haw} \qquad \text{(Equation 24)}$$

Conclusion.

The thermal agitation of the ΛCDM model is half that of the expanding flat De Sitter universe for the Hubble sphere. This ratio can only be attributed to one difference: the presence of matter in the ΛCDM model compared to the vacuum of the De Sitter universe.

Reférences :

1. Ulf Leonhardt 2021 EPL 135 10002